

# Proceedings



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I · R · E

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(Including the WAVES AND ELECTRONS Section)

**May, 1947**

Volume 35

Number 5

PROCEEDINGS OF THE I.R.E.

Analysis of Problems in Dynamics  
by Electronic Circuits

Duct Propagation at 10 and 3 Cen-  
timeters

Microwave Oscillators Using Disk-  
Seal Tubes

Microwave Omnidirectional An-  
tennas

Q Circles—Part II

Voltage-Regulator-Tube Charac-  
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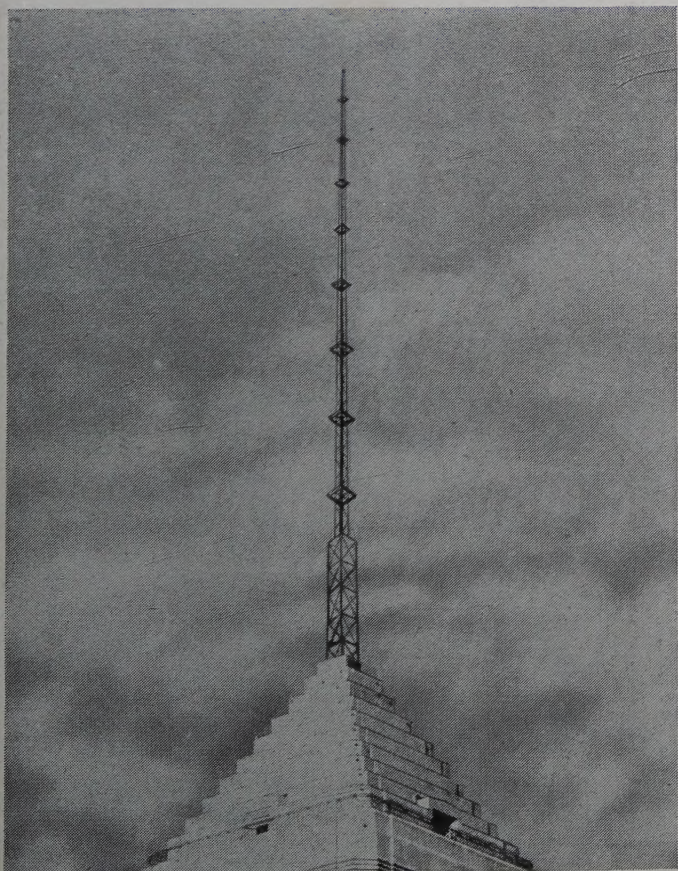
Waves and Electrons  
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Impedance Measurement on Trans-  
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Microwave Power Measurement  
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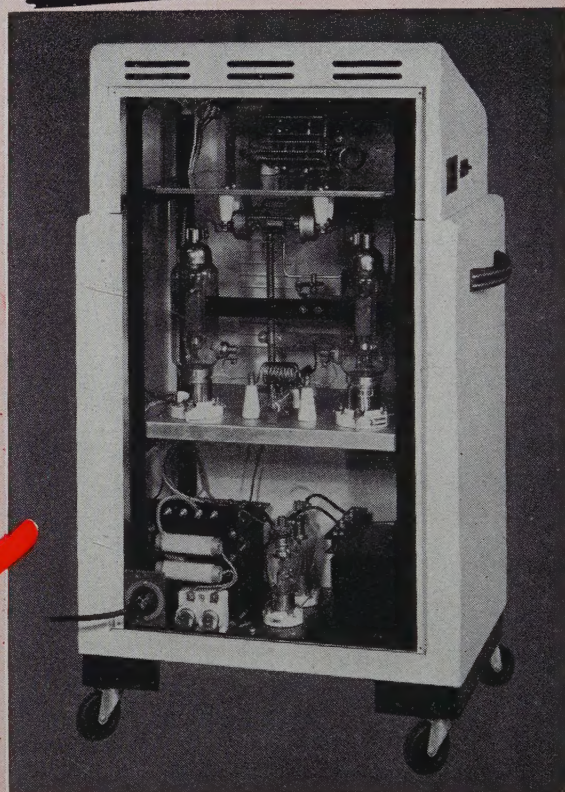


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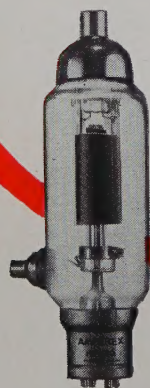
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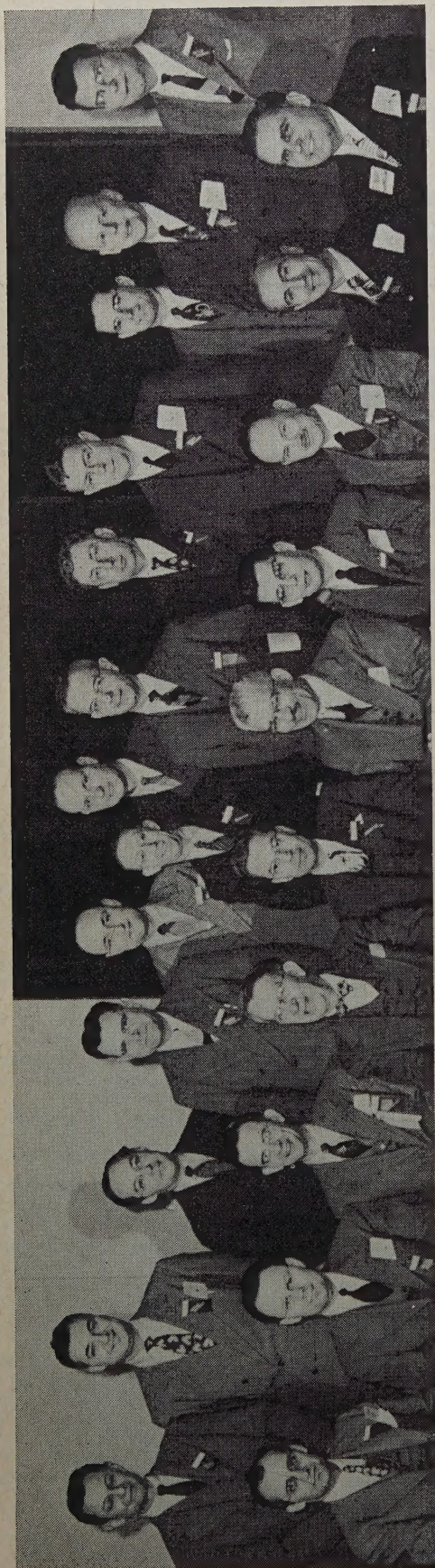
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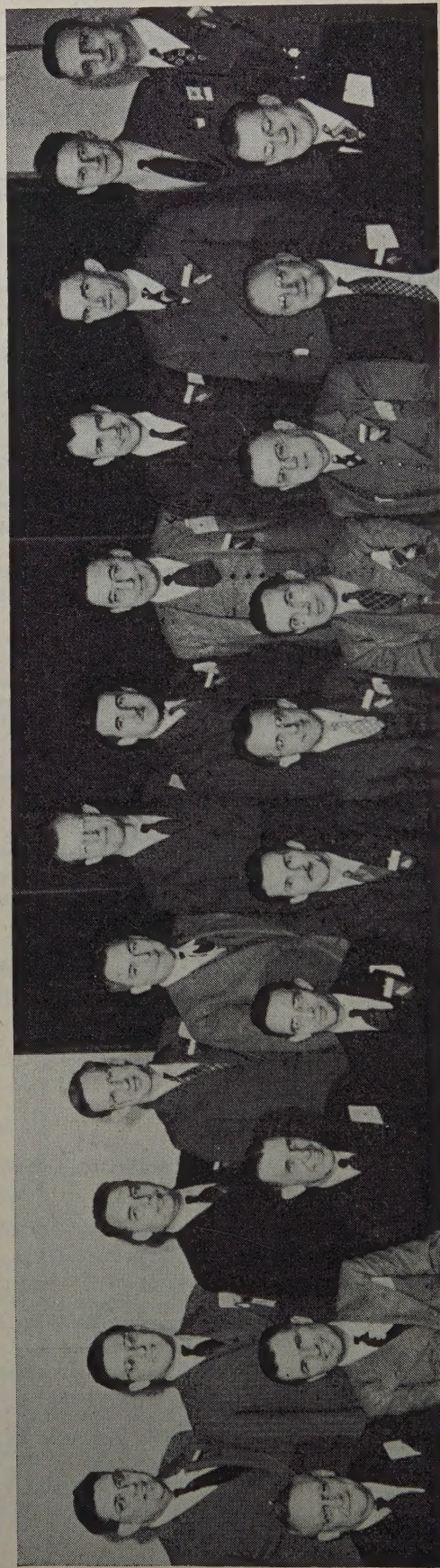


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Of recent years, an active debate has been carried on relative to the justification of work in the field of "pure science" as opposed to the desirability of endeavor in the domain of "applied technology." One group of thinkers has felt that the pursuit of truth for its own sake was not only more satisfying intellectually and more stimulating to the dignity of the investigator but also, in the long run, more likely to benefit science, engineering, and society through the discovery of *basic* facts susceptible of many later useful applications. Other thinkers have insisted that "pure science" was nothing more than a meaningless exploration in a social and economic vacuum, and that it constituted a sort of cloistered and self-indulgent scientific "doodling" by the research worker and was thus unworthy of encouragement. Society, these thinkers proclaim, is entitled to ask the expensively trained investigator, using costly facilities and the help of his fellow workers, to produce results markedly slanted toward practically immediate and major social benefit. As correlative disputes, there have been argued such questions as the following. Should research be institutionally subsidized by government? Should the engineer enter the political and industrial-management arenas to preside, at least in part, over the social destinies of his mental products? Can and should a research or development worker become an industrial leader or a political figure of prominence? And so the conflicts rage.

It is accordingly interesting to bring one viewpoint to our readers in the following forceful guest editorial by a leading engineer who is chairman of the Institute's Papers Procurement Committee and also vice-president of the Emerson Radio and Phonograph Corporation.—*The Editor.*

## The Position of the Engineer in Our Postwar Society

DORMAN D. ISRAEL

Despite extravagant promises made during the election campaign last fall, and even though the people had told many of the incumbent leaders that they had "had enough," every thinking person realizes that many social reforms of the recent decade will not be repudiated. It is no more possible to forget them than to eliminate the existence of the scientific advances made during the same period.

Engineers, along with other scientists, have given astounding technological developments to the world. Entrusting the unqualified exploitation of these innovations to others is to display a brand of naivete long since unbecoming to our profession.

The transitions in the development of the mechanical, economic, and social aspects of our age have become so closely related that it is hardly possible to locate the points of fusion. It follows that the engineer who usually led the chronology of major advances is likewise—if only by reason of his solid familiarity with the background and details—best set to carry on with the ensuing exploitive phases. That he has not been so appointed by society is only proof that the engineer has failed to realize and be equal to his total responsibility. It is well known that the human race cannot afford another failure in the management of the social and economic development of our technology.

The postwar job before the engineer transcends that of the pure scientist, and involves faithful attention to his regular duties as well as to each of the following:

1. Adequate time, effort, and skill must be devoted to the maintaining and further development of our national security. To expect a few self-sacrificing members of our profession to do all of this is to invite disaster. Each of us must make it our task to find a way to contribute from our skill in a definite manner.
2. The product for which he is responsible must not only be technically satisfactory but it must, furthermore, use materials and processes in a manner conducive to the encouragement of all workers associated with this product.
3. Methods, and particularly their mechanization, must be developed and put into effect to the optimum degree.
4. Plans always must be under way and continually in operation whereby new developments are being gradually introduced as though by normal evolution, thus avoiding sudden and abortive changes so damaging to worker morale and product quality.
5. The engineer's achievements, as such, are of no value. The social and economic phases of the production and use of the product—in other words, the service rendered—are what count. The evaluation by society of a product is only in terms of the service rendered.

# Analysis of Problems in Dynamics by Electronic Circuits\*

JOHN R. RAGAZZINI†, MEMBER, I.R.E., ROBERT H. RANDALL‡, AND  
FREDERICK A. RUSSELL§, MEMBER, I.R.E.

**Summary**—This paper describes a method for obtaining an engineering solution for integrodifferential equations of physical systems using an electronic system. The components consist of standard plug-in feed-back amplifier units. As the interconnections are wires, resistors, and capacitors, no complicated mechanical layout problem is involved and a generally flexible analyzer need not be set up, for it is a simple matter to assemble the particular circuit for any system of equations for which solutions are desired. The system should, therefore, be of interest to those involved in a study of the dynamics of physical systems.

## I. INTRODUCTION

THE FORMULATION of electrical analogs of dynamic problems in fields other than electrical has long been used to obtain solutions for such problems.<sup>1</sup> Then, in most cases, a physically realizable network may be synthesized to fit the equations and a network used to obtain the electrical outputs representing the solution of the equations.<sup>2</sup> For complicated problems this method does not usually result in a network whose individual parameters correspond to the individual parameters of the original system, so that experimentation in the nature of varying the parameters is not simple. This objection is largely overcome through the generous use of isolating amplifiers within the electrical network. Until the modern methods of feed-back stabilization were developed, the use of amplifiers introduced complicating circuit elements which altered with variation in tube characteristics. The other method of attack on problems of this type has been through the use of the mechanical differential analyzer<sup>3</sup> having as its basic tool an ingenious mechanical integrator, recently improved through the use of a polarized-light servo-operated torque amplifier.<sup>4,5</sup>

The technique described herein employs as its basic tool a stabilized feed-back amplifier of standard design,<sup>6</sup>

which by mere external changes in connection will serve as integrator, differentiator and sign changer. Professor J. B. Russell of Columbia University first brought these techniques to the attention of the authors in the circuits employed in the Western Electric M-IX antiaircraft gun director.<sup>7</sup> As an amplifier so connected can perform the mathematical operations of arithmetic and calculus on the voltages applied to its input, it is hereafter termed an "operational amplifier." The operations can be performed to any desired degree of precision, providing power supplies of excellent regulation and circuit components of high precision are used. For most engineering computations, ordinary circuit components are adequate.

## II. OPERATIONAL AMPLIFIERS

The term "operational amplifier" is a generic term applied to amplifiers whose gain functions are such as to enable them to perform certain useful operations such as summation, integration, differentiation, or a combination of such operations. In view of the fact that

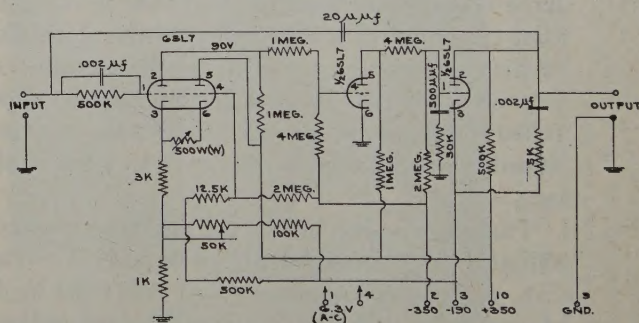


Fig. 1—Direct-current amplifier for use in electronic computers.

many operations involve steady or slowly changing inputs, the inherent frequency response of such amplifiers must extend down to zero frequency. The base unit in the operational amplifier is generally a direct-current amplifier having an odd number of stages. The unit shown in Fig. 1 was developed specifically for general laboratory use. However, any well-designed, stable, direct-current amplifier having an odd number of stages, or an equivalent phase shift, is adaptable to the uses which will be described.

<sup>7</sup> Instruction booklet prepared by the Bell Telephone Laboratories for the Western Electric M-IX antiaircraft gun director.

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† Columbia University, New York, N. Y.

‡ City College of New York, N. Y.

§ Newark College of Engineering, Newark, N. J.

<sup>1</sup> M. F. Gardner and J. L. Barnes, "Transients in Linear System," John Wiley and Sons, New York, N. Y., 1942.

<sup>2</sup> H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., Inc., New York, N. Y., 1945.

<sup>3</sup> V. Bush, "The differential analyzer," *Jour. Frank. Inst.*, vol. 212, pp. 447-448; October, 1931.

<sup>4</sup> H. P. Kuehni and H. A. Peterson, "A new differential analyzer," *Trans. A.I.E.E., (Elec. Eng.)*, vol. 63, pp. 221-228; May, 1944.

<sup>5</sup> T. M. Berry, "Polarized light servo-system," *Trans. A.I.E.E. (Elec. Eng.)*, vol. 63, pp. 195-197; April, 1944.

<sup>6</sup> E. L. Ginzton, "DC amplifier techniques," *Electronics*, pp. 98-102; March, 1944.

The basic equation of the operational amplifier may be derived by reference to Fig. 2. Here, if it is assumed that the box marked  $A$  is a direct-current amplifier and

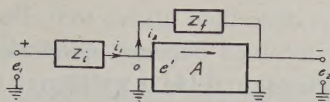


Fig. 2—Block diagram for basic feedback computation.

that the input at  $o$  is connected directly to the grid of the first tube, the currents at the junction  $o$  must add up to zero. Thus,

$$i_1 = i_2$$

or,

$$\frac{e_1 - e_1'}{Z_i(p)} = \frac{e_1' + e_2}{Z_f(p)} \quad (1)$$

where the voltages  $e_1$ ,  $e_2$ , and  $e_1'$  are functions of time and  $Z_i(p)$  and  $Z_f(p)$  are used in the manner of the Heaviside impedance operators. It is noted that, if  $A$  is the voltage gain of the amplifier,

$$e_2 = A e_1'. \quad (2)$$

Using this relation and simplifying (1),

$$e_2 = \frac{Z_f(p)}{Z_i(p)} \left[ \frac{1}{1 + \frac{1}{A} \left( 1 + \frac{Z_f(p)}{Z_i(p)} \right)} \right] e_1. \quad (3)$$

If the amplifier gain  $A$  is made sufficiently large (5000 is a practicable value), the bracketed term will approach unity, so that, with but negligible error,

$$e_2 = \frac{Z_f(p)}{Z_i(p)} e_1. \quad (4)$$

Equation (4) is the basic equation for the high-gain direct-current feed-back amplifier, and is the justification for the term "operational" amplifier, as will be shown in the succeeding paragraphs.

It will be noted in Fig. 2 that the output voltage  $e_2$  is of opposite polarity from that of the input voltage  $e_1$ . Hence, if the impedances  $Z_f(p)$  and  $Z_i(p)$  are equal resistances, the amplifier will perform the simple operation of sign-changing. In fact, sign-changing will be included in all the operations which can be performed by the amplifier.

If the impedances of (4) are unequal resistances, a scale change will be accomplished, for

$$e_2 = \frac{R_f}{R_i} e_1. \quad (5)$$

It will be noted that the accuracy of the scale change depends only on the accuracy of the resistors, and does not depend on the amplifier components, so long as the

amplifier gain remains large for all frequencies of interest.

Often it is desired to vary the scale of a given input, or to make the output adjustable. Three methods of accomplishing this operation are shown in Fig. 3. The circuits are self-explanatory and all of them may be used interchangeably. It is to be noted that circuits (a) and (b) fully realize the low output impedances which result in degenerative feed-back amplifiers. On the other hand, the output impedance of circuit (c) may reach a value as high as one quarter the resistance of the output potentiometer. This output impedance may be important in producing computation errors if the circuit following has a low impedance. In case (a) the gain varies linearly with the resistance of resistor  $R_f$ ; in case (c) the output voltage will be linear with change in resistor  $R_c$ ; but in case (b) the gain will vary as the reciprocal of the resistance  $R_i$ . These facts are of particular importance when the parameter is to be varied by a servomechanism whose output angle is most readily made linearly variable with the applied voltage.

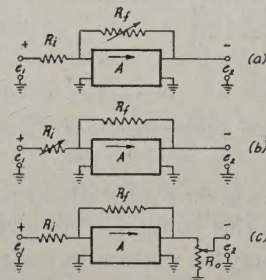


Fig. 3—Three methods for obtaining variable changes in scale or gain.

The more important operations which can be performed by virtue of (4) are those of the calculus, which make systems of these amplifiers capable of solving differential equations. Thus, if  $Z_f(p)$  is a capacitor whose impedance expressed operationally is  $1/Cp$ , and  $Z_i(p)$  is a resistance  $R$ , (4) becomes

$$e_2 = \left( \frac{1}{RC} \right) \frac{1}{p} e_1. \quad (6)$$

This amplifier is now an integrator which can accurately integrate input voltages with respect to time. It should be noted that the multiplying factor  $1/RC$  may be made unity by choosing  $R$  as 1 megohm and  $C$  as 1 microfarad, and that the sign of the output voltage  $e_2$  is negative as compared to the input voltage  $e_1$ . If the input voltage is constant, the output voltage will rise linearly with time up to the limit of the amplifier. If the input voltage is removed, the output voltage  $e_2$  will remain constant at the integrated value. To remove this output voltage it is necessary to provide a switch which short-circuits the capacitor  $C$ , thus returning the output voltage to zero.

If  $Z_f(p)$  is now made a resistance  $R$ , and  $Z_i(p)$  is made a capacitor whose impedance is expressed operationally

as  $1/Cp$ , (4) becomes

$$e_2 = (RC)p e_1. \quad (7)$$

The operational amplifier is now a differentiator whose output is the derivative of the input voltage. If a voltage of constantly increasing magnitude is applied to the input of this differentiator, the output will be a constant value. One of the disadvantages of the differentiator is its very good high-frequency response. For example, if the voltage fed to the differentiator is produced by a coarse potentiometer, the output voltage will contain high-magnitude pulses produced by each step of the contact arm.

Another basic operation which can be performed by the operational amplifier is the important one of summing the voltages obtained from any number of independent sources. A circuit to provide the summation of

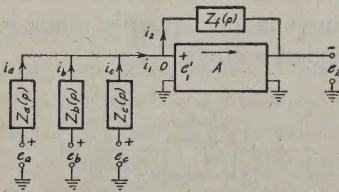


Fig. 4—Circuit for adding functions of three variables.

three input voltages is shown for illustration in Fig. 4. If it is assumed that the voltage  $e_1'$  is virtually zero, the current equation may be written in the same manner as was used to derive (4):

$$i_a + i_b + i_c = i_2. \quad (8)$$

So,

$$\frac{e_a}{Z_a(p)} + \frac{e_b}{Z_b(p)} + \frac{e_c}{Z_c(p)} = \frac{e_2}{Z_f(p)}. \quad (9)$$

Rearranging,

$$e_2 = \frac{Z_f(p)}{Z_a(p)} e_a + \frac{Z_f(p)}{Z_b(p)} e_b + \frac{Z_f(p)}{Z_c(p)} e_c. \quad (10)$$

If all the impedances are equal resistances, (10) reduces to a summation of the three input voltages:

$$e_2 = e_a + e_b + e_c. \quad (11)$$

This summation may be accomplished for any number of input voltages.

It is evident that the summation may be made algebraic (that is, may include subtractions) if the input voltages are of proper sign, and they may be made so by the use of sign-changing amplifiers where required. It is easy to see that scale-changing may be made to accompany the summation if the resistances  $R_a$ ,  $R_b$ , etc., are not selected to equal  $R_f$ , but ratios of  $R_f/R_a$ ,  $R_f/R_b$ , etc., are chosen to give the desired individual multiplying factors. Furthermore, it is possible to use the general equation (10) and perform calculus operations on the

input voltages at the same time they are summed and altered in magnitude. The only restriction is that the impedance  $Z_f(p)$  is common to all the operations involved.

The operations described above may be summarized and the process involved becomes generalized and capable of further extension if (4) is written in the form

$$e_2(t) = \frac{F_1(p)}{F_2(p)} e_1(t) \quad (12)$$

or

$$F_1(p)e_1(t) = F_2(p)e_2(t). \quad (13)$$

In this form, the equation may be used for the setting up of complicated systems, examples of which will be described in the ensuing pages. The functions  $F_1(p)$  and  $F_2(p)$ , as shown in Fig. 5, may be produced not only by

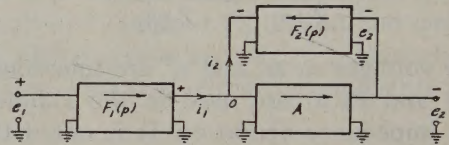


Fig. 5—Generalized circuit for operational computation.

passive circuits consisting of impedances, but also with additional operational amplifiers.

### III. SYNTHESIS OF COMPUTERS FOR LINEAR FIRST-DEGREE EQUATIONS

The techniques for setting up an electronic computer for an equation of a physical system can be divided into two classes, which will be referred to as the "in-line" and "current-junction" methods. One or a combination of these methods will apply to a particular problem. Usually there are many possible circuit arrangements, one of which will be more convenient to synthesize, more economical of equipment, or will permit easier adjustment of the desired parameters.

Before an equation is synthesized, it should be written in a form such that the independent variable is time, and the literal coefficients represent positive numerical values. A simple differential equation which fulfills the above conditions without rearrangement is that of the mechanical dynamics of a d'Arsonval meter or oscillograph element with constant rotational inertia  $J$ , viscous friction damping factor  $D$ , and stiffness factor  $K$ . The relationship between the angular displacement  $\theta(t)$  and the applied torque  $\tau(t)$  is given by the familiar second-order equation:<sup>8</sup>

$$J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + K\theta = \tau(t) \quad (14)$$

or

$$(Jp^2 + Dp + K)\theta - \tau(t) = 0. \quad (15)$$

<sup>8</sup> R. E. Doherty and E. G. Keller, "Mathematics of Modern Engineering," vol. I, p. 17, John Wiley and Sons, New York, N. Y., 1936.

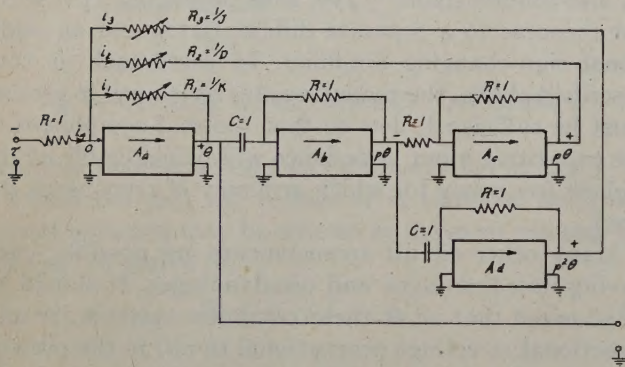


$$F_1(p)x + F_1'(p)x' + F_1''(p)x'' + \dots - F_2(p)y - F_2'(p)y' - F_2''(p)y'' - \dots = 0. \quad (17)$$

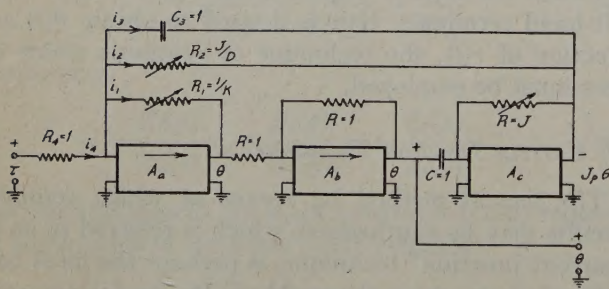
If the variables  $x, x', x'', \dots, y$  are voltages such as  $e_1(t)$  and  $e_2(t)$ , then the various expressions  $F(p)$  are admittances, and the synthesizing process consists of summing algebraically the currents flowing into the junction  $o$ , of an amplifier such as that shown in Fig. 5, and setting the result equal to zero. The various currents are mutually independent since the grid input terminal  $o$  is virtually at zero potential (since  $e_1'$  is negligibly small) and hence effectively grounded.

The technique is illustrated in Fig. 7(a) for (15), which is now to be solved for  $\theta(t)$  in terms of  $\tau(t)$ . Amplifier  $A_a$  is first assumed to produce an output voltage equal to  $\theta(t)$ . In order that  $\theta(t)$  may exist at this point, it is necessary to feed four currents (one for each term of the equation) to its grid input terminal, junction  $o$ , thus solving the equation

$$-\tau(t) + K\theta + Dp\theta + Jp^2\theta = 0. \quad (18)$$



(a)



(b)

Fig. 7—(a) Example illustrating "current-junction" technique of synthesis.

(b) Simplified computer for above example.

The current  $i_4$ , representing the term of the independent variable  $\tau(t)$ , will exist if we let the input resistor be a unit value and apply a voltage proportional to  $-\tau(t)$  at the left-hand terminals. The second term is  $K\theta$ , obtained by feeding back the voltage proportional to  $\theta$  through a conductance proportional to  $K$ , or a resistance,  $R_1 = 1/K$ . Thus a current  $i_1(t) = K\theta$  will also flow at junction  $o$ . Voltages proportional to  $p\theta$  and  $p^2\theta$ , neces-

sary to produce currents  $i_2(t)$  and  $i_3(t)$ , are produced by the in-line technique described previously. When the currents  $i_1, i_2, i_3, i_4$  are applied to junction  $o$ , the output of amplifier  $A_a$  must then be proportional to  $\theta(t)$ , as assumed at the start. The computer of the above example may be somewhat simplified, at the expense of having resistor  $R_2$  dependent on two of the coefficients,  $J$  and  $D$ , as shown in Fig. 7(b).

The choice of the computer system is sometimes determined by the variations of the coefficients of the equations. If the coefficients such as  $J, D$ , and  $K$  are merely adjustable but remain constant as inputs are applied, practically any system is workable. However, occasionally a coefficient is not constant but a function of either time or one of the variables of the system. In that case a circuit must be chosen so that the value of the coefficient is proportional to the angle of rotation of a potentiometer shaft, so that a simple servo-driven unit may be used.

#### IV. PROBLEMS INVOLVING SIMULTANEOUS EQUATIONS

Simultaneous integrodifferential equations arise when problems of coupled physical systems are encountered. Frequently it is inconvenient or impracticable to solve these equations mathematically for the desired unknown, and then set up a computer for the resulting equation. In addition, a solution may be desired for each of the unknown quantities. In such cases it is convenient to synthesize a computer for each of the equations of the set, and then to interconnect the circuits in a manner such that the equations are satisfied simultaneously.

The steps required for synthesis of a computer for simultaneous linear equations may be tabulated as follows:

(a) Each equation is written in a form such that one of the dependent variables, with its coefficients, stands alone on the right-hand side of the equation, this dependent variable being different for each equation of the set.

(b) By one of the methods previously described a separate computer is set up for each equation, assuming that all the variables save the one on the right are independent variables, so that the computer will produce an output voltage proportional to the dependent variable on the right-hand side of the equation.

(c) Functions of the output of each computer required to provide the proper input functions for the other computers are noted. Circuits are added to the output of each to provide these additional output functions.

(d) The heretofore-separate systems are cross-connected so that each input terminal receives its proper voltage from one of the outputs of the system. Only the input terminals for the true independent variable or variables will then be left free, and to these external driving functions will be applied, as in the cases discussed previously.

(e) Scales of computation are adjusted so that each amplifier will have a voltage output of reasonable magnitude.

As an example consider the airplane shown in Fig. 8, for which it is desired to make a study of the pitching characteristics when the elevators are manipulated. A pair of axes fixed in the airplane will be used as references, with the customary positive directions as shown.

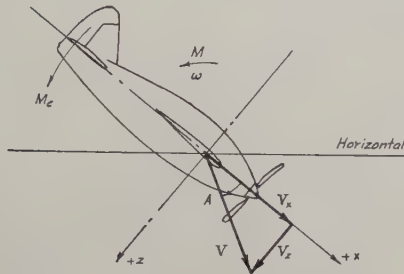


Fig. 8—Diagram illustrating notation for equations (19), (20), (21), and (22).

At the instant illustrated the airplane is pointing at a negative angle  $\theta$  from the horizontal, and is moving with a resultant linear velocity  $V$  in a direction at an attack angle  $A$  from the longitudinal axis. The velocity  $V$  is considered to have two rectilinear components,  $V_x$  and  $V_z$ .

It is desired to synthesize a computer which will solve for the disturbances or changes in angular velocity  $\omega$  and the disturbances or changes in the linear velocities  $v_x$  and  $v_z$  caused by small moments  $M_o$  applied to the airplane by means of the elevators. The airplane is assumed to have a mass  $m$  and moment of inertia  $J$  about the  $y$  axis, perpendicular to the plane of the paper.

Let it be assumed that initially (at  $t=0$ ) the airplane is flying with velocities and angular positions indicated by upper-case symbols with the subscript  $o$ . Then equations for the changes or disturbances in linear and angular velocities (indicated by lower-case symbols) can be written for the symmetrical motions of the airplane. Three equations of motion are required, which are:<sup>9</sup>

(1) The sum of the  $x$ -directed forces equals zero;  $\sum f_x = 0$ .

(2) The sum of the  $z$ -directed forces equals zero;  $\sum f_z = 0$ .

(3) The sum of the angular moments about the center of gravity and the  $y$  axis equals zero;  $\sum M = 0$ .

With certain approximations,<sup>9</sup> these three equations can be written in the following form:

$$(p + k_{xx})v_x + V_o(\sin A_o)\omega + g(\cos \theta_o) \frac{\omega}{p} = k_{xz}v_z \quad (19)$$

$$-(p + k_{zz})v_z + V_o(\cos A_o)\omega$$

$$- k_{zw}w + g(-\sin \theta_o) \frac{\omega}{p} = k_{zx}v_x \quad (20)$$

$$-(k_{Mz}p + k_{Mz})v_z + k_{Mx}v_x + \frac{M_o}{J} = (p + k_{M\omega})\omega \quad (21)$$

where  $V_o$ ,  $A_o$  and  $\theta_o$  are respectively the initial resultant velocity, attack angle, and angle from the horizontal (chosen negative);  $v_x$ ,  $v_z$ , and  $\omega$  are the increments in downward, forward, and angular velocities, respectively, for which solutions are desired; and the coefficients (all arranged so as to be numerically positive) are:

$$\begin{aligned} k_{xx} &= -\frac{1}{m} \frac{\partial f_x}{\partial v_x} & k_{zz} &= -\frac{1}{m} \frac{\partial f_z}{\partial v_z} \\ k_{xz} &= \frac{1}{m} \frac{\partial f_x}{\partial v_z} & k_{zx} &= -\frac{1}{m} \frac{\partial f_z}{\partial v_x} \\ k_{x\omega} &= -\frac{1}{m} \frac{\partial f_x}{\partial \omega} & k_{Mz} &= -\frac{1}{J} \frac{\partial M}{\partial v_z} \\ k_{Mz} &= -\frac{1}{J} \frac{\partial M}{\partial v_z} & k_{Mx} &= \frac{1}{J} \frac{\partial M}{\partial v_x} \end{aligned} \quad (22)$$

$$k_{M\omega} = -\frac{1}{J} \frac{\partial M}{\partial \omega}$$

$g$  = gravitational constant.

(The symbol  $\ddot{v}_z$  represents the downward acceleration.)

The equations as written above are in the form required for synthesis of the computer; and this process can now be carried out as outlined. Three separate computers are planned, one for each of the three equations, arranged to solve for the variable on the right-hand side.

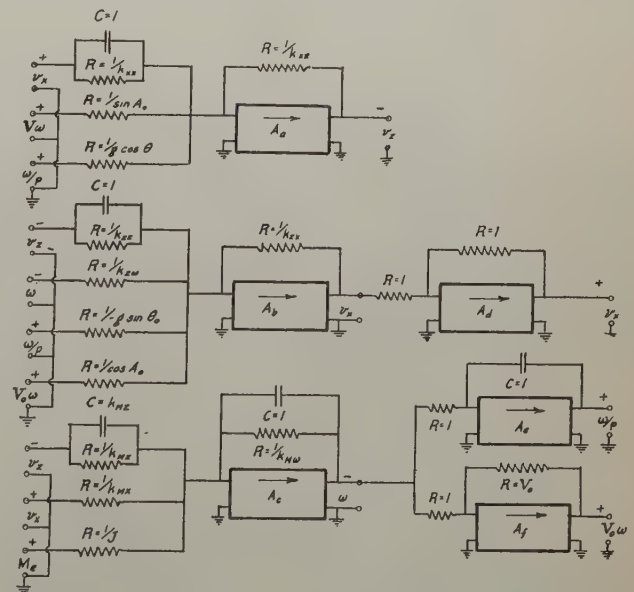


Fig. 9—Electronic computer for symmetric motions of airplane.

In Fig. 9, the output of amplifier  $A_a$  is chosen to be  $-v_x$ ; the feed-back network is chosen so as to multiply by the coefficient  $k_{xx}$ . Similarly, amplifiers  $A_b$  and  $A_c$  produce  $-v_z$  and  $-\omega$ , respectively; the feed-back networks provide for multiplication by  $k_{zz}$  and  $(p + k_{M\omega})$ , respectively. The required inputs are then assumed to be available, and input impedances are selected so that these inputs will be multiplied by the proper coefficients.

<sup>9</sup> W. F. Durand, "Aeronautical Theory," vol. 5, division N, by B. Melvill Jones, chap. 5, J. Springer, Berlin, 1934-1936.

It will be noted that, with the chosen feed-back impedances, it is possible to use capacitors to differentiate but not to integrate input terms. For this reason, it is necessary to assume as inputs the integrated quantities, such as  $\omega/p$ , where such are required. It is also convenient to assume that the multiplication by  $V_o$  is to be taken care of elsewhere, so that  $V_o\omega$  is available for an input. It is now evident that the inputs required include, in addition to the three outputs  $-v_x$ ,  $-v_z$ , and  $-\omega$ , the quantities  $v_x$ ,  $\omega/p$ , and  $V_o\omega$ . Amplifiers  $A_x$ ,  $A_z$ , and  $A_f$  are added to the circuits in order to produce these terms.

The next step consists of interconnecting the three computers so that correspondingly labeled input and output terminals are connected; these wires are not shown on the diagram. When these connections are complete, the only input terminal left free is that of the independent variable  $M_e$ , the applied elevator moment. This moment may be assumed proportional to rudder angle, and may therefore be an input obtained from a potentiometer supplied with a direct voltage and attached to the elevator control operated by the "pilot" of the simulated airplane, or by an automatic pilot which it is desired to study.

There are, of course, further practical considerations, which have not been mentioned. Stabilization of each loop of the computer circuit must be accomplished, a problem to be discussed later. Scales must be adjusted so that the output voltages of the  $v_z$  and  $\omega$  computers will be of the same order of magnitude as that of the  $v_x$  computer.

#### V. EQUATIONS WITH NONCONSTANT COEFFICIENTS AND OF HIGHER DEGREE

In the analysis of some systems, the equations may be of higher degree and, in addition, some of the coefficients may be functions either of time or of some of the variables. In view of the fact that the quantities involved are usually voltages, simple servomultiplier and divider circuits will be described.

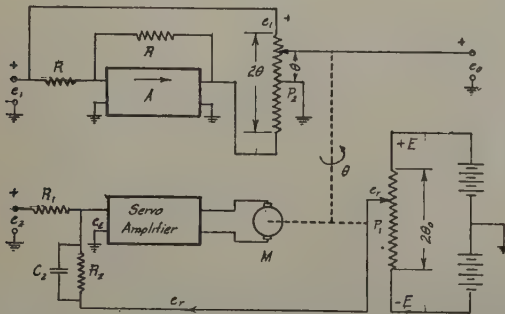


Fig. 10—Servomultiplier circuit.

As shown in Fig. 10, the servomechanism, being a degenerative device, causes the error voltage  $e_e$  to be brought to zero. The angle  $\theta$  at which the servomechanism just closes its error is given by the relation

$$\frac{e_r}{R_2} = \frac{e_2}{R_1} \quad (23)$$

and

$$e_r = \left( \frac{\theta}{2\theta_o} \right) 2E = \left( \frac{\theta}{\theta_o} \right) E \quad (24)$$

where  $\theta$  is the shaft rotation and  $2\theta_o$  is the total angle subtended by the follow-up potentiometer  $P_1$ . Solving for the shaft rotation,

$$\theta = \left( \frac{R_2}{R_1} \right) \left( \frac{\theta_o}{E} \right) e_2. \quad (25)$$

To produce multiplication with the second voltage  $e_1$ , a second potentiometer  $P_2$ , is supplied on one side with the voltage  $e_1$  directly, and on the other side with the voltage  $e_1$  through a sign changer. The shaft of this potentiometer is connected to that of the servomechanism, so that its output is

$$e_o = \left( \frac{e_1}{\theta_o} \right) \theta, \quad (26)$$

substituting the value of  $\theta$  from (25),

$$e_o = \left( \frac{R_2}{R_1 E} \right) e_1 e_2. \quad (27)$$

By proper choice of  $R_2$ ,  $R_1$ , and  $E$ , a scale factor for the multiplication may be produced to specifications. The capacitor  $C_2$  is merely for stabilization of the servosystem and does not enter in the computation, except during transient conditions.

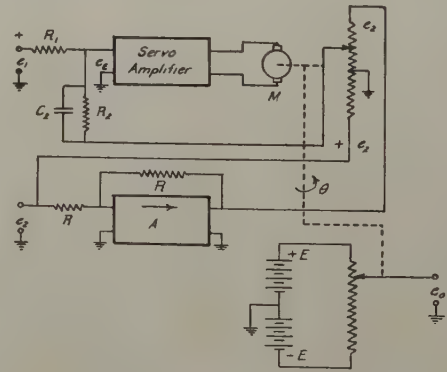


Fig. 11—Servodivider circuit.

In a similar manner, it is possible to divide one voltage by another using a servodriven potentiometer. A schematic circuit of this system is shown in Fig. 11. The servosystem closes the error voltage  $e_e$  to zero, so that

$$\frac{e_1}{R_1} = \frac{2\theta e_2}{2\theta_o R_2}. \quad (28)$$

Solving for the shaft rotation  $\theta$ ,

$$\theta = \left( \frac{R_2 \theta_o}{R_1} \right) \frac{e_1}{e_2}. \quad (29)$$

The output voltage  $e_o$  is obtained from a potentiometer on the same shaft supplied with  $\pm E$  volts to ground. Thus,

$$e_o = \left( \frac{2E}{2\theta_o} \right) \theta = \left( \frac{ER_2}{R_1} \right) \frac{e_1}{e_2}. \quad (30)$$

It is observed that the output voltage  $e_o$  is the quotient of  $e_1$  and  $e_2$  multiplied by a scale factor. Thus it is possible to multiply or divide a term in any equation by another term with any suitable scale constant by use of these small servodriven potentiometers.

Frequently it is desired to insert a function of a given variable in the computer system. For instance, if a study of aerodynamic systems is made in which a coefficient is a function of wind velocity, it is possible to adapt one of the servodriven systems to perform this operation. One technique is to construct a servomechanism similar to the multiplier of Fig. 10, and to drive a cam through an angle  $\theta$  proportional to an independent variable  $e_2$ . The desired function may be cut on a cam which drives another linear potentiometer whose output voltage is the function of the independent variable. The same result may be obtained without a cam by winding a potentiometer whose voltage output is the desired function of shaft angle.

For instance, referring to the electronic computer for the symmetric motions of an airplane in Fig. 9, the quantity  $V_o$  is the initial airspeed which is constant. The term  $V_o\omega$  produced by amplifier  $A_f$  is used in the computer for inputs which, for more accurate computations, should use the magnitude of the instantaneous velocity  $V$ , the vector value of which is given by the equation

$$\bar{V} = \bar{V}_o + \bar{v} \quad (31)$$

where  $\bar{v}$  is the change in velocity from the initial value  $\bar{v}_o$ . To make this correction in the computer, it would be necessary to vary the feed-back resistor  $R_1$  by means of a servodriven potentiometer whose shaft rotation is proportional to  $|V|$ .

Inputs to the computer can also be obtained by manual curve tracing. If a function of the independent variable is plotted and the plot rotated by means of the servomotor, the curve can be manually followed and the voltage output from a potentiometer used in the system. Such methods have been used in differential analyzers in the past and can be used in electronic computers as well.

Practical physical problems offer numerous examples of system variables which have fixed upper limits. For example, an angle specifying the position of a boat rudder may vary between zero and a fixed maximum. Similar limits apply to airplane controls. A mechanical system involving spring forces will often involve maximum spring extensions.

The purely mathematical solution of the dynamics of such systems where such limiting action occurs is often difficult. The electronic representation of such limits is

comparatively simple and can be introduced into the system wherever the limits occur. Fig. 12 shows one such circuit. Whenever the input voltage  $e_1$  exceeds the bias voltage  $E$ , one or the other diode will conduct, depending on the polarity of  $e_1$ . When this occurs the output voltage will be limited closely equal to the bias voltage, provided the resistance  $R$  is high compared to the diode resistance. A limiter of this type may be inserted in the computers previously described wherever needed.

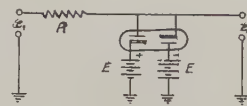


Fig. 12—Circuit for introducing nonlinearity in form of limits.

## VI. OPERATING CONSIDERATIONS

The frequency response of any operational amplifier may be obtained by replacing the operator  $p$  by the quantity  $j\omega$ . Thus the frequency response of an integrator is given by

$$\frac{E_2}{E_1} = \left( \frac{1}{RC} \right) \frac{1}{j\omega}. \quad (32)$$

This expression indicates a gain characteristic which drops 6 decibels per octave. On the other hand, the frequency response of a differentiator is given by

$$\frac{E_2}{E_1} = (RC)j\omega, \quad (33)$$

which indicates a gain characteristic that rises 6 decibels per octave.

These frequency characteristics may be used as a convenient test of amplifier accuracy in producing a given operation. For instance, if the gain curve of a differentiator indicates a linearly rising gain of 6 decibels per octave up to a given frequency, but beyond this frequency shows considerable deviation from linearity, it may be concluded that as a differentiator it is correct up to this particular frequency. If voltages containing frequency components of higher value are applied at the input, error in differentiation will result. Extending the frequency response of the base amplifier will correct the situation.

Without further explanation, it is evident that by proper choice of  $Z_f(p)$  and  $Z_i(p)$  many operational expressions may be set up accurately. The accuracy which will result depends on the maintenance of the high values of gain  $A$ , regardless of the rate of change or frequency component content of the input functions. A frequency range of from zero (direct current) to the highest frequency at which accurate computation is desired must be maintained in the amplifiers. In addition, the accuracy of the external circuit constants used must be assured.

For the impedance functions  $Z_f(p)$  and  $Z_i(p)$ , capacitors and resistors are generally used. It is evident that,

while theoretically usable, inductances are avoided, since at the relatively low frequencies involved it is difficult to obtain as pure or accurate an impedance function as can be obtained with a capacitor of low loss and low retentivity.

Wherever the computer involves the use of capacitors other than small ones inserted to achieve stability, considerable difficulty may be found due to retentivity of charge. It has been recommended by the Bell Telephone Laboratories that capacitors with polystyrene dielectric be used, as they are relatively free of this effect.

A perfect representation of the equation for a stable mechanical system should, of course, be stable electri-

response are, of course, attenuated as a result. In some cases it may be necessary to choose a new time scale<sup>10</sup> in which these attenuated high frequencies are well above the upper frequencies of response of the dynamical system being simulated.

While the amplifier units described earlier are reasonably drift free, residual drifts may often be annoying, particularly where small input voltages and long time intervals are involved. These effects can be readily minimized by changing the voltage scale and the time scale so that the useful voltages are large compared to the drift voltages and so that the experimental "run" is completed before errors have had time to accumulate.

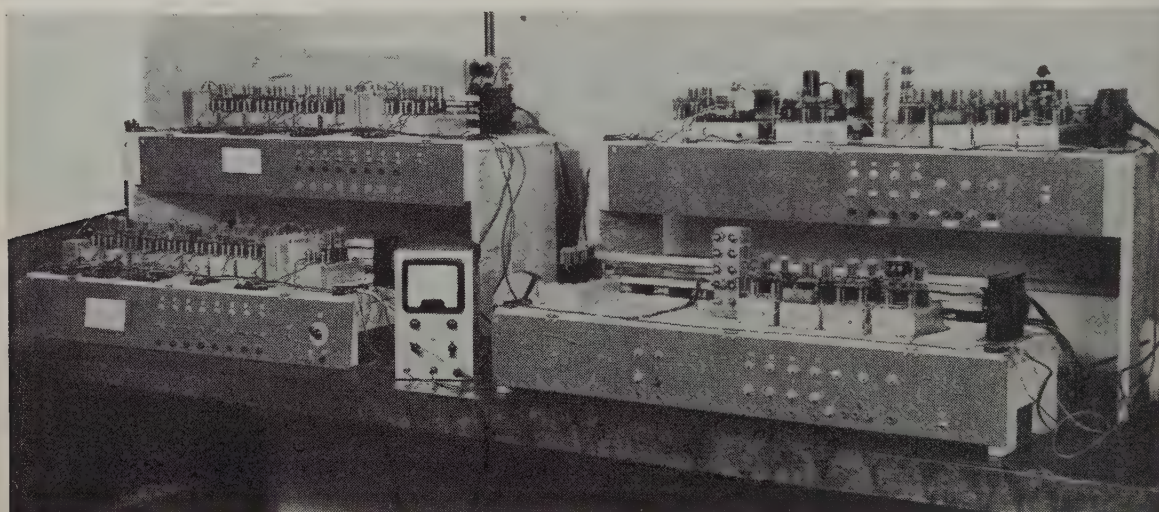


Fig. 13—Typical laboratory electronic-computer arrangement. One set of amplifiers in the lower bay on the left is the computer described in Fig. 9 for aircraft symmetrical motions.

cally. Unavoidable progressive phase shifts, both within individual amplifier units and appearing accumulatively when several units are connected in a loop, may lead to oscillation at some upper frequency. A careful redesign of the complete system will often minimize the number of separate amplifiers used in any single loop so that oscillation does not occur. The nature of the system itself which is being represented will often automatically stabilize the electronic equivalent, as, for instance, when a mechanical system having large mass will act as a low-pass filter, eliminating from the feed-back loop those higher frequencies for which phase shift may occur.

The individual amplifiers may exhibit instability when the over-all gain of the amplifier is too high. This condition may be overcome by inserting a small capacitor across the feed-back impedance of differentiators and scale changers, or by inserting a resistor in series with the capacitor of an integrator, thereby limiting the frequency response. The higher harmonics in the system

Drift and stability for these systems are functions of the direct-current power supply, which should be extremely well regulated and of low dynamic output impedance.

A typical laboratory electronic-computer arrangement is shown in Fig. 13.

## VII. ACKNOWLEDGMENT

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<sup>10</sup> J. L. Gardner and M. F. Barnes, "Transients in Linear System," vol. 1, p. 226; John Wiley and Sons, New York, N. Y., 1942.

# Wave Theoretical Interpretation of Propagation of 10-Centimeter and 3-Centimeter Waves in Low-Level Ocean Ducts\*

C. L. PEKERIS†

**Summary**—The data on the transmission of 10- and 3-centimeter waves in low-level ocean ducts obtained by the Naval Research Laboratory expedition to Antigua, British West Indies, were analyzed by wave theory. The analysis was made for a distribution of modified index of refraction with height (shown by curve  $C'$  in Fig. 1). Below the horizon, the 10-centimeter wave was found to propagate by the first normal mode with a theoretical decrement of 1 decibel per nautical mile, as against an observed value of about 0.8 decibel per nautical mile in the first 80 miles from transmitter.<sup>1</sup> Theory verified the observed constancy of decrement with height for this wavelength. Beyond 80 miles the observed rate of attenuation dropped to a low value of 0.2 decibel per nautical mile. This change of slope in the intensity curve is probably due to the emergence of scattered radiation after the direct diffracted beam had been depleted. With the exception of one point there is quantitative agreement between the observed and theoretical distribution of intensity with height (see Fig. 9) for the 10-centimeter wave.

The 3-centimeter wave was found to propagate below the horizon by the first and second modes, with theoretical decrements of zero and 0.5 decibel per nautical mile, respectively. The latter agrees with the observed values at high elevations, but near the surface, where theoretically attenuation should be negligible, the observed rate of attenuation exceeds the theoretical value by about 0.3 decibel per nautical mile. This is probably due to attenuation by scattering from horizontal inhomogeneities in the distribution of refractive index, and from the rough surface. Theory verifies the observed increase of decrement and decrease of intensity with height above about 10 feet for the 3-centimeter band.

## I. INTRODUCTION

IN THE SPRING of 1945, the Naval Research Laboratory organized an expedition to Antigua, British West Indies, for the purpose of studying propagation of microwaves over the ocean under conditions of low-level "ducts." A duct is a state of the atmosphere favorable to propagation of microwaves, which is brought about by a suitable variation with height of the refractive index of the air. When, for example, the index of refraction of the air decreases with elevation for some distance above the surface, the rays which start at the source at a slight upward inclination with the horizontal are bent downwards, and eventually suffer reflection at the surface, from where they are propagated further by additional reflections. The radiation which would under normal conditions escape to space is thus guided, to a greater or smaller degree, close to the surface, and is propagated out to great

ranges. Conditions which favor the establishment of a duct are a diminishing humidity, or an increasing temperature with elevation, or both. At Antigua the temperature was observed to decrease with elevation, but the rapid fall of the moisture content with height in the surface layer brought about a persistent surface duct extending to heights of 20 to 40 feet. As a result, it was observed that both 10- and 3-centimeter waves were propagated out to ranges of about 200 miles.

The island of Antigua is particularly suited for the study of propagation in oceanic surface ducts because of the persistent atmospheric structure that is characteristic of the northeasterly trade-wind belt. The air masses to the northeast of the island have had a long sojourn over the ocean, and are therefore horizontally homogeneous and representative of maritime conditions. Furthermore, the propagation measurements made were unusually extensive for this type of experiment in that continuous one-way transmission data were obtained at four or more elevations and for two positions of the transmitting antenna. The simultaneous measurements of propagation characteristics of the 10- and 3-centimeter bands offered, as will be shown later, an additional advantage in the rational interpretation of the data.

## II. METHOD OF ANALYSIS

In this section we shall give an outline of the method used in analyzing the transmission data by wave theory. The basic theory was developed in recent years by Booker, Burrows, Freehafer, Furry, Hartree, Pearcey, Pryce, and, independently in another field of application, by the writer. Our account will be limited principally to a statement of results, for the detailed proofs of which the reader is referred to the original memoirs.<sup>2</sup>

When the fractional change of the index of refraction  $\mu$  over a distance of a wavelength is small, the electromagnetic field is determined by a single Hertzian potential  $\Psi$  which satisfies the wave equation

$$\nabla^2 \Psi + k^2 \mu^2 \Psi = 0, \quad k = 2\pi/\lambda \quad (1)$$

where a time factor  $e^{i\omega t}$  has been assumed. For vertical polarization  $\Psi$  represents the vertical component of the electric Hertzian potential, while for horizontal polarization it represents the vertical component of the magnetic Hertzian potential. The observed field intensity is proportional to  $|\Psi|$  in each case. Equation (1) is to be

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† Columbia University Mathematical Physics Group, on leave of absence from M.I.T. This paper is based on work done for the Naval Research Laboratory under contract N5ori-90 with the Navy's Office of Research and Inventions.

<sup>1</sup> The band actually used in the experiments was 9.1 centimeters rather than 10 centimeters. For a wavelength of 9.1 centimeters the theoretical decrement is 0.85 decibels per nautical mile.

<sup>2</sup> See W. H. Furry, "Theory of characteristic functions in problems of anomalous propagation," Radiation Laboratory Report 680, February, 1945, where references to other papers will be found.

solved subject to the following boundary conditions:

- (a)  $\Psi$  should reduce to the form  $e^{-ikR}/R$  for small distances  $R$  from the source,
- (b)  $\Psi$  should vanish at the ground, and
- (c)  $\Psi$  should represent an outgoing wave at great distances from the source.

The boundary condition (b) is not strictly true, but it is a good approximation for the low-order normal-mode solutions of (1).

The quantity  $\mu$  appearing in (1) represents the index of refraction of the air, which will be assumed to vary only with height above the ground, and the boundary condition (b) is to be satisfied at the spherical surface of the earth. The analysis can be simplified by the use of a device due originally to Schelleng, Burrows, and Ferrell<sup>3</sup> and later developed by M. H. L. Pryce,<sup>4</sup> whereby space is transformed so as to make the surface of the earth plane and the rays curved. The transformation can be accomplished by using in (1), instead of  $\mu$ , the so-called modified index of refraction  $N$  defined by

$$N = \frac{r\mu(r)}{b\mu(b)} \quad (2)$$

where  $r$  denotes the distance from the center of the earth and  $b$  is the radius at some reference level. Taking  $b=a$ , the radius of the earth, we have

$$\frac{r}{a} = 1 + \frac{h}{a}, \quad (3)$$

$$\frac{\mu(r)}{\mu(a)} = 1 + \dot{\mu}h + \mu'(h), \quad (4)$$

$$N \simeq 1 + h\left(\dot{\mu} + \frac{1}{a}\right) + \mu'(h). \quad (5)$$

Here  $h$  denotes height above ground,  $\dot{\mu}$  is the vertical gradient of the index of refraction in the *standard atmosphere*, and  $\mu'(h)$  denotes the *refraction anomaly* or the deviation of the refractive index from standard conditions. The term  $h\dot{\mu}$  in (5) has the effect of modifying the radius of the earth  $a$  to an effective value  $a_e$  determined by

$$\frac{1}{a_e} = \frac{1}{a} + \dot{\mu}. \quad (6)$$

The value of  $a_e/a$  is generally taken as 4/3. In practice it is convenient to use the quantity  $M$ , defined by

$$M - M_0 = 10^6(N - 1). \quad (7)$$

Substituting in (5) and using the dependence of  $\mu$  on atmospheric pressure  $p$  (millibars), temperature  $T$  (absolute scale), and the partial pressure of water vapor  $e$  (millibars), one finds that

$$M = \frac{79p}{T} + \frac{3.8 \times 10^5 e}{T^2} + 0.048h \quad (8)$$

<sup>3</sup> J. C. Schelleng, C. R. Burrows, and E. B. Ferrell, "Ultra-short-wave propagation," *Proc. I.R.E.*, vol. 21, pp. 427-463; March, 1933.

<sup>4</sup> Unpublished report. See also J. E. Freehafer, "The effect of atmospheric refraction on short radio waves," Radiation Laboratory Report 447, 1943; and C. L. Pekeris, "Accuracy of the earth-flattening approximation in the theory of microwave propagation," *Phys. Rev.*, vol. 70, pp. 518-522; October, 1946.

where  $h$  is expressed in feet. Here the term  $0.048h$  is  $10^6(h/a)$ ; under standard conditions the sum of the first two terms in (8) varies with height like  $-0.012h$ , giving

$$M_{\text{std}} = M_0 + 0.036h. \quad (9)$$

Barring certain pathological cases which we need not consider here,<sup>4</sup> it is possible to obtain a solution of (1) satisfying the boundary conditions in terms of *normal modes* as follows:

$$\Psi = -i\pi \sum_1^\infty H_0^{(2)}(k_m d) U_m(h_1) U_m(h_2) \quad (10)$$

where  $d$  denotes horizontal distance from source (i.e., distance along a great circle),  $h_1$  and  $h_2$  the heights of transmitter and receiver above ground, and the  $k_m$  are characteristic complex numbers whose imaginary parts are negative. The height-gain functions  $U_m(h)$  must satisfy the differential equation

$$\frac{d^2 U_m}{dh^2} + (k^2 N^2 - k_m^2) U_m = 0, \quad k = 2\pi/\lambda \quad (11)$$

and are to be normalized so that

$$\lim_{(k) \rightarrow 0} \int_0^\infty U_m^2(h) dh = 1. \quad (12)$$

The method of deriving the normal-mode solution (10) of the wave equation (1) (by residues) is due originally to H. Lamb, who in a classical paper written in 1904<sup>5</sup> gave the first exact solution of the problem of radiation from a point source in an homogeneous elastic half-space. The problem treated by Lamb possesses only one normal mode (the Rayleigh wave); later H. Jeffreys<sup>6</sup> used the same method in a problem of a layered medium involving an infinite number of modes (love waves). A detailed development of the normal-mode expansion theorem in cases of media with continuous or discontinuous variations of the refractive index was given by the writer in May, 1943.<sup>7</sup> The problem was also treated by T. Pearcey,<sup>8</sup> W. H. Furry,<sup>9</sup> and G. L. Roe.

In order to meet the boundary condition (b) we must have

$$U_m(0) = 0. \quad (13)$$

Condition (c) is already met at large *horizontal* distances by the choice of the Hankel function of the second kind, because

$$H_0^{(2)}(k_m d) \rightarrow \sqrt{\frac{2}{\pi k_m d}} e^{-ik_m d + i\pi/4}. \quad (14)$$

<sup>5</sup> H. Lamb, "On the propagation of tremors over the surface of an elastic solid," *Phil. Trans. Roy. Soc. A*, vol. 203, pp. 1-42; January, 1904.

<sup>6</sup> H. Jeffreys, "The formation of love waves," *Beitrage zur Geophysik*, vol. 50, pp. 336-350; June, 1931.

<sup>7</sup> See reference 5, p. 2, of Radiation Laboratory Report 680, and C. L. Pekeris, "Theory of propagation of sound in a half-space of variable sound velocity under conditions of formation of a shadow zone," *Jour. Accous. Soc. Amer.*, vol. 18, pp. 295-315; October, 1946.

<sup>8</sup> See reference 6, p. 2, in Radiation Laboratory Report 680.

<sup>9</sup> W. H. Furry, Radiation Laboratory Report 680, February, 1945.

In order to satisfy condition (c) at large heights we require that *the base of  $U_m(h)$  should decrease with  $h$  for large  $h$* . The last condition and condition (13) can be satisfied only for a particular set of characteristic values  $k_m$ , and the resulting solutions of (11) represent the normal modes.

The normal modes can be roughly visualized as originating from constructive interference between plane waves incident at a grazing angle  $\alpha_m$ , and their reflections from the ground, the  $\alpha_m$  being determined from

$$k_m = k_\mu(h_1) \cos \alpha_m. \quad (15)$$

The existence of particular directions for which the reflected waves interfere *constructively* with the incident waves bears some analogy with diffraction spectra from gratings, where the spectra of various orders appear at particular angles.<sup>10</sup> These directions are such that the difference in optical path from two neighboring lines in the grating is one wavelength. The analogy with diffraction spectra is closer in the case of trapping, i.e., when the modified index has a minimum at some level, resulting in a channeling of the radiation through that level. For the purpose of bringing out this analogy we will assume a model of a surface duct consisting of a layer of constant modified index of refraction topped by a perfect conductor. The wave energy is then completely trapped between the conducting ground and the conducting ceiling. The field can then be represented as being due to the source and its images in the two conducting surfaces, the number of images being infinite in this case. *These images correspond to the lines in a diffraction grating.* Under actual conditions, the specular reflection from the idealized conducting ceiling is replaced by continuous reflection from levels in which the modified index decreases with height, and the infinite series of discrete images is replaced by a corresponding continuous distribution of images. The  $\alpha_m$  are under actual conditions complex numbers, so that their interpretation as grazing angles becomes strained, especially under standard or substandard refraction conditions where the  $\alpha_m$  are distinctly nonreal.

In solving (11) one finds that the two terms in the factor multiplying  $U_m$  nearly cancel each other. Hence it is convenient to write

$$k_m^2 = k^2(1 - \Lambda_m) \quad (16)$$

$$N^2(h) \equiv 1 + y(h); y(h) = 2 \times 10^{-6} [M(h) - M_0], \quad (17)$$

whereby (11) is transformed into

$$\frac{d^2 U_m}{dh^2} + k^2 [y(h) + \Lambda_m] U_m(z) = 0. \quad (18)$$

Equation (18) can be standardized by the introduction of nondimensional or *natural units of height and horizontal distance*. Let

$$q \equiv \text{standard slope of } N^2 \text{ curve} = 3/(2a), \quad (19)$$

<sup>10</sup> J. C. Slater, "Microwave Transmission," p. 284, McGraw-Hill Book Co., New York, N. Y., 1942.

where  $a$  is the radius of the earth. Then the natural units of height  $H$  and of horizontal distance  $L$  are defined by

$$H = (k^2 q)^{-1/3}, \quad L = 2(kq^2)^{-1/3}. \quad (20)$$

With

$$z = h/H, \quad x = d/L, \quad Y(z) = (k/q)^{2/3} y(h), \quad (21)$$

$$D_m = (k/q)^{2/3} \Lambda_m = B_m + iA_m, \quad (22)$$

(18) takes on the form

$$\frac{d^2 U_m(z)}{dz^2} + [Y(z) + D_m] U_m(z) = 0. \quad (23)$$

In terms of natural units and by the use of (14) one finds, on inserting the time factor  $e^{i\omega t}$ , that

$$\Psi = e^{i(\omega t - 2\pi d/\lambda)} \cdot \frac{2\sqrt{\pi}}{L} \frac{1}{\sqrt{x}} \sum_1^\infty e^{-A_m x + iB_m x} U_m(z_1) U_m(z_2). \quad (24)$$

Here  $U_m(z) = \sqrt{H} U_m(h)$ , so that, by (12),

$$\int_0^\infty U_m^2(z) dz = 1. \quad (25)$$

The normal-mode solution (24) is useful beyond the horizon and whenever only a few modes are excited.

Of particular interest to the interpretation of the Antigua experiment is the case of a linear-exponential  $M$  curve:

$$M = M_0 + 0.036h + ge^{-ch}, \quad (26)$$

$$Y(z) = z + \alpha e^{-\gamma z}. \quad (27)$$

In Table I are given some numerical constants which are useful in computing natural units and in treating the exponential model. A value of 6371 kilometers for the mean radius of the earth was adopted. By the "decrement" is meant the rate of attenuation in the horizontal direction, *excluding the  $1/d$  divergence factor, when only*

TABLE I  
CONVERSION FACTORS FOR NATURAL UNITS

Wave-length	$H$	$L$	$\alpha$	$\gamma$	Decrement
centimeters	feet	nautical miles			decibels per nautical mile
$\lambda$	$7.243\lambda^{2/3}$	$3.304\lambda^{1/3}$	$3.848g\lambda^{-1/3}$	$7.243\lambda^{2/3}c$	$2.629\lambda^{-1/3}A_m$
3	15.07	4.766	1.850g	15.07c	1.822A <sub>m</sub>
10	33.62	7.120	0.8290g	33.62c	1.220A <sub>m</sub>

*one mode is excited.* This term applies directly to the case when only the first mode persists at moderate ranges, but it is also useful in cases when the first few modes are completely trapped (attenuation zero) but the variations in the field are controlled primarily by the first leaky mode. The horizon is expressed by the equation

$$d = 1.063(\sqrt{h_1} + \sqrt{h_2}) \text{ nautical miles} \quad (28)$$

where  $h_1$  is the height of transmitter and  $h_2$  the height of a point on the horizon at a distance  $d$  from the transmitter, both expressed in feet.

In representing coverage diagrams we use the scale of *path attenuation*, which is defined as the ratio between

the power radiated by a doublet at one terminal of the path to the maximum power that can be recovered from a similar antenna at the other terminal, when the orientation of the antennas and the impedance of the load are adjusted for maximum power transfer. The formula for path attenuation is

$$P.A. = -20 \log_{10} \left( \frac{3\lambda |\Psi|}{8\pi} \right) \text{ decibels} \quad (29)$$

where  $\Psi$  is the Hertzian potential given by (24).

### III. THE $M$ CURVE OBSERVED DURING THE ANTIGUA EXPERIMENT

Observations of the vertical distribution of temperature and humidity were made on board ship and on a halyard rig which ran from a pole at the top of a 90-foot wooden tower, located 60 feet from the shore, to the water's edge. On account of the state of the sea and the difficulties in aeration of the radiation shield when running with the wind, the ship data were of limited accuracy. However, when heading into the wind, representative psychrometric data were obtained at an elevation of 20 feet. These showed that the air at 20 feet was colder than the water. Our knowledge of the refraction conditions that prevailed during the Antigua experiment is therefore based mostly on the soundings made at the shoreline. The shore data showed that during the entire period of observations a simple surface duct existed over the water. The strength of the surface duct in the open sea must have been greater than was observed on the shore because of the modifying influence of the island. From observations made inland it appears that the duct is probably completely destroyed within one-quarter to one-half mile from shore. In judging the refraction conditions at sea one therefore has to increase suitably the  $M$  values in the surface layer as observed near the tower. An example of such an adjustment is shown by the  $M$  curve  $C'$  in Fig. 1.

The individual soundings showed considerable scatter both in the temperature and in the humidity readings. Representative  $M$  curves were computed from smoothed temperature and humidity data extending over a period of a given weather regime. Fig. 1 shows representative  $M$  curves under conditions of normal winds, low winds, and high winds. The observed values are plotted as dots, while the continuous curves show the degree to which the observed data can be approximated analytically by a linear-exponential expression.

The wave theoretical interpretation of the transmission data obtained in the Antigua experiment was made on the basis of the  $M$  curve  $C'$  in Fig. 1:

$$M - M_0 = 0.036h + 12.06e^{-0.0714h}, \quad (30)$$

$h$  being expressed in feet. When expressed in natural units (see (21) and Table I), we obtain

$$Y(z) = z + 10e^{-2.4z} \quad (31)$$

for the 10-centimeter band, and

$$Y(z) = z + 22.31e^{-1.076z} \quad (32)$$

for the 3-centimeter band. The curve  $C'$  was adopted as representative for the open sea under normal wind conditions, after consultation with members of the Antigua

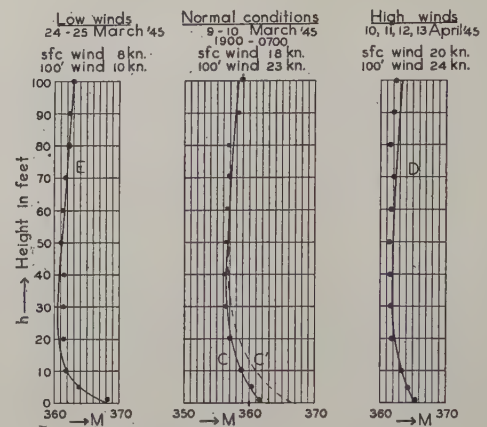


Fig. 1—Representative  $M$  curves at Antigua.  
• = Average observed points.

Curve	Equation of curve	Value of $M$ at water surface
$C'$	$M = 354.7 + 0.036h + 12.06e^{-0.0714h}$	
$C$	$M = 354.7 + 0.036h + 7.6e^{-0.0720h}$	389.3
$D$	$M = 359.8 + 0.036h + 5.85e^{-0.0681h}$	391.3
$E$	$M = 359.4 + 0.036h + 9.0e^{-0.137h}$	394.4

expedition as to the magnitude of the correction to be applied to the observed shore data shown by curve  $C$  in Fig. 1. As explained above, this correction allows for the modifying influence of the island on the approaching air masses. That such a correction was necessary became apparent independently from a preliminary analysis of the transmission data for the 10-centimeter band. Using the phase-integral method, which though approximate is sufficient for the purpose in hand, it was found that for the 10-centimeter band the theoretical horizontal decrements are 2.1, 1.5, and 1.6 decibels per nautical mile for the  $M$  curves  $E$  (low),  $C$  (normal), and  $D$  (high), respectively. The observed decrements, when averaged over all transmitter heights and receiver heights, were 1.8, 0.9, and 0.6 decibels per nautical mile, respectively. While the effect of wind on transmission of the 10-centimeter band, especially the adverse effect of low winds, is qualitatively accounted for, the theoretical decrements for the  $M$  curves obtained on the shore line are considerably in excess over the values observed at sea. Curve  $C'$  was judged to represent the maximum correction for the land effect which is consistent with the rather reliable  $M$  values measured on board ship at elevations greater than 20 feet. Subsequently it was found that the theoretical decrement for curve  $C'$  is in agreement with the observed values.

### IV. THE NORMAL MODES

Having adopted the  $M$  curve given in (30), (31), and (32), our next step is to compute the characteristic values  $D_m$  and the height-gain functions  $U_m$  from (23),

subject to the normalizing condition (25) and to the boundary conditions stated on page 454. With these, the field is then computed from (24) and (29). At present there is no general method which is both convenient and exact for solving (23) in case  $Y(z)$  is a linear-exponential function of the form (27). However, several approximate methods have been developed, some of which are as yet unpublished, by which useful solutions can be obtained without an undue amount of labor in limited regions of the  $\alpha, \gamma$  plane. The accuracy of the approximate methods is judged either by criteria of convergence of successive approximations in an individual method, or by the degree of agreement between results obtained from two different methods, when more than one method is applicable. In this section we shall give an account of the computation of the field intensity for the  $M$  curve (30), both for the 3-centimeter and 10-centimeter bands. A detailed exposition of the methods used in the solution, some of which are applicable to more general types of continuous  $M$  curves, will be given in another paper.

### 1. The Normal Modes for the 10-Centimeter Band

For a wavelength of 10 centimeters we have

$$Y(z) = z + 10e^{-2.4z} \quad (31)$$

The computation of the characteristic value  $D_1$  for the first mode in this case proved to be most difficult because the parameters  $\alpha$  and  $\gamma$  happen to lie in a region where none of the available analytical methods are particularly effective. In Table II are listed several approximations to  $D_1$  obtained by different methods, together with supplementary information indicating the probable error of each determination.

The value adopted in the computations of the field intensity was  $D_1 = -1.25 + 0.84i$ . The reason for choosing this value is that, when using the form  $U_1 = A(z + bz^2)e^{-az}$

TABLE II

APPROXIMATE DETERMINATIONS OF THE CHARACTERISTIC VALUE OF THE FIRST MODE  $D_1$  FOR THE CASE  $Y(z) = z + 10e^{-2.4z}$

Method	Approximation	$D_1$
Phase integral	Leading term	$-1.41 + 0.72i$
	Including second term	$-1.30 + 0.62i$
Perturbation, <sup>11</sup> using linear combinations of standard functions.	Five standard modes	$-1.20 + 0.47i$
	Six standard modes	$-1.31 + 0.52i$
New phase integral <sup>12</sup> method.	Leading term; including first-order perturbation	$-1.09 + 0.56i$
		$-1.11 + 0.84i$
Variational method <sup>13</sup>	Using $U_1 = Aze^{-az}$	$-1.44 + 0.90i$
	Using $U_1 = A(z + bz^2)e^{-az}$	$-1.25 + 0.84i$

<sup>11</sup> C. L. Pekeris, "Perturbation theory of the normal modes for an exponential  $M$  curve in nonstandard propagation of microwaves," *Jour. Appl. Phys.*, vol. 17, pp. 678, 684; August, 1946.

<sup>12</sup> C. L. Pekeris, "Asymptotic solutions for the normal modes in the theory of microwave propagation," *Jour. Appl. Phys.*, vol. 17, pp. 1108-1124; December, 1946.

<sup>13</sup> C. L. Pekeris and S. Ament, "Characteristic values of the first normal mode in the problem of propagation of microwaves through an atmosphere with a linear-exponential modified index of refraction," to be published in *Phil. Mag.*

in the variational method, one obtains a value of  $D_1$  for the standard case which differs from the correct value by less than 1 per cent, and that this form of  $U_1$  would be expected to yield even better results as trapping is approached. However, from the values tabulated in Table II it would appear that an error of as much as 0.2 cannot be precluded. Such an error in the imaginary part of  $D_1$  would imply an uncertainty of about 0.25 decibel per nautical mile in the decrement. Since this is within the range of fluctuations of daily values of the observed decrement in a given weather regime, it did not seem worth while to seek a more refined value for  $D_1$ .

Having adopted a value for  $D_1$ , the height-gain function  $U_1$  was obtained by numerical integration of (23). For this purpose it was necessary to determine  $\dot{U}_1(0)$ , the derivative of  $U_1$  at the surface. The latter was obtained as follows. The constant  $A$  in

$$U_1 = A(z + bz^2)e^{-az} \quad (33)$$

was determined from the normalization condition

$$\int_0^\infty U_1^2(z) dz = 1, \quad (25)$$

and  $\dot{U}_1(0)$  was then computed from the relation

$$\dot{U}_1(0)^2 = - \int_0^\infty U_1^2(z) (1 - \alpha\gamma e^{-\gamma z}) dz, \quad (26)$$

which follows directly from the differential equation

$$\frac{d^2 U_1(z)}{dz^2} + [z + \alpha e^{-\gamma z} + D_1] U_1(z) = 0. \quad (34)$$

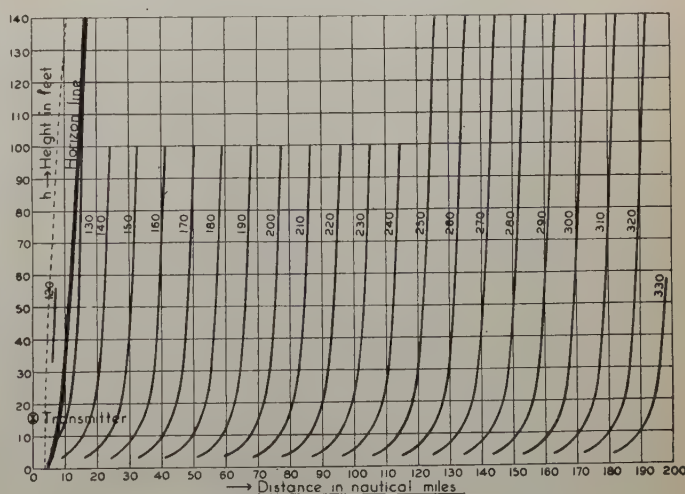


Fig. 2—Theoretical coverage diagram for a transmitter height of 16 feet.

$$\begin{aligned} \text{Wavelength} &= 10 \text{ centimeters} \\ M - M_0 &= 0.036h + 12.06 e^{-0.0714h} \\ Y(Z) &= Z + 10 e^{-2.4Z} \end{aligned}$$

Numbers denote path attenuation in decibels.

Expression (33) was used in the integrand of (26). The value of  $\dot{U}_1(0)$  thus obtained was  $1.613e^{0.6961i}$ . Using the form  $U_1 = Aze^{-az}$  and the same method, one arrives at the value of  $1.658e^{0.6966i}$ .

The decrement of the first mode is 1.0 for the 10-centimeter band while that of the second mode is close to 4.

Hence the field of this band is determined primarily by the first mode alone, the second and higher modes disappearing after a short range beyond the horizon. The coverage diagram shown in Fig. 2 was computed for

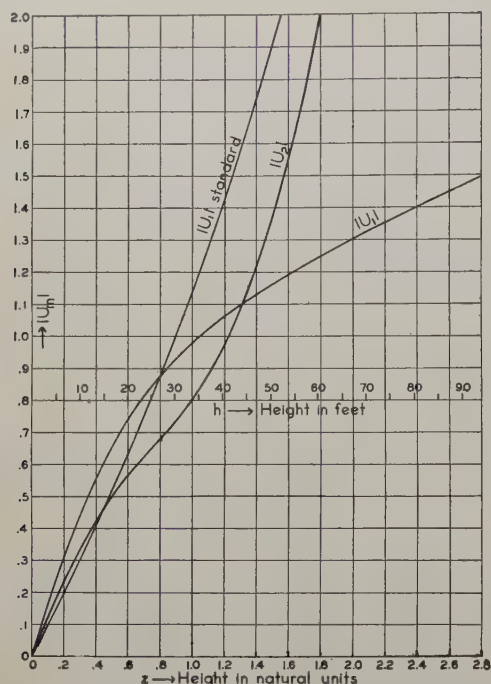


Fig. 3—Height-gain functions  $|U_1|$  and  $|U_2|$  for the first and second modes.

Wavelength = 10 centimeters  
 $M - M_0 = 0.036h + 12.06e^{-0.0714h}$   
 $Y(Z) = Z + 10e^{-2.4Z}$   
 Unit of  $Z = 33.62$  feet

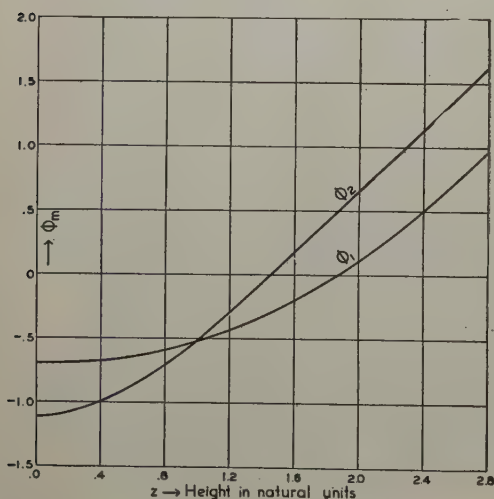


Fig. 4—Phases  $\phi_1$  and  $\phi_2$  of the first and second modes.

$U_m(Z) = |U_m(Z)|e^{-i\phi_m(Z)}$   
 Wavelength = 10 centimeters  
 $M - M_0 = 0.036h + 12.06e^{-0.0714h}$   
 $Y(Z) = Z + 10e^{-2.4Z}$

such ranges and elevations that the second mode did not contribute more than 25 per cent of the total intensity.<sup>14</sup> It will be seen that this region extends inwards up to the horizon, and to elevations of 100 feet. It follows that, in our application, neither the decrement nor

the height-gain function of the second mode needed to be known with high accuracy.

Using a linear combination of six standard modes in the perturbation method,<sup>11</sup>  $D_2$  was found to be  $-2.01 + 3.11i$ ; linear combinations of four and five standard modes yielded values  $-2.02 + 3.18i$  and  $-2.01 + 3.14i$ , respectively. Hence, in this case, the perturbation method leads to fairly accurate values for the characteristic value. The value of  $D_2$  obtained by the phase-integral method was  $-2.19 + 3.01i$ ; for the standard case  $D_2 = -2.04 + 3.54i$ . In the final computations of the field a value of  $-2.01 + 3.11i$  for  $D_2$  was adopted, and the height-gain function of the second mode was computed by the phase-integral method using Hankel functions of order  $1/3$  (R. E. Langer,<sup>15</sup> H. Jeffreys<sup>16</sup>). The height-gain function thus obtained is, of course, only approximate, but for the reasons explained above it is adequate for our particular application. Figs. 3 and 4 show the height-gain functions of the first two modes for the 10-centimeter band.

## 2. The Normal Modes for the 3-Centimeter Band

For a wavelength of 3 centimeters we have

$$Y(z) = z + 22.31e^{-1.076z}. \quad (32)$$

In this case the first mode is completely trapped,  $D_1$  being equal to  $-8.01 + 5.5 \times 10^{-10}i$ , as determined by the phase-integral method (Gamow, Furry<sup>2</sup>). The height-gain function  $U_1$  was computed by a phase-integral method involving Bessel functions of order  $1/3$ .<sup>12</sup> It was accomplished by developing solutions of the Langer type near each turning point and then joining them at the duct height.

The second mode was found to be on the verge of trapping, yet with a decrement that was not too small to be negligible. For this mode the new phase integral method was found to be effective. Using the leading term in this method, a value of  $-3.66 + .18i$  for  $D_2$  was obtained; when a first-order perturbation was included the value changed to  $-3.63 + 0.26i$ . This value is probably accurate to within 0.02, judging from the success of the method in reproducing exact values of  $D_1$  which were determined in the same range of parameters  $\alpha$  and  $\gamma$  by the use of Hartree's differential analyzer.<sup>18</sup>

With an adopted value of  $-3.63 + 0.26i$  for  $D_2$ , the height-gain function  $U_2(z)$  was computed by integrating the differential equation (23), and using a value for  $\dot{U}_2(0)$  as determined from the new phase-integral method.  $U_2(z)$  was also computed from the leading term in the phase-integral solution, and the results were found to agree closely up to 30 feet elevation, but between 30 and 90 feet the two solutions began to diverge, reaching a maximum discrepancy in amplitude of 25 per cent at 90 feet. In view of the fact that the perturbation term

<sup>15</sup> R. E. Langer, "On the connection formulas and the solutions of the wave equation," *Phys. Rev.*, vol. 51, pp. 669-676; April, 1937.

<sup>16</sup> H. Jeffreys, "Asymptotic solutions of linear differential equations," *Phil. Mag.*, vol. 33, pp. 451-456; June, 1942.

<sup>14</sup> This 25 per cent line is given by the - - - - curve.

was not included in the second computation of  $U_2(z)$ , the result can be considered as satisfactory.

The third mode in the 3-centimeter band leaks heavily, with a decrement close to 4 decibels per nautical mile. Hence its decrement and height-gain function were computed only approximately, merely for the purpose of determining the region where its contribution to the

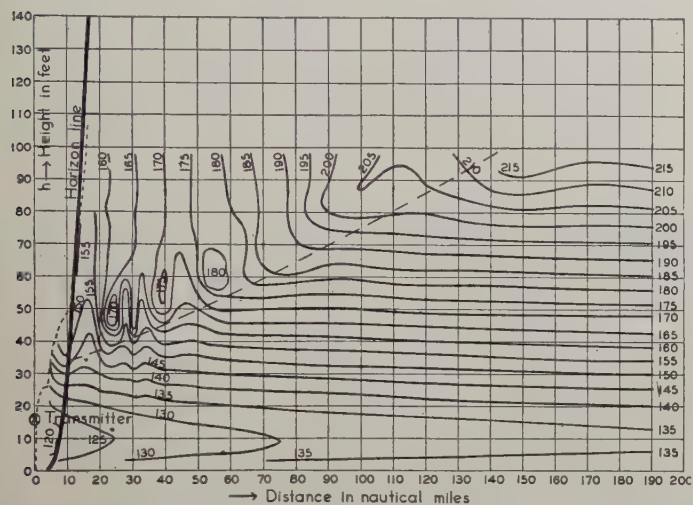


Fig. 5—Theoretical coverage diagram for a transmitter height of 16 feet.

$$\begin{aligned} \text{Wavelength} &= 3 \text{ centimeters} \\ M - M_0 &= 0.036h + 12.06 e^{-0.0714h} \\ Y(Z) &= Z + 22.31 e^{-1.076Z} \end{aligned}$$

Numbers denote path attenuation in decibels.

--- Approximate line of demarcation between regions controlled by the first mode or by the second mode.

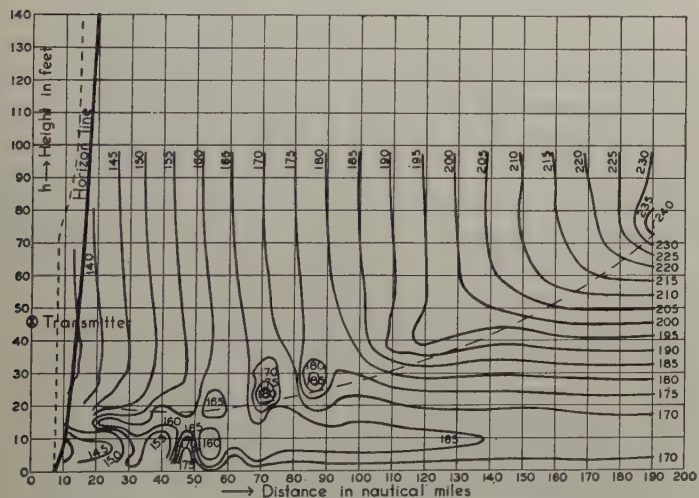


Fig. 6—Theoretical coverage diagram for a transmitter height of 46 feet.

$$\begin{aligned} \text{Wavelength} &= 3 \text{ centimeters} \\ M - M_0 &= 0.036h + 12.06 e^{-0.0714h} \\ Y(Z) &= Z + 22.31 e^{-1.076Z} \end{aligned}$$

Numbers denote path attenuation in decibels.

--- Approximate line of demarcation between regions controlled by the first mode or by the second mode.

total field intensity is less than 25 per cent. This region is extensive and is shown in Figs. 5 and 6.<sup>14</sup> A value of  $D_3 = -3.13 + 2.16i$  was obtained from the phase-integral method, and  $U_3(z)$  was also computed by this method using the Hankel function of order  $1/3$ . Figs. 7 and 8 show the height-gain functions of the first 3 modes for the 3-centimeter band.

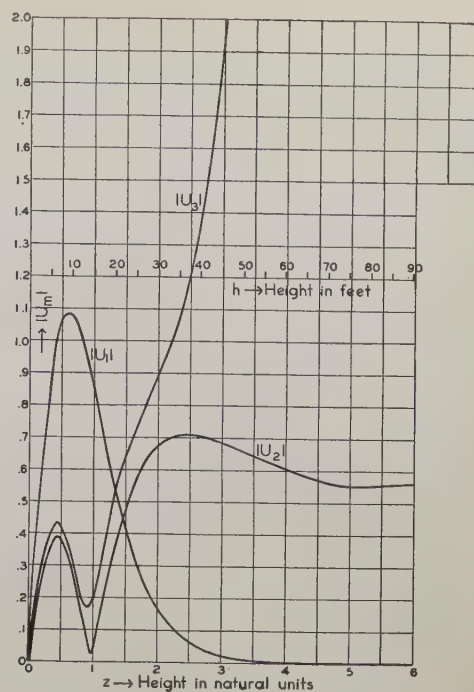


Fig. 7—Height-gain functions  $|U_m|$  for the first three normal modes.

$$\begin{aligned} \text{Wavelength} &= 3 \text{ centimeters} \\ M - M_0 &= 0.036h + 12.06 e^{-0.0714h} \\ Y(Z) &= Z + 22.31 e^{-1.076Z} \\ \text{Unit of } Z &= 15.07 \text{ feet} \end{aligned}$$

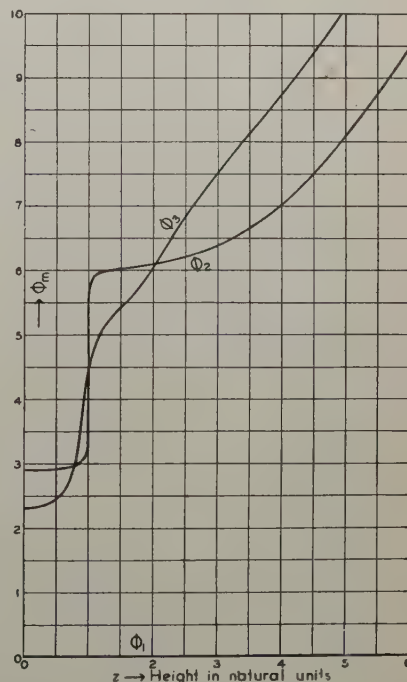


Fig. 8—Phases of the first three normal modes.

$$\begin{aligned} U_m(Z) &= |U_m(Z)| e^{-i\Phi_m(Z)} \\ \text{Wavelength} &= 3 \text{ centimeters} \\ M - M_0 &= 0.036h + 12.06 e^{-0.0714h} \\ Y(Z) &= Z + 22.31 e^{-1.076Z} \end{aligned}$$

## V. DISCUSSION OF THE THEORETICAL COVERAGE DIAGRAMS

With the aid of the computed normal modes, the potential  $\Psi$  was computed from (24) at several ranges  $x$ , and the path attenuation below the horizon at these

ranges was determined from (29). At each range we thus obtained a "cross section" giving the variation of field intensity with height. By graphical interpolation, the cross sections were used to map out contour lines of equal intensity, and the results are shown by the coverage diagrams in Figs. 2, 5, and 6.

The coverage diagram for the 10-centimeter band shown in Fig. 2 has the simple appearance which is characteristic of propagation by a single (first) mode. The contribution from the second mode is negligible in most of the mapped region; at a range of 21.3 miles ( $x=3$ ) and a height of 134 feet ( $z=4$ ) the omission of the second mode changes the level by about 0.1 decibel. To the same degree of accuracy the variation of intensity with height is given by the square of the ordinate  $|U_1|$  plotted in Fig. 3. Between 20 and 100 nautical miles the contour lines in Fig. 2 give an attenuation of 1.1 decibels per nautical mile, of which 1.0 decibel per nautical mile is due to the decrement of the first mode and 0.1 decibel per nautical mile to the  $(1/d)$  divergence factor. It did not seem worth while to plot a coverage diagram for a transmitter height of 46 feet, as was done in the case of the 3-centimeter band, because it would have an appearance similar to Fig. 2. The only difference would be in the numerical value of path attenuation which would be less by  $20 \log_{10}(1.12/.635) = 5$  decibels at each level, as follows from the curve  $|U_1|$  in Fig. 3.

In contrast to Fig. 2, the coverage diagrams for the 3-centimeter band shown in Figs. 5 and 6 exhibit more of a structural pattern, on account of the interference between the first two modes which are excited in comparable strengths. For this wavelength the decrement of the first mode is zero, that of the second mode is only 0.47 decibel per nautical mile, while the third mode has a decrement close to 4 decibels per nautical mile. The contribution from the latter mode is negligible in most of the mapped region; at a range of 19 nautical miles ( $x=4$ ) and a height of 98 feet ( $z=6.5$ ) the omission of the third mode changes the level by less than 0.3 decibel when the transmitter is situated at an elevation of 16 feet, and by an even smaller amount when the transmitter is located at 46 feet.

Figs. 5 and 6 exhibit two distinct regions, separated roughly by the dashed line, in which propagation takes place either primarily by the first mode (lower right) or primarily by the second mode (upper left). The former region is characterized by low attenuation and a rapid decrease of intensity with elevation, while the second region is characterized by a rate of attenuation of about 0.6 decibel per nautical mile and a nearly uniform distribution of intensity with height. For the transmitter height of 16 feet (Fig. 5) the latter region is small, because at that level the amplitude of the first mode is high and the amplitude of the second mode is near a minimum (see Fig. 4). On the other hand, for a transmitter height of 46 feet the region in which propagation takes place via the second mode is more extensive, because the first mode is poorly excited at that level.

## VI. COMPARISON OF OBSERVED WITH THEORETICAL FIELD INTENSITIES

### 1. The 10-Centimeter Band

A. *Rates of Attenuation.* In Table III are listed the average observed rates of attenuation under various wind conditions.<sup>17</sup> It is seen that the observed rates of

TABLE III  
OBSERVED AVERAGE RATES OF ATTENUATION, IN DECIBELS PER NAUTICAL MILE, FOR THE 10-CENTIMETER BAND AT RANGES LESS THAN 80 MILES

Condition	(Transmitting Antenna, 16 feet) Recording Antenna Height (feet)				(Transmitting Antenna, 46 feet) Recording Antenna Height (feet)			
	14	24	54	94	14	24	54	94
General Average	0.84	1.00	1.03	1.07	1.31	1.10	1.26	1.24
Low Wind	1.38	1.49	1.55	1.44	2.42	2.30	1.96	1.84
Normal Wind	0.86	0.92	0.86	0.97	0.86	0.80	0.97	0.97
High Wind	0.46	0.57	0.57	0.57	0.69	0.46	0.80	0.80

attenuation are fairly constant with elevation, and that in the case of normal winds, for which the theoretical field intensities were computed, they average about 0.9 decibel per nautical mile. The theoretical rate of attenuation is about 1.1 decibels per nautical mile, of which 1.0 is due to the decrement of the first mode and 0.1 to the  $(1/d)$  divergence factor. The discrepancy between the theoretical and observed values is of the order of the uncertainty of either, so that for ranges less than about 80 miles the agreement in magnitude between observed and predicted rates of attenuation can be considered as satisfactory.

In agreement with observation, wave theory predicts a rate of attenuation which is independent of height.

As was pointed out in Section III, the observed poor transmission under conditions of low winds is qualitatively accounted for on the basis of wave theory.

The above considerations apply only to the field at ranges less than about 80 miles. Beyond 80 miles a rate of attenuation of about 1 decibel per nautical mile should still theoretically persist, whereas observationally it is found that the rate drops to a low value of about 0.2 decibel per nautical mile. The signal also becomes erratic at these ranges, and the intensity assumes a more or less uniform distribution with height. Both of these features suggest that the received signal does not originate from the diffracted direct beam, but is contributed by some secondary source of irradiation, such as, perhaps, scattered radiation.

B. *Variation of Field Intensity With Height.* One important result discovered in the Antigua experiment was that in the case of the 10-centimeter band the intensity *increased* with elevation from 14 to 94 feet. This is in contrast with the vertical distribution of intensity for the 3-centimeter band which was characterized by a *drop* of intensity with elevation above 14 feet. In Fig. 9, the curve shows the theoretical variation of intensity with height for the 10-centimeter band at a range of

<sup>17</sup> M. Katzin, R. W. Bauchman, and W. Binnian, "3- and 9-centimeter propagation in low ocean ducts," to be published in Proc. I.R.E.

21 miles and for a transmitter height of 46 feet. The dots represent average intensity levels observed at a range of 20 miles, when referred to the level of 94 feet as datum. The agreement between theory and observations is seen to be good from 24 up to 94 feet, but at 14 feet the observed mean level is below the theoretical by about 6 decibels. The variation of intensity with elevation for a transmitter height of 16 feet was found to be the same as for the 46-foot position of the transmitter, in agreement with theory, but the levels were everywhere about 5 decibels below the theoretical values, in conformity with the observed low intensity near 14 feet.

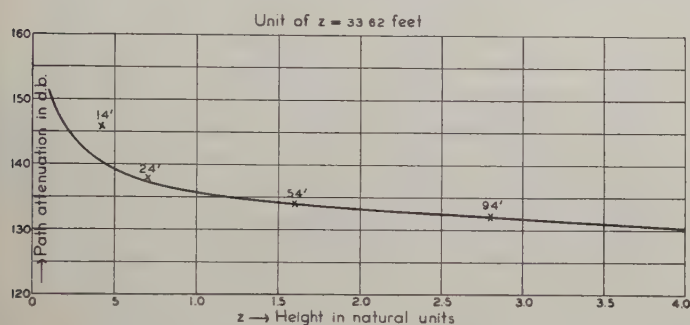


Fig. 9—Theoretical and observed vertical distribution of intensity of Antigua experiment at a range of 20 miles.

Wavelength = 10 centimeters

$M + M_0 = 0.036h + 12.06 e^{-0.0714h}$

$Y(Z) = Z + 10 e^{-2.4Z}$

Unit of  $Z = 33.62$  feet

— Theoretical

x Average observed levels relative to level at 94 feet

## 2. The 3-Centimeter Band

*A. Rates of Attenuation.* Referring to Table IV, the tabulated values of observed rates of attenuation of the 3-centimeter band are, with but three exceptions, lower than the values for the 10-centimeter band given in Table III. There is also a definite indication of an increasing rate of attenuation with elevation. Thus, in the case of normal winds, the increase of rate of attenuation between 14 and 94 feet, if at all real, is around 13 per cent for the 10-centimeter band, while for the 3-centi-

TABLE IV  
OBSERVED AVERAGE RATES OF ATTENUATION, IN DECIBELS PER NAUTICAL MILE, FOR THE 3-CENTIMETER BAND

Condition	(Transmitting Antenna, 16 feet) Receiving Antenna Height (feet)					(Transmitting Antenna, 46 feet) Receiving Antenna Height (feet)				
	6	14	24	54	94	6	14	24	54	94
General Average	0.44	0.44	0.61	0.76	0.66	0.51	0.51	0.55	0.91	0.82
Low Wind	0.80	0.97	2.30	1.84	0.74	0.71	0.86	0.74	0.86	1.95
Normal Wind	0.42	0.34	0.51	0.69	0.69	0.46	0.51	0.51	0.74	0.74
High Wind	0.23	0.17	0.23	0.34	0.46	0.51	0.34	0.28	0.46	0.40

meter band it is of the order of 50 to 100 per cent. Both of these features are in qualitative agreement with theory.

When we come to make a quantitative comparison between the observed and computed rates of attenuation for the 3-centimeter band, we find that some schematization of the complex structure of the coverage

diagrams shown in Figs. 5 and 6 is necessary. For this purpose the computed "cross sections" at 19 miles ( $x=4$ ) and 95 miles ( $x=20$ ) were used to determine average theoretical rates of attenuation between these ranges, and the results are shown in Table V. In the regions where, theoretically, transmission takes place via the second mode, such as transmitter 16 feet, receiver 94 feet, and transmitter 46 feet, receiver 54 feet or 94 feet, the computed rate of attenuation of about 0.6 compares favorably with the observed rate of about 0.7 decibel per nautical mile, for normal winds. On the other hand, the very low rates of attenuation pre-

TABLE V  
THEORETICAL AVERAGE RATES OF ATTENUATION IN DECIBELS PER NAUTICAL MILE BETWEEN THE RANGES OF 19 AND 95 NAUTICAL MILES FOR THE 3-CENTIMETER BAND

	Receiving Antenna Height (feet)				
	6	14	24	54	94
Transmitting Antenna 16 feet	0.09	0.09	0.09	0.24	0.58
Transmitting Antenna 46 feet	0.28	0.13	0.41	0.56	0.57

dicted by theory in regions where transmission takes place via the first mode (arising from the divergence factor only) are less than the observed values by about 0.3 decibel per nautical mile. This small excess of observed-rate attenuation could probably be attributed to scattering due both to horizontal inhomogeneities in the distribution of refractive index and to the roughness of the sea surface. The first mode is particularly susceptible to the influence of these secondary factors because, first, any small attenuation they bring about stands out against the theoretically vanishing decrement, and, second, because this mode channels the radiation in a layer centered around 9 feet above the surface (see Fig. 4), which is subject to disturbances by the sea waves running from 4 to 8 feet.

*B. Variation of Field Intensity With Height.* The influence of scattered radiation on transmission in regions governed by the first mode is reflected also in the observed distribution of intensity with height for the 3-centimeter band. Fig. 5, as well as Figs. 6 and 4, show that above an elevation of about 20 feet the intensity of the first mode should theoretically fall off with height at a rate of about 1 decibel per foot. Scattered radiation, if comparable in intensity to the direct diffracted beam, would tend to illuminate the dark regions at the expense of the bright ones, and thus bring about a less rapid variation of intensity with height. Furthermore, we must take into consideration the fact that on account of the rolling and pitching of the ship the heights of the transmitters varied, so that in comparing observed with theoretical vertical distributions of intensity one should average out the coverage diagrams over the corresponding height swung through by the transmitters. Although this averaging was not actually carried out, it is clear

that it would result in a more even distribution of intensity with height, and that it would probably eliminate the interference patterns shown on the coverage diagrams. In the case of the 10-centimeter band, where theoretically the intensity does not vary rapidly with height, the effect of the varying heights of transmitters is of course less.

For the reasons given above no quantitative comparison of observed with computed vertical distribution of intensity with height for the 3-centimeter band was attempted. At a range of 20 miles, Fig. 5 shows that for a transmitter height of 16 feet the intensity decreases with elevation between 6 and 24 feet at an average rate of about 0.45 decibel per foot; this compares favorably with the observed mean rate of decrease of about 0.35 decibel per foot at this range. However, as was mentioned before, the rapid theoretical fall of intensity between 24 and 50 feet, amounting to a rate of about 1 decibel per foot, is not verified observationally. The quantitative comparison with theoretical of observed vertical distribution at a range of 20 miles for a trans-

mitter height of 46 feet is further complicated by the changing interference pattern at that range shown in Fig. 6, and by the fact the observed distribution was deduced by interpolation from data taken at smaller and larger ranges than 20 miles.

*Theory, however, does verify the important qualitative difference in intensity distribution with height between the 10-centimeter and 3-centimeter bands, namely, that in the latter the intensity generally decreases with height above 6 feet. The observed optimum position of transmitter for the 3-centimeter band of between 6 and 15 feet is in agreement with theory.*

## VII. ACKNOWLEDGMENT

The extensive computations required in this investigation were carried out at the Columbia University Division of War Research, Mathematical Physics Group, by Mrs. B. Brown and Misses D. Weinstock, L. Selig, F. Jones, and E. Herman, under the supervision of Miss Alice Osterberg.

# Microwave Oscillators Using Disk-Seal Tubes\*

A. M. GUREWITSCH†, SENIOR MEMBER, I.R.E., AND J. R. WHINNERY†, SENIOR MEMBER, I.R.E.

**Summary**—Some practical aspects of microwave oscillators using disk-seal tubes are discussed. Typical design and performance data are given.

A general small-signal oscillator theory is presented and applied to the re-entrant disk-seal-tube microwave oscillator.

It is shown how information on frequency of oscillation, tuning, and frequency stability can be obtained.

## INTRODUCTION

THE USEFUL frequency range of space-charge-controlled tubes has been considerably extended during the war. A notable advance has been accomplished by the "disk-seal" tube. This tube is a triode of novel design.<sup>1</sup> It is formed by plane-parallel electrodes spaced sufficiently close to minimize harmful transit-time phenomena. The disk-seal construction enables the tube to be an integral part of the circuit.

Photographs of three different disk-seal tubes, now on the market, the 2C40, 2C43, and 2C39, are shown in Fig. 1. Their respective rated frequency ranges are at present as follows:

- 2C40—up to 3370 megacycles
- 2C43—up to 3370 megacycles
- 2C39—up to 500 megacycles.

It may be mentioned that these tubes are being con-

stantly improved. For example, some of the developmental 2C39's have operated in oscillators as high as 3000 megacycles. These tubes may be used as oscillators or amplifiers in their respective frequency ranges.

The design of the disk-seal triode is particularly adapted to a so-called grid-separation or grid-return circuit. In such a circuit the output load is placed between the plate and the grid, as contrasted to the conventional arrangement where the output load is placed between

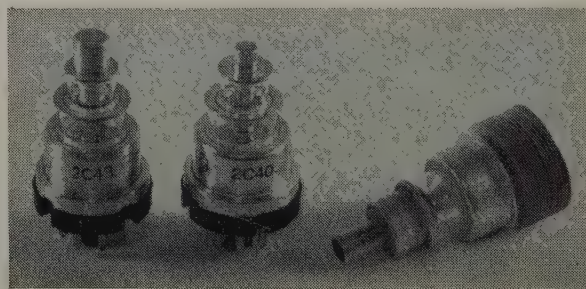


Fig. 1—Commercial disk-seal tubes.

the plate and cathode (see Figs. 2 and 3). It is readily seen that, if the cathode-grid resonator and the grid-plate resonator are not coupled by external means, the only way in which energy can be exchanged between the two resonators is through the electron beam and the electromagnetic coupling through the grid. In amplifiers the latter coupling should usually be small to avoid

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† General Electric Company, Schenectady, N. Y.

<sup>1</sup> E. D. McArthur, "Disk-seal tubes," *Electronics*, vol. 18, pp. 98-102; February, 1945.

undesirable regeneration. In oscillators or neutralized amplifiers, an additional coupling (feedback) of some kind between the output and input is necessary. This coupling is frequency sensitive and is one of the important problems in the design of tunable oscillators.

The circuit shown in Fig. 3 may be considered as the basic oscillator circuit using grid-separation ideas. Many somewhat different physical realizations of the circuit are possible, the best one for any given problem depend-

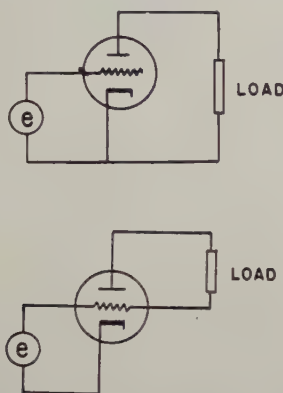


Fig. 2—Cathode-return and grid-return (grid-separation) circuits.

ing upon the application. A few of the considerations in choosing a design are:

- (1) Continuous-wave or pulsed operation
- (2) Tuning range
- (3) Number of controls permissible
- (4) Stability with variations in applied voltages
- (5) Stability with temperature
- (6) Stability with changes in load
- (7) Physical size
- (8) Importance of power output versus life, efficiency, or other factors.

Although, in continuous-wave operation, oscillations have been observed for wavelengths as low as 5 centimeters, reliable operation of the 2C40 tube is limited at present to the range above about 9 centimeters. In pulsed operation of 2C43 tubes, the lowest wavelengths so far observed were around 4 centimeters. Reliable operation can again be expected for wavelengths above 9 centimeters.

#### GENERAL CONSIDERATIONS IN OSCILLATOR DESIGN

We will first discuss some practical aspects of oscillator design and give a few typical examples; later, a general oscillator theory with some applications will be given.

A problem common to all circuits described here is that of establishing satisfactory contact between the tube elements and the circuit. Particular care must be taken in the case of the plate contact to assure both reliable electrical contact and thermal contact. The latter is necessary to prevent excessive seal temperatures which would lead to premature tube failures. Resilient

materials such as beryllium copper, phosphor bronze, or hard brass should be used. Pulling or shearing force upon the seals should be minimized.

A reasonable number of fingers with firm, positive contact pressure have been found superior to either a small number of fingers or a large number of flimsy fingers. The width of the slots between fingers must also be kept small to minimize unwanted leakage.

The problem of supplying the tube with the necessary direct-current potentials is usually solved by the well-known techniques of by-pass capacitors and resonant chokes. These may, in certain designs, be used also for tuning purposes; in other designs, sliding plungers, dielectric slugs, and similar tuning elements are employed. In the design of oscillators for pulsed operation, care should be taken not to distort the pulse form by the use of plate chokes or plate by-pass capacitors which introduce too large capacitive loading upon the pulser.

#### SOME EXAMPLES

##### A. "End-to-End" Oscillator

At first, we shall consider the most straightforward design, the so-called "end-to-end" oscillator. This oscillator type illustrates best the fundamental grid-separation circuit. The oscillator consists of two tunable cavities of the concentric-line type which are separated by the grid plane, as shown in Fig. 3. The tuning is accom-

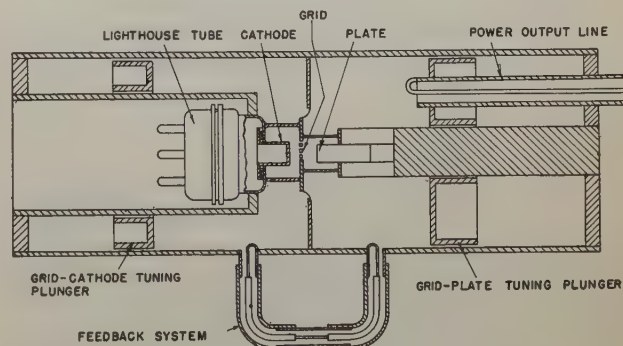


Fig. 3—A typical grid-separation circuit.

plished by a plunger in each resonator. The feedback is made by a feed-back system of concentric lines and loops. Various other feed-back designs are shown in Figs. 4, 5, and 6. The feedback is essentially a link through which the energy is exchanged between the two resonators, and, in conjunction with the resonators themselves, must satisfy certain phase and amplitude relations which will be considered in detail later. The feed-back circuits shown in Fig. 6 are very universal, in that phase and amplitude adjustments may be made more or less independently. Such circuits are recommended in cases where optimum performance is desired, rather than simplicity of operation, or for experimental study of oscillators. Figs. 4 and 5 show a few simpler arrangements which will perform satisfactorily if not too large a tuning

range is desired. For oscillators to be used over a wide tuning range, the feedback, which is generally frequency sensitive, may be coupled mechanically to the plunger as is shown, for example, in Fig. 7. The two resonators can be operated in a mode corresponding to an odd

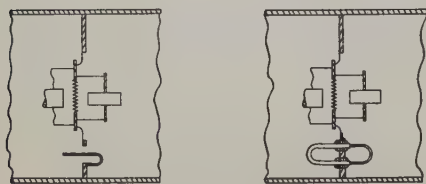
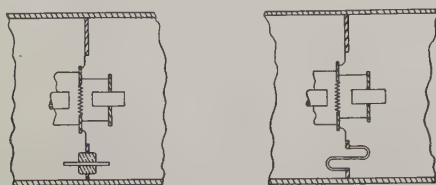


Fig. 4—Simple feed-back device.

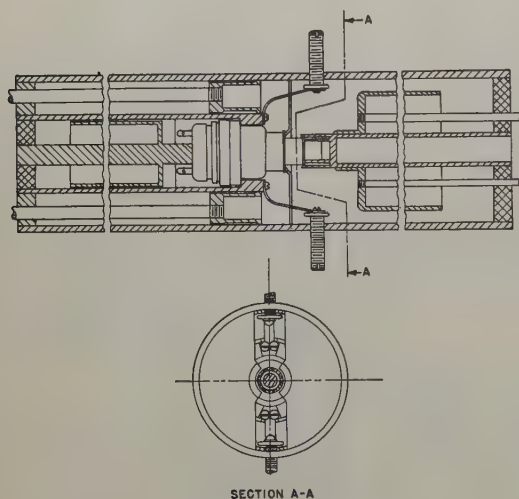


Fig. 5—Feedback with fine adjustment.

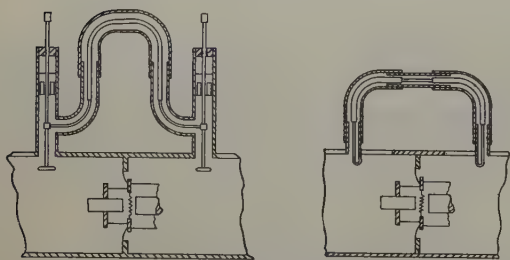


Fig. 6—Feedback with a wide-range adjustment.

number of quarter-wavelengths, corrected by the feedback, end effects, and electronics of the tube. All these factors will be considered in more detail later. However, the consideration of the frequency range of operation, physical size of the oscillator, and similar factors will determine the most appropriate mode of operation.

The typical "end-to-end" oscillator of Fig. 3 shows the B-supply line filtered for ultra-high frequency by means of quarter-wavelength chokes designed for the range of operation. The power is usually taken out from such an oscillator by a concentric line terminated by a loop pro-

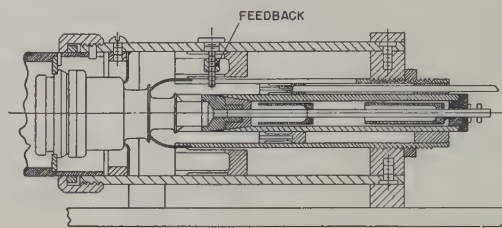


Fig. 7—Oscillator with feedback mechanically coupled to tuning plunger.

truding into the grid-plate resonator, as shown in the sketch.

A tuning-range curve and the physical dimensions of an "end-to-end" oscillator are given in Fig. 8. This type of oscillator was successfully used both for continuous-wave and pulsed operation. The advantage of this design is the accessibility of the two resonators. The disadvantage is its size (length) and the difficulty of ganging the two plungers.

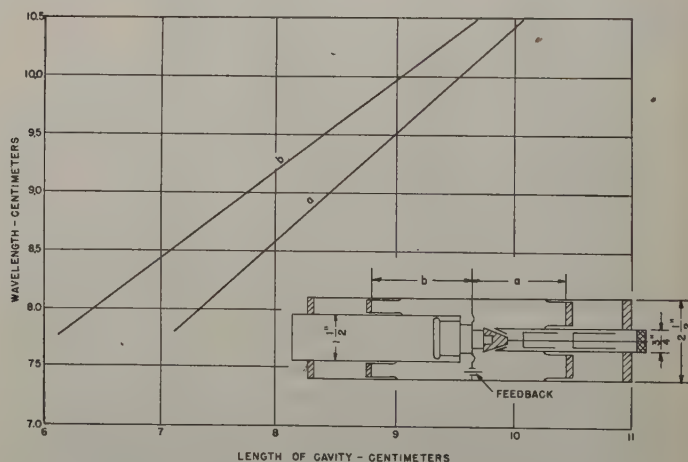


Fig. 8—Tuning curve and design dimensions for an "end-to-end" oscillator.

### B. The Folded-Back Oscillator

Electrically, the folded-back oscillator is very similar to the "end-to-end" oscillator described previously. This oscillator can be considered as derived from the "end-to-end" oscillator by folding back the grid-cathode cavity over the plate-grid cavity or vice versa. The former possibility is illustrated in Fig. 8. The problem of feedback is similar to that in the "end-to-end" oscillator. Both types of circuits have at times in the past been used as oscillators without external feedback, the feedback in this case being provided by the interelectrode coupling in the tube, but results were usually not satisfactory and varied greatly from tube to tube. A feedback suitable

for fairly wide-range, single-control operation can be made as follows: a few fingers from the cathode plunger are removed and a screw attached to it protruding

is shown using a 2C39 tube. The advantages of the folded-back design are small physical size and ease of ganging the two plungers for tuning.

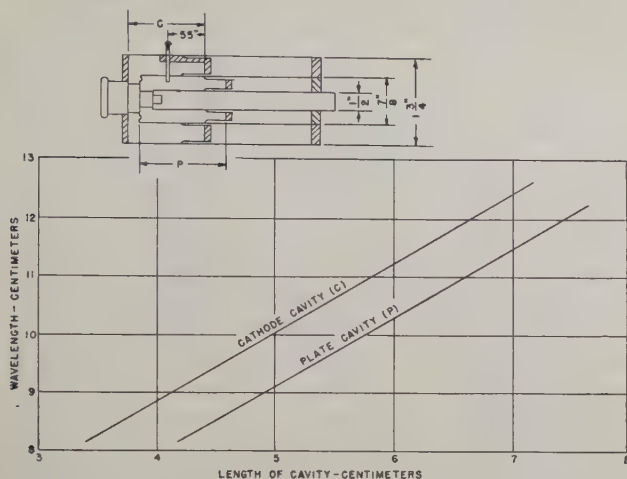


Fig. 9—Tuning curve and design dimensions for a "folded-back" oscillator with feedback coupled to tuning plunger.

through a slot in the grid cylinder into the plate-grid cavity. It was found that the proper location and adjustment of this screw assures satisfactory operation of the oscillator in the range of wavelengths from 9 to 12 centimeters (with a 2C40) without any readjustment of feedback. The tuning curve and design dimensions are given in Fig. 9. The tuning curve and design dimen-

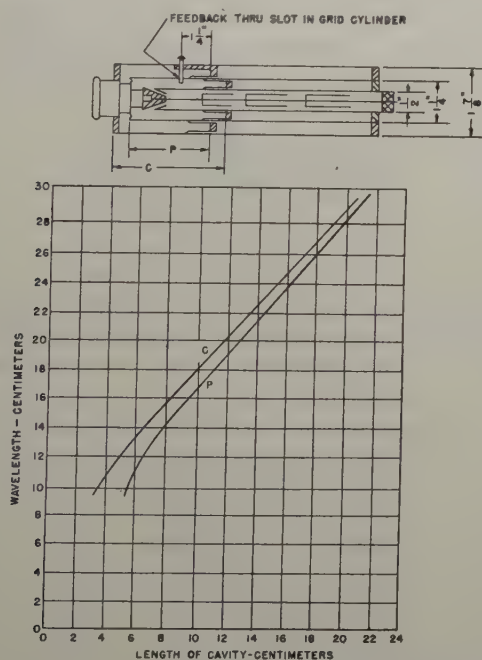


Fig. 10—A wide-range (9 to 30 centimeters) "folded-back" oscillator.

sions for an oscillator of similar type with a range of 9 to 30 centimeters is shown in Fig. 10. If only a small tuning range is required, a simpler feedback of fixed type can be used as in Fig. 11. In Fig. 12, an oscillator

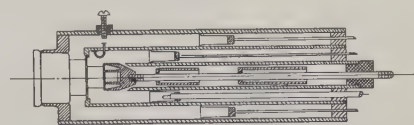


Fig. 11—"Folded-back" oscillator with fixed feedback.

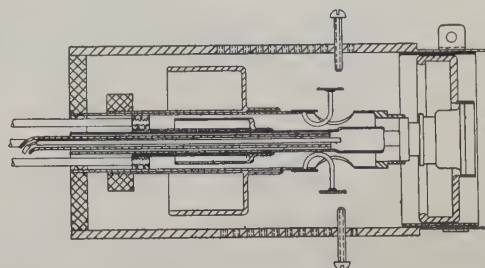


Fig. 12—An oscillator using a 2C39 tube.

### C. The Re-entrant Oscillator

This oscillator is considerably different from either of the oscillators described previously. It is simpler mechanically, easier to make oscillate, and it may be made tunable by a single control over a considerable range of frequency. The advantages of this circuit over the one previously described became evident in our early development of the circuit. Different modifications of this oscillator as applied to 2C40, 2C43, and 2C39 tubes are shown in Figs. 13, 14, and 15. As can be seen, this oscillator consists of an outer cylinder which makes a contact with the base of the tube, if a 2C40 or

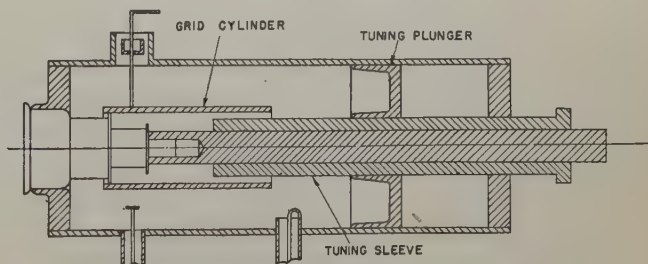


Fig. 13—Re-entrant oscillator with tuning device.

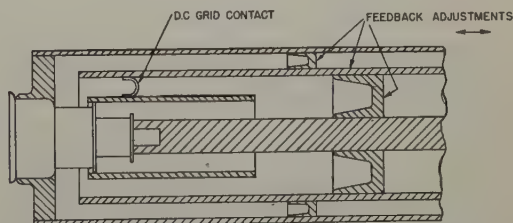


Fig. 14—A modified re-entrant oscillator.

a 2C43 is used, and with the plate if a 2C39 is used. An inner rod makes contact with the plate (or cathode in a 2C39 oscillator). Finger or capacitive plungers close up the resonant space. The grid connection consists of a

cylinder which clamps on the grid ring of the tube. A direct-current connection for grid bias is made at the proper point, or points, of the grid cylinder. The frequency of oscillation ordinarily depends upon the length of the grid cylinders; the plunger has to be placed at a

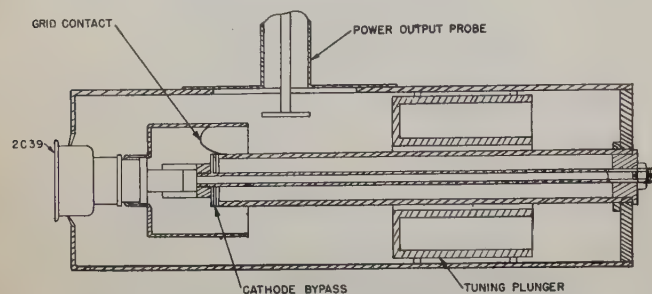


Fig. 15—A re-entrant oscillator using a 2C39 tube.

proper position to obtain optimum performance. There are, however, other modes of operation in which the plunger position is more important than the grid cylinder length in setting the frequency.

The following data represent physical dimensions of re-entrant oscillators built in this laboratory for different frequencies. Referring to Fig. 16:

$f$ (mega-cycles)	$l_g$ (inches)	$l_p$ (inches)	$l_d$ (inches)	$d_p$ (inches)	$d_o$ (inches)	$d_a$ (inches)
3300	1.37	2.0	1.17	9/16	13/16	1.5
3000	1.75	2.4	1.55	9/16	13/16	1.5
1500	3.8	6.2	2.35	9/16	13/16	1.5
1000	5.6	9.4	1.76	9/16	13/16	1.5

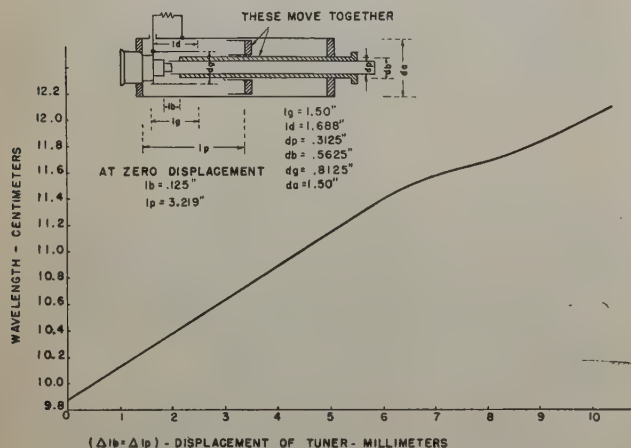


Fig. 16—Tuning curve and design dimensions for a re-entrant oscillator.

This circuit may be made tunable in various ways. For example, the grid cylinder may be made telescopic and its length varied as the plunger is moved. The motion along the anode of the plate-contact assembly including the plunger will also result in tuning (see Figs. 13, 16, and 17). For an increased tuning range, a plate extension as shown in Fig. 18 has been used in some practical applications. In Figs. 16 and 17, the tuning curves for re-entrant oscillators using the above method are given. Also are given the characteristic dimensions of the circuits. Fig. 13 shows an oscillator which has a moveable tuner on the plate rod and a separate plunger.

The tuner on the plate rod is first put as near the plate of the tube as possible and the plunger adjusted for optimum power. Then the plate tuner and the plunger are locked rigidly together and moved together for tuning. A tuning curve for such an oscillator and the

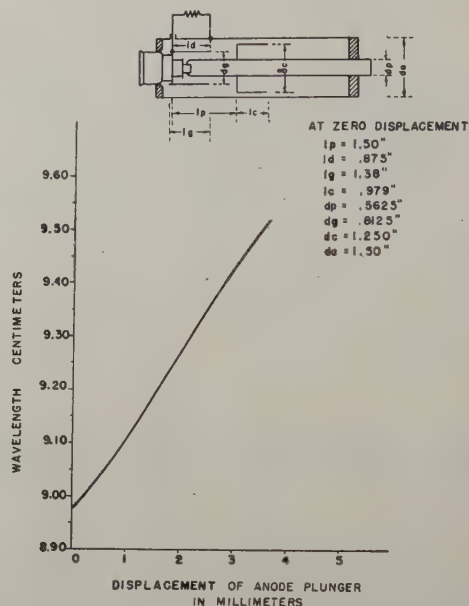


Fig. 17—Tuning curve and design dimensions for a re-entrant oscillator ( $\lambda = 9$  to 9.6 centimeters).

characteristic dimensions of the circuit are given in Fig. 17. The problem of power extraction is not much different from that in other circuits. It is of course one of matching the desired load impedance to the impedance that the oscillator requires for optimum operation. Points for power extraction will preferably be



Fig. 18—Plate extension.

chosen which are physically convenient. Several possibilities are shown in Fig. 13. For pulsed operation with the 2C43, the data given below are typical. Referring to Fig. 16:

$f$ (mega-cycles)	$l_p$ (inches)	$l_c$ (inches)	$l_d$ (inches)	$l_b$ (inches)	$d_a$ (inches)	$d_b$ (inches)	$d_c$ (inches)	$d_g$ (inches)
3300	2.7	2.28	2.2	5.4	1 15/32	9/16	1 1/4	13/16
3000	3.5	2.5	3.0	6.2	1 15/32	9/16	1 1/4	13/16

A plate voltage of 2.5 to 3 kilovolts peak was used with these circuits. A power output in the order 1 kilowatt may be obtained with 3 kilovolts on the plate, 1000 pulses per second, and 1 microsecond pulse duration (2C43 tube).

#### THEORETICAL STUDIES OF SOME OSCILLATORS

Analysis of oscillator performance at the lower radio frequencies commonly falls into two parts. In one part, which might be called the small-signal analysis,

the tube is assumed to have a known and constant transconductance and plate resistance, and the relation between these constants and the circuit constants required for oscillations is studied by Kirchoff's Laws. The analysis is primarily useful for determining the frequency of oscillation and the necessary circuit adjustments for oscillation to begin. It does not give information on the power output, efficiency, or best tube operating point, which all depend upon the solution of the large-signal problem. The second part of the analysis is done, usually approximately, from the tube characteristics in a manner similar to the calculation of the corresponding quantities for a power amplifier.

Our approach to the analysis of microwave oscillators using disk-seal tubes has followed this same general division. The large-signal problem for analysis of power output, efficiency, and the optimum operating conditions for a given tube should be solvable once the method of analysis for a power amplifier is known, and so our efforts in this direction have been concentrated on the power-amplifier problem, as described in another paper.<sup>2</sup> The small-signal analysis has, however, been applied to a number of microwave oscillators using disk-seal tubes, with a number of useful results. As explained above, this analysis should be most successful in giving information concerning the required relations between the circuit and the tube to give oscillation at a given frequency. Since information on frequency is given by the analysis, tuning and frequency-stability problems can also be studied. The quantitative study of certain of these problems will be described in this paper.

### A. General Equivalent Circuit

Representative types of microwave oscillators using disk-seal tubes with coaxial resonant circuits have been described earlier in this paper. In any of these, there may be a resonant tank circuit for the cathode-grid region, another for the grid-plate region, and a means for providing feedback between the two. The feedback may consist only of the inherent cathode-plate capacitance of the tube, or may be obtained by coupling loops, probes, or holes between cavities, a transmission line with matching controls connected between cavities, or the transmission line built into the circuit in such a way that there is a less-clear separation between the cavities and the feedback, as in the re-entrant oscillator. Several of these techniques have been sketched, for example, in Figs. 3, 4, and 11. The output load, if properly matched, may be coupled in at nearly any point in either cavity.

It is convenient to make a division of functions in order to place the study of all of the types of microwave triode oscillators on a common basis. The cavities and feed-back arrangements, including the tube interelectrode capacitances if desired, may be considered as a linear four-terminal network connecting the grid-plate

region of the tube to the cathode-grid region. The effects from the tube electronics are considered by including a certain input loading conductance  $g_i$  between cathode and grid terminals and a certain transadmittance<sup>3</sup>  $Y_m$  having both magnitude and phase angle, relating the alternating component of the grid-plate current to the alternating component of the cathode-grid voltage. More complete equivalent circuits for the tube electronics at high frequencies have been presented by Llewellyn and Peterson,<sup>4</sup> but the above quantities are usually sufficient for oscillator calculations. Some values of input loading and magnitude of transadmittance for typical disk-seal tubes have been measured by N.T. Lavoo.<sup>5</sup> The phase angle of transadmittance may be estimated from parallel-plane tube transit-time analysis,<sup>6</sup> since this quantity is believed to be least sensitive to differences between the actual and ideal triode that render calculated values of input conductance and magnitude of transadmittance unreliable. Finally, the output loading (plus losses in the cavity) is included as a conductance referred to the grid-plate terminals. The remaining interconnecting four-terminal network may then be considered as loss-free.

The resulting general equivalent circuit is shown in Fig. 19. The equations for the four-terminal network

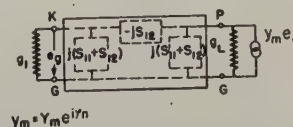


Fig. 19—General equivalent circuit for triode oscillator.

may be in any one of the many possible forms, making use of the impedance parameters, admittance parameters, the  $ABCD$  constants, the  $T$  or  $\pi$  equivalents, or any of the others. In Fig. 19, the network is interpreted in terms of the admittance parameters and  $\pi$  equivalent. This particular representation is convenient in identifying one portion of the network with the cathode-grid cavity, one with the grid-plate cavity, and one with feedback between cavities.

### B. The Oscillation Equation and Curves

The requirement for oscillation in the system of Fig. 19 can be set down in any one of the equivalent ways used for studying oscillators. One of these states that the voltage fed through the network to the cathode-grid terminals must be equal in magnitude and phase to the initially assumed cathode-grid voltage, or it can simply be said that the requirements that  $e_g$  and  $Y_m e_g$  be consistent in this closed system supply certain relations between the parameters of that sys-

<sup>3</sup> B. J. Thompson, "Review of ultra-high-frequency tube problems," *RCA Rev.*, vol. 3, pp. 146-155; October, 1938.

<sup>4</sup> F. B. Llewellyn and L. C. Peterson, "Vacuum-tube networks," *Proc. I.R.E.*, vol. 32, pp. 144-166; March, 1944.

<sup>5</sup> To be published in *Proc. I.R.E.*

<sup>6</sup> F. B. Llewellyn, "Electron-Inertia Effects," The University Press, Cambridge, Mass., 1941.

<sup>2</sup> H. W. Jamieson and J. R. Whinnery, "Power amplifiers with disk-seal tubes," *Proc. I.R.E.*, pp. 483-489; July, 1946.

tem. In general, two relations will result, one from real parts and one from imaginary parts of the equations.

$$\frac{s_{22}g_1}{Y_m^2} = \frac{\left(\frac{s_{12}}{Y_m}\right)\left[\left(\frac{s_{11}}{g_1}\right)\left(\frac{s_{12}}{Y_m}\right) - \left(\frac{s_{11}}{g_1}\right)\sin\phi_n - \cos\phi_n\right]}{1 + \left(\frac{s_{11}}{g_1}\right)^2} \quad (1)$$

$$\frac{g_L g_1}{Y_m^2} = \frac{\left(\frac{s_{12}}{Y_m}\right)\left[\sin\phi_n - \left(\frac{s_{11}}{g_1}\right)\cos\phi_n - \left(\frac{s_{12}}{Y_m}\right)\right]}{1 + \left(\frac{s_{11}}{g_1}\right)^2} \quad (2)$$

Equation (1) can be thought of as the tuning or frequency-determining equation. Actually, frequency is expressed only implicitly by the equation, the equation being written in such a way that  $s_{22}$ , the output susceptance, is given in terms of the other tube and circuit parameters.

Equation (2) relates the allowed load conductance to the tube and circuit parameters. The circuit will operate as an oscillator only over the region for which  $g_L$  is positive, since a negative value of  $g_L$  would mean that an additional external radio-frequency power source would have to be supplied to make the circuit self-consistent.

### C. Application to the Re-entrant Oscillator

1. *Equivalent Circuit:* Typical re-entrant oscillators were discussed earlier and shown for example in Fig. 13. For purposes of discussing an equivalent circuit, let us divide this into several regions, as indicated in Fig. 20, which shows the significant radio-frequency aspects

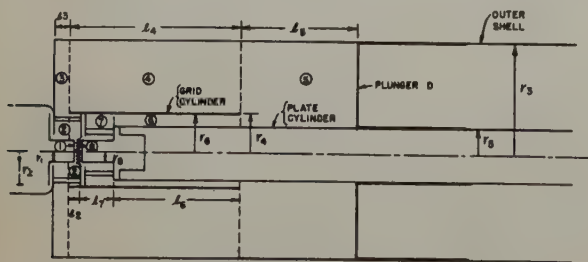


Fig. 20—Re-entrant oscillator.

of the oscillator simplified by the omission of direct-current connections, by-passes, and chokes. The regions are:

- (1) Input region of the tube between cathode face and grid
- (2) Region between cathode post of the tube and grid cylinder, up to end of grid cylinder
- (3) Region between cathode post and outer shell, from end of grid cylinder to end of cavity
- (4) Region between grid cylinder and outer shell
- (5) Region between plate cylinder and outer shell, from end of grid cylinder to plunger

- (6) Region between plate rod and grid cylinder, up to plate disk of tube
- (7) Region between anode post of tube and grid cylinder
- (8) Output region of tube between grid and face of anode.

The most elementary equivalent circuit for a basic re-entrant oscillator, neglecting all end effects, is shown in Fig. 21. Here a transmission line represents the cathode-grid cavity (4) of Fig. 20, a second transmission line represents the grid-plate cavity (6), and a



Fig. 21—Elementary equivalent circuit for a basic re-entrant oscillator.

third transmission line in series between the other two represents the region (5) between the end of the grid cylinder and the plunger.

A more complete equivalent circuit is shown in Fig. 22. The elements in the equivalent circuit correspond-

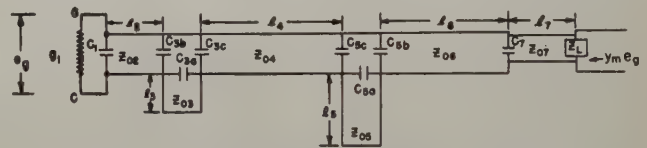


Fig. 22—A more complete equivalent circuit for a re-entrant oscillator.

ing to the divisions of Fig. 20 are given below, with methods for obtaining approximate element values.

1. The input of the tube presents electronic loading conductance  $g_1$  and hot capacitance  $C_1$  between cathode face and grid. For a 2C40,  $C_1$  is believed to be in the range of 1 to 1.5 micromicrofarad<sup>7</sup> and  $1/g_1$  in the range 200 to 400 ohms for small signals in the 10-centimeter region.

2. A small section of line of length  $l_2$  and characteristic impedance  $Z_{02} = 60 \ln(r_2/r_1)$ .

3. A small section of the line of length  $l_3$  and characteristic impedance  $Z_{03} = 60 \ln(r_3/r_1)$  in series between (2) and (4). Discontinuity capacitances<sup>8</sup>  $C_{3b}$ ,  $C_{3a}$ ,  $C_{3c}$  are shunted across lines 2, 3, and 4, respectively, at the junction. These capacitances are usually of the order of from a few tenths to one micromicrofarad. It should be noted that  $C_{3a}$  is negative.

4. Section of transmission line of length  $l_4$ , characteristic impedance  $Z_{04} = 60 \ln(r_3/r_4)$ .

5. Transmission line of length  $l_5$ , characteristic impedance  $Z_{05} = 60 \ln(r_3/r_5)$  in series with lines (4) and (6). Again discontinuity capacitances  $C_{5c}$ ,  $C_{5a}$ ,  $C_{5b}$  are

<sup>7</sup> Note that  $C_1$  does not correspond exactly to published values of input capacitance, since  $C_1$  is only the capacitance between grid and face of cathode plus a fringing or discontinuity capacitance.

<sup>8</sup> J. R. Whinnery and H. W. Jamieson, "Coaxial-line discontinuities," Proc. I.R.E., vol. 32, pp. 695-709; November, 1944.

shunted across lines 4, 5, and 6, respectively (see 3 above). These are usually of the order of a few tenths of a micromicrofarad, and  $C_{5a}$  is negative.

6. Transmission line of length  $l_6$ , characteristic impedance  $Z_{06} = 60 \ln(r_6/r_5)$ .

7. Transmission line of length  $l_7$ , characteristic impedance  $Z_{07} = 60 \ln(r_6/r_a)$ . A discontinuity capacitance  $C_7$  is shunted across the junction between lines (6) and (7).

8. Here load impedance  $Z_L$  (this is the quantity calculated) is shunted between grid and plate and the  $Y_{me}$  current generator is applied.

Once the equivalent circuit is set up, and a frequency selected, it can be reduced if desired to an equivalent network and the general results of (1) and (2) applied.<sup>9</sup> In the procedure usually followed, we select the complete set of dimensions in the oscillator and calculate the equivalent circuit. The approximate wavelength of oscillation is estimated either from experience or from an approximate equation to be given later. Several values of wavelength are then taken in this vicinity, and the value of load conductance and susceptance  $s_{22}$  are calculated, say by (1) and (2). The wavelength of oscillation is that for which the calculated value of  $s_{22}$  from (1) corresponds to the known value in the actual equivalent circuit of the cavity plus grid-plate capacitance. The oscillator will oscillate at this frequency only if the value of  $g_L$  calculated from (2) is at the same time positive. If desired, we may now go back and change some dimensions in the oscillator, set down the new equivalent circuit, and repeat the procedure, resulting in the new wavelength of oscillation and allowable load conductance. When enough points are obtained in this manner, the two quantities, frequency and allowable load, may be plotted as a function of the parameter which is being varied.

**2. Some Conclusions from the Re-entrant Analysis:** Study of the equivalent circuits and application to specific oscillators has led to certain general important conclusions. The most important probably is the division into functions; that is, it is found that in most oscillators studied, proportions are such that the regions 6, 7, and 8 of Fig. 20 essentially constitute the output tank circuit, so that tuning arrangements in this region are most effective. The remainder of the circuit has its main effect in changing feedback, thus explaining why varying the plunger  $D$  does affect strength of oscillations markedly, but is not as effective in tuning as might be expected. These and other conclusions are explained in more detail in the following:

*a. The tank circuit and approximate determination of oscillator frequency:* Because of the large reflection due to the series line (5) (see Fig. 20) there is a large standing wave in the line (6), and so this line, the region (7) around the plate post of the tube, and the grid-plate

capacitance contain most of the circulating energy, and so constitute the major part of the tank circuit. Thus the high-impedance point appears in the line (6) near the end of the grid cylinder. A rough approximation to the wavelength of the oscillator may then be had by assuming that this is an infinite impedance point; that is, the line (6) may be considered as a half-wave resonant line foreshortened by the tube inductance and capacitance. Approximate resonant frequency may then be obtained by solving the transcendental equation:

$$\left( \omega L_7 - \frac{1}{\omega C_8} \right) = Z_{06} \cot \beta l_6 \quad (3)$$

where

$L_7$  = inductance =  $2 \times 10^{-9} l_7 \ln(r_6/r_8)$  henry

$C_8$  = output capacitance (order of  $1.7 \times 10^{-12} Fd$ )

$Z_{06} = 60 \ln(r_6/r_5)$  ohms

$\beta$  = phase constant =  $\omega/3 \times 10^{10}$

$l_7$  and  $l_6$  being measured in centimeters.

Some comparisons are shown in Table I in which measured wavelengths are compared with values calculated from (3). The measurements were made on

TABLE I

$\lambda$ (measured)	$l_6$ (centimeter)	$r_5$ (centimeter)	$r_6$ (centimeter)	$r_a$ (centimeter)	$l_7$ (centimeter)	$\lambda$ calculated from Eq. (3)
9.2	2.08	0.714	1.032	0.275	1.03	11.35
10	3.05	0.714	1.032	0.275	1.03	12.6
20	8.25	0.714	1.032	0.275	1.03	19.95
29	12.80	0.714	1.032	0.275	1.03	28.35
9.1 pulsed	1.30	0.714	1.032	0.275	1.03	9.9
10 pulsed	2.10	0.714	1.032	0.275	1.03	11.35

several different oscillators with dimensions given earlier. The above comparisons stress the fact that this is to be considered only as an approximation. The values for the long wavelengths (20 and 29 centimeters), provide good checks, however.

*b. The feed-back circuit:* With the regions 6, 7, and 8 of Fig. 20 functioning mainly as the tank circuit, the remainder of the circuit acts primarily as the feed-back circuit. To be certain of optimum conditions for oscillation, both magnitude and phase of feedback should be under control, requiring two independent variable parameters in this feed-back circuit. The single plunger  $D$  is often the only variable control provided, and this may affect both phase and magnitude of feedback, but not independently.

Calculations made from the equivalent circuit with plunger position varied give curves of the form indicated in Fig. 23. Here wavelength and allowable load conductance calculated as described earlier are plotted as functions of plunger position. The range over which load conductance is positive is the range of plunger positions over which oscillations may exist, since the oscillator should stop oscillating on either side when allowable load conductance becomes negative. Of course, if there is already a fixed conductance  $g_{L0}$  (referred to the grid-plate region of the circuit) arising from circuit

<sup>9</sup> This is not necessary, of course, because all oscillation calculations can be performed directly on the equivalent circuit as originally set up in Fig. 22, if desired.

losses or external loading, the range of oscillations will then be only over the region for which  $g_L$  exceeds  $g_{L0}$ , as is also indicated in Fig. 23.

The maximum value of  $g_L$  found on curves as in Fig. 23 may be used as a rough indication of the excellence of an oscillator, for it does at least indicate the amount that the oscillator could be loaded and yet oscillate. (A true comparison requires solution of the large-signal

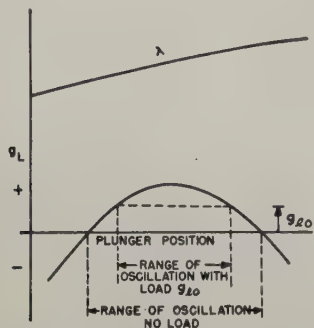


Fig. 23—Wavelength and allowable load conductance plotted as functions of plunger position.

problem.) Thus, if some parameter in the feed-back circuit, independent of plunger setting, is changed, different curves of  $g_L$  versus plunger position will be obtained, some having higher peaks than others and so probably representing better oscillators. This is indicated qualitatively in Fig. 24.

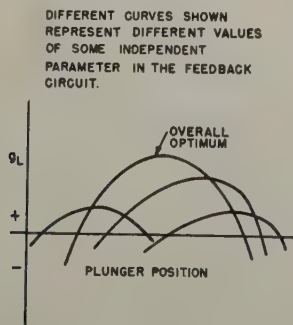


Fig. 24—Load conductance as a function of plunger position with some quantity in the feed-back circuit as a parameter.

It follows from the above considerations that adjustment in the feed-back circuit of any parameter independent of the plunger position may have these effects:

1. Slight change in wavelength of oscillation.
2. Change in plunger position for maximum output.
3. Marked effect on value of output at optimum plunger setting.

In practical oscillators, these effects have been especially noticeable when numbers and positions of grid cylinder supports were changed. In certain experimental oscillators, another independent plunger control on the cathode side of the cavity was also provided to give a second independent parameter in the feed-back circuit.

In Fig. 25 are shown some curves similar to those in Fig. 23, but actually calculated by the methods de-

scribed earlier for a given 30-centimeter re-entrant built in this laboratory. The range of plunger positions over which the actual oscillator operated was not measured accurately, but agreed within a few millimeters with the calculated range. The calculated change in wavelength over this range also agreed closely with the experimental. The setting of plunger maximum output agreed within 3 millimeters with that calculated. In

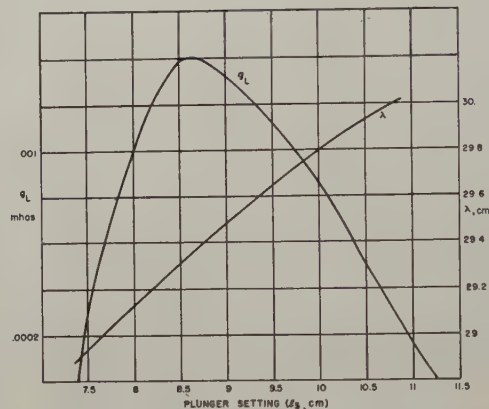


Fig. 25—Wavelength and allowable load conductance as functions of plunger position calculated for a 30-centimeter re-entrant oscillator.

Fig. 26 the length of the grid-cathode line  $l_4$  was selected as a parameter to vary in the calculations. (Although it could not be conveniently varied physically by itself, it served to prove the point in the calculations.) The different curves with different optimum plunger settings and values of maxima at these settings are observed as

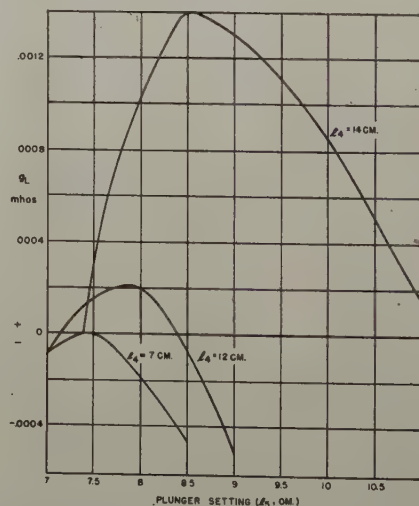


Fig. 26—Load conductance as a function of plunger setting with grid-cathode line  $l_4$  as a parameter.

described qualitatively above. It should be noted that there is no plunger setting for which the case with  $l_4 = 7$  centimeters would oscillate.

*c. Tuning:* It has been pointed out that tuning operations which act on the region of the plate-grid line are much more effective than in other parts of the circuit because of the relatively large standing wave in this region. As a matter of fact, the rough formula for

resonant wavelength given in (3) is dependent only upon that region.

As an example of relative effectiveness of quantities on the input and output, several curves are shown. In Fig. 27, calculated curves of wavelength as a function of cathode-grid capacity and of grid-plate capacitance are shown for the same 30-centimeter re-entrant used as an example above. The grid-plate capacitance is thus

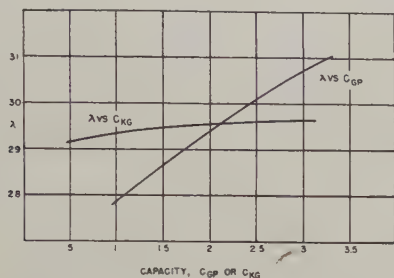


Fig. 27—Calculated curves of wavelength as a function of cathode-grid capacitance and of grid-plate capacitance.

seen to be roughly five times as effective in tuning as the cathode-grid capacitance. Also in Figs. 28 and 29 are shown points taken with a great many tubes in a re-entrant oscillator used for acceptance tests by the Electronic Tube Division of this Company. Operating wavelength as a function of measured grid-plate capacitance is shown in Fig. 28, and as function of measured cathode-

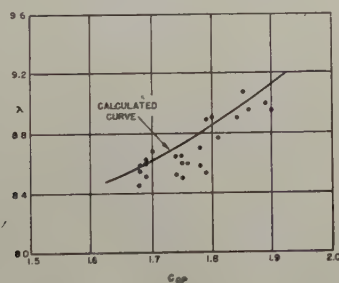


Fig. 28—Operating wavelength as a function of measured grid-plate capacitance.

grid capacitance in Fig. 29. In Fig. 28 is also shown a calculated curve using the equivalent circuit. Although the data is of a gunshot nature, agreement is satisfactory, and again the relative effectiveness of grid-plate capacitance over cathode-grid capacitance is demonstrated.

Although tube capacitances have been used as examples, other quantities give similar conclusions on relative effectiveness of tuning in the plate-grid region and in other parts of the circuit. For example, polystyrene disks movable in position have been found much more effective in tuning when placed in the plate line (6) of Fig. 20 than when placed in the grid line (4).

The following tuning means have been used in practice, all of which act on the region in the vicinity of the plate line:

1. Introduction of dielectric in plate-grid line movable in position.

2. Grid cylinder adjustable in length.

3. Sliding plate cylinder on plate cap of tube, thus adding a gap which may be considered as adding additional series inductance to this part of the circuit (Fig. 13).

Although the plunger *D* of Fig. 20 does give some tuning, it is not an ideal tuning means because:

1. Tuning range is relatively small (order of 3 to 6 millimeters in the 10-centimeter range).

2. Since the major function of the plunger is adjustment of feedback to an optimum, its use for tuning is not satisfactory.

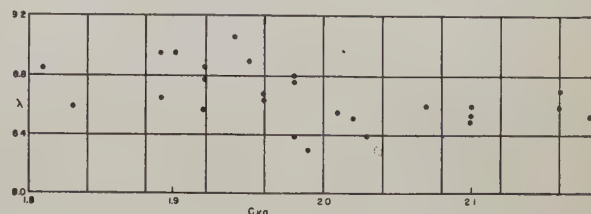


Fig. 29—Operating wavelength as a function of measured cathode-grid capacitance.

Oscillators have been made in which the range of tuning obtainable with the plunger alone is increased. These have larger-diameter grid cylinders, so that there is then not so much of a discontinuity between lines 5 and 6. More of the circulating tank-circuit energy is then in the region 5 and so may be affected by the plunger position.

*d. Frequency shift with voltage:* Analyses performed on re-entrant oscillators have revealed one major contribution to frequency shift with changes in voltage. This arises from the change in the phase angle of transadmittance with tube voltages. That is, the phase

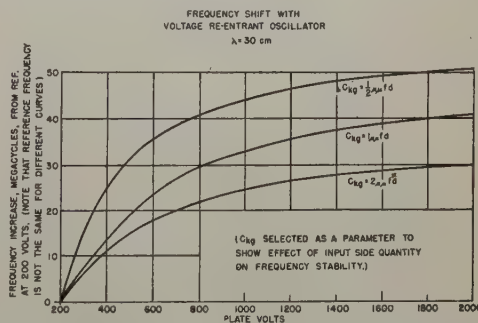


Fig. 30—Frequency shift with voltage re-entrant oscillator ( $\lambda = 30$  centimeters).

delay in the tube due to finite transit time of electrons is a definite part of the over-all phase shift about the circuit. As voltages change, so do the transit times, and therefore the phase delay through the tube (phase angle of transadmittance). Frequency must then shift slightly to maintain the phase through the feed-back path proper for oscillations. In Fig. 30 are plotted some curves of frequency shift with voltage calculated for a given 30-centimeter re-entrant including only this factor, and with several parameters varied. Measured

curves of frequency shift with voltage in this laboratory resulted in curves which were of the same general shape, had frequency changing in the same direction (increasing frequency with increasing voltage), and were of the same order of magnitude of frequency shift, although the measured amounts seemed to vary from time to time depending upon contacts, circuit adjustments, and other doubtful factors.

Note that, although operating frequency is determined mainly by quantities on the plate-grid side of the cavity, the amount of *frequency shift* with voltage may in certain cases be influenced more by quantities on the cathode-grid side. This is because the effect arises from changes in phase in the tube which must be compensated for by a slight change in frequency to give the corresponding change in phase around the entire feedback circuit—an effect which is thus influenced by the whole feed-back circuit. Among calculated examples and measured frequency shifts, many have been observed in which change of a parameter on the cathode-grid side affected the amount of frequency shift with voltage more than the corresponding change on the grid-plate side.

*e. Readjustment of plunger from continuous-wave to pulse operation:* It has often been noted experimentally that if the plunger position was set for optimum operation under continuous-wave conditions, the oscillator would not operate at its optimum under pulse conditions (or might not even oscillate at all) unless the plunger were readjusted. This phenomenon is also revealed theoretically in the above analysis and is traceable to the change in phase angle of transadmittance with voltage, as was the frequency shift with voltage described above. Thus, if plots of allowable load conductance versus plunger position are made with two values of phase angle of transadmittance, one corresponding to a continuous-wave and one to a pulse voltage, two curves may be obtained somewhat as indicated in Fig. 31. The

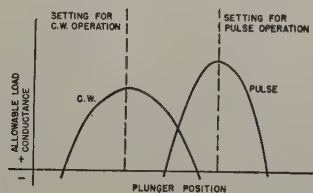


Fig. 31—Required plunger settings per optimum operation for continuous-wave and pulse operation.

required plunger shift from continuous-wave to pulse operation is shown by such curves. Although only a few cases were checked analytically, calculated, and observed, plunger shifts were in the same direction and agreed very well in magnitude. Phase angles of transadmittance were estimated from parallel-plane tube theory.

*f. Change in frequency with filament voltage:* In addition to frequency changes with plate voltage, it is known that there is a frequency shift occurring with changes in filament temperature. This shift is traceable to the

changes in interelectrode spacings with temperature, a factor which can effect frequency in at least four ways:

1. Through change in grid-plate capacitance.
2. Through change in cathode-grid capacitance.
3. Through change in phase angle of transadmittance affecting frequency through the mechanism described above.
4. Through change in cathode-plate capacitance, affecting feedback.

Curves of the three interelectrode capacitances (measured at low frequency) versus filament voltage were measured for a number of tubes, and typical curves are reproduced in Fig. 32.<sup>10</sup> It is seen that grid-plate capacitance decreased with increasing temperature, cathode-

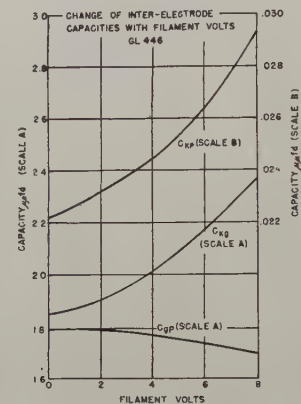


Fig. 32—The three interelectrode capacitances (measured at low frequency) as functions of filament voltage.

grid capacitance increased at a more rapid rate, and cathode-plate capacitance also increased, suggesting that there was both expansion in the cathode post and a motion of the grid screen away from the plate. The decrease in  $C_{gp}$  tended to increase frequency while the increase in  $C_{kg}$  tended to decrease frequency, in this case giving a remarkable degree of compensation. (Although the change in  $C_{kg}$  is considerably greater than the change in  $C_{gp}$ , its relative effectiveness on frequency is considerably less.)

Calculations indicated that the third factor (change in phase of transadmittance) was appreciable, although considerably less than either of the first two factors. This also acts to increase frequency with increasing temperature, since the spacing of the low-voltage (cathode-grid) region is decreased, thus giving the same direction of effect as an increase in voltage (see Fig. 30).

The importance of the fourth effect is not known, since  $C_{kp}$  has not been included in re-entrant analyses except for spot checks to justify its neglect, but it is believed to be small.

A rough calculation was carried through for frequency shift versus filament voltage, including only the first three factors and utilizing the data of Fig. 32. Frequency shift versus  $C_{gp}$  was estimated from curves on several re-entrant oscillators calculated and measured

<sup>10</sup> These are not representative of present production types.

in the 10-centimeter range; such as that of Fig. 28. It was assumed that the relative effectiveness of a given change in  $C_{kg}$  and  $C_{gp}$  was the same as found in Fig. 27, that is, about 1 to 5.5. It was also assumed that the changes in  $C_{kg}$  and  $C_{gp}$  represented proportional changes in interelectrode spacings, so that changes in transit angle could be calculated and the corresponding frequency changes estimated from Fig. 30. Finally, the total shift was taken as the sum of the above three components. This information is summarized in Table II. Here the compensation between the several factors is evident for changes due to  $C_{gp}$  alone or  $C_{kg}$  alone would give shifts of the order of 75 megacycles over the entire range, while the net shift is only 8.5 megacycles.

TABLE II

Filament volts	$\Delta C_{kg}$ (in micro-farads)	$\Delta f$ due to $\Delta C_{kg}$ (in mega-cycles)	$\Delta C_{gp}$ (in micro-farads)	$\Delta f$ due to $\Delta C_{gp}$ (in mega-cycles)	$\Delta f$ due to phase of $Y_m$ (in mega-cycles)	Total $\Delta f$ (in mega-cycles)
0	0	0	0	0	0	0
2	0.06	-8.1	-0.005	3.75	1.7	-6.4
4	0.21	-28.5	-0.02	15	3.5	-10
6	0.35	-47.6	-0.05	37.5	6.0	-4.1
8	0.56	-76.0	-0.10	75	9.5	8.5

The net shift from Table II is plotted versus filament volts in Fig. 33, together with a curve of measured frequency shift taken on an 11-centimeter re-entrant with

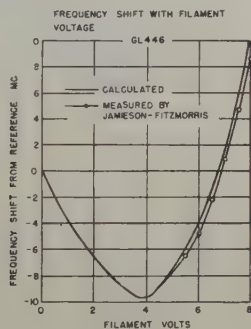


Fig. 33—Frequency shift as a function of filament voltage.

GL-446 tube. The closeness of agreement is surprising, considering that no maneuvering was done to make the agreement look good. Actually it is much better than is justified by the accuracy of the method. However, it does lend support to the factors chosen as important factors in this shift, and brings out these conclusions for the example studied:

1. Frequency shifts due to changes in  $C_{kg}$  and  $C_{gp}$  were both large, of the same order of magnitude, and opposite in direction, so that compensation occurred to a large extent.

2. Frequency shift from change in phase angle of transadmittance with changing spacings was smaller than either of the above changes alone, but was comparable to the net frequency shift because of the partial cancellation of the two large changes.

3. Since net frequency shift is the difference between fairly large shifts of opposite sign, changes in the circuit

or tubes may change the character of the net shift considerably. Thus frequency might increase with filament voltage over the entire range, decrease, or change direction somewhere in the range, as in the calculated curve of Fig. 33 where this change occurs at about 4 volts. With only slight changes in the conditions, the voltage corresponding to this change in direction could change considerably.

#### D. General Study of Frequency Stability and Tuning

Some detailed studies have been made of the dependence of oscillator frequency on small changes in the parameters of various parts of the oscillator circuit in order to investigate the effect of various quantities on frequency stability and tuning. The procedure followed was the obtaining of the total differential  $dS_{22}$  from the "frequency-determining" (1) in terms of differential changes in all other parameters. Each differential change is in turn divided into a part representing an independent variation in that parameter, and a second part representing the change because of frequency shift (if the particular element is frequency sensitive). From the resulting equation, the per-unit frequency shift can be solved for in terms of the per-unit changes of the various circuit and tube parameters. The procedure is straightforward, but the resulting equations are long enough so that space limitations do not permit their reproduction here. Conclusions derived from such studies can perhaps best be given by discussion of results from one particular study.

In the study of a certain pulsed 12-centimeter re-entrant oscillator by the above described procedure, it was found that the grid-plate capacitance was the most important factor in affecting frequency, an increase of 1 per cent in this capacitance causing roughly a 0.2 per cent decrease in frequency. The coefficient showing the importance of the part of  $S_{22}$  external to the tube was very nearly as large as that from the grid-plate capacitance, as might be expected. The next most important factors were the feed-back susceptance  $S_{12}$  external to the tube and the phase angle of transadmittance, 1 per cent changes in these causing roughly 0.03 per cent frequency shifts. The cathode-grid capacitance has about one tenth the effect of the grid-plate capacitance. The cathode-plate capacitance was of small importance in affecting frequency because its effects were masked by the larger feedback external to the tube in this re-entrant design. Magnitude of transadmittance and the tube input conductance were also found to have little effect on frequency, as might be guessed, and the input susceptance external to the tube was also ineffective because of the larger susceptance from cathode-grid capacitance which paralleled it.

If the above results were to be used in studying frequency stability under changes of voltage, load, temperature, etc., the changes of interest would first have to be converted to corresponding changes in elements of the equivalent circuit.

# Microwave Omnidirectional Antennas\*

HENRY J. RIBLET†, ASSOCIATE, I.R.E.

**Summary**—This paper describes briefly a number of radiating elements which have proved useful in the design of high- and low-gain omnidirectional antennas at wavelengths of 3 and 10 centimeters.

## INTRODUCTION

THE SUCCESS of microwave radars for military purposes has induced considerable interest in the development of omnidirectional microwave antennas. During the war these antennas, in conjunction with suitable transponders, were used for homing and identification purposes, and now they are seeing service with communication systems.

Active work on microwave omnidirectional antennas began at the Radiation Laboratory in 1942. A large number of individuals have contributed to the development of these antennas. This paper summarizes the work done at the Radiation Laboratory and is, therefore, a report on the activities of a group of men in the Laboratory rather than an account of the efforts of the author alone.

The use of centimeter wavelengths places a different emphasis on design than is encountered in low-frequency applications, so that, in deciding the place of these antennas in the general art, the following considerations should be kept in mind:

(a) High frequencies involve close tolerances when measured in inches, so that designs which are inherently reproducible are required.

(b) High-gain antennas can be made without excessive size, and this fact, coupled with relatively narrow radar bandwidths, shifts the design emphasis in many cases from bandwidth to pattern quality.

(c) Antennas with gains, relative to an isotropic antenna, ranging from  $3/2$  to 20 are of importance, so that a premium is placed on radiating elements or bays with easily variable impedances.

(d) Feed lines are large compared with radiating elements, so that older antenna forms usually have to be materially altered in order to be usable.

(e) Pressurizing and weatherizing means are electrically large, and hence become an integral part of the design.

The first efforts toward high-gain microwave omnidirectional antennas were in experiments with a biconical horn antenna.<sup>1</sup> However, it was soon realized that, for

the narrow-band problems encountered in radar practice, an array arrangement was much more economical of space and was more easily weatherized.

The considerations listed above have placed an emphasis on the development of a radiating element which, for a given polarization and frequency, has the following properties: (a) gives by itself a uniform azimuth pattern; (b) is small, mechanically reliable, and susceptible of quantity production; (c) when coupled to a line, has an impedance, either pure series or shunt, which is sufficiently flexible so that it can be used in a single-element antenna or in a high-gain antenna with many bays; and (d) has by itself an elevation pattern which does not differ too greatly from that of a dipole.

When an element is found which satisfies these conditions, for a given application, the design of arrays with a multiplicity of elements is straightforward. In all the applications which have been encountered, the peak of the radiation pattern should be on or near the horizontal plane. This condition is insured if pure series or shunt elements are spaced  $\lambda g/2$  apart on the feed line. In this case the pattern can easily be calculated and the impedance behavior is simple, since, if the elements are in shunt or series on the line and are spaced  $\lambda g/2$  apart, it will be found that they are essentially in parallel or series with each other and with the shorting stub.

## GROUND AND SHIP ANTENNAS

Fig. 1 shows a die-cast element which has been very useful for horizontal polarization in the 10-centimeter

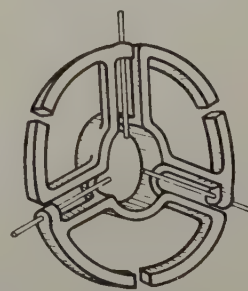


Fig. 1—Tridipole element.

band. It illustrates well the properties which a microwave omnidirectional element should possess. Its azimuthal pattern has a uniformity which is better than 2 to 1 in power<sup>2</sup> from 9 to 11 centimeters. It has been possible to eliminate the need for insulators or dielectric supports by the use of three-wire feeds. The flexibility of impedance is made possible by the fact that the exciting pins may be fastened directly to the inner conduc-

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† Formerly, Radiation Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts; now, Submarine Signal Company, Boston, Massachusetts.

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<sup>1</sup> W. L. Barrow, L. J. Chu, and J. J. Jansen, "Biconical electromagnetic horns," *Proc. I.R.E.*, pp. 769-779; December, 1939.

<sup>2</sup> All pattern data will be given in terms of power.

tor or capacitively coupled thereto, Fig. 2 shows a completed antenna using these elements. It has separate

ments, capacitive coupling has been convenient. In the latter case, over a 2 per cent band the standing-wave

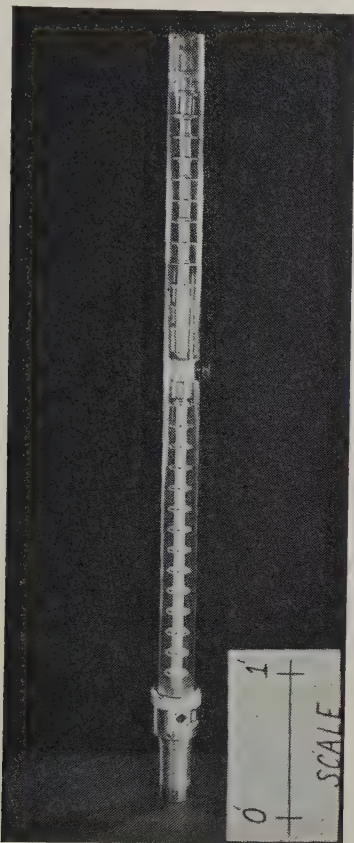


Fig. 2—Tridipole omnidirectional antenna.

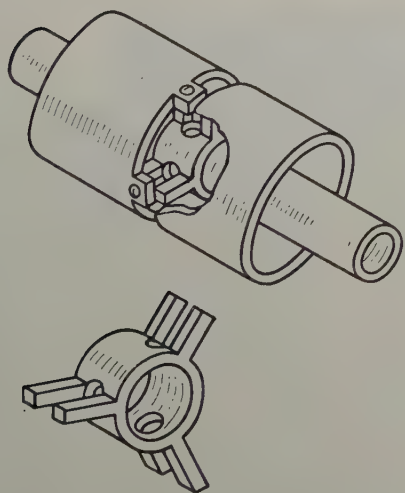


Fig. 3—Cylindrical element.

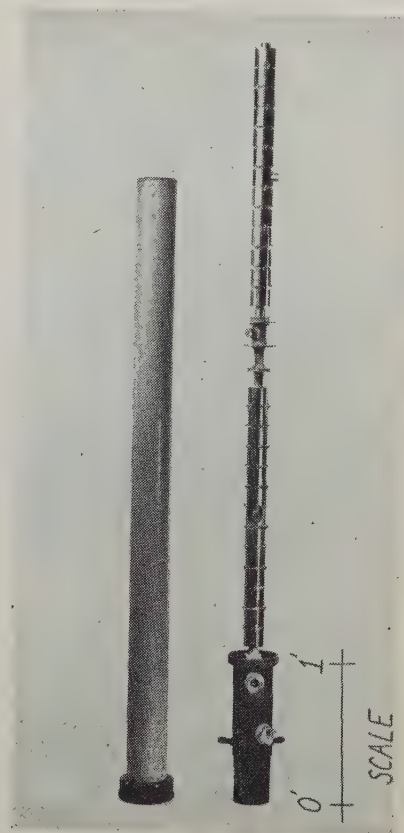


Fig. 4—Omnidirectional antenna using  $\lambda/2$  cylinders.

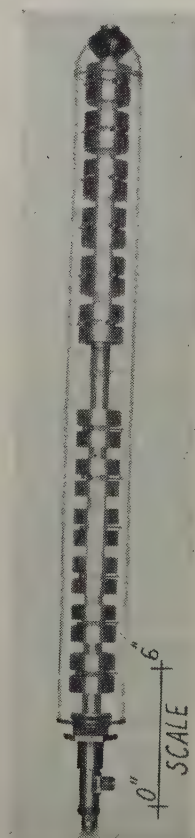


Fig. 5—Omnidirectional antenna using  $\lambda/4$  cylinders.

transmitting and receiving sections. Experience indicates that one, two, or three elements may be directly coupled to the line with reasonable bandwidths, i.e., standing-wave ratio  $< 1.4$  over a 10 per cent frequency band.<sup>3</sup> For an antenna with as many as fourteen ele-

<sup>3</sup> All standing-wave ratios are measured in terms of voltage.

ratio  $< 1.4$ , while its elevation pattern, which has a half width of 8 degrees and side lobes less than 5 per cent, is unchanged. These elements have been die-cast in three sizes for the 10-centimeter band and in two sizes for 1000 and 700 megacycles. The latter have been provided with capacitive stubs at both the input terminals and ends of the dipoles, making possible a reduction in the relative diameter of the elements and greater pattern uniformity.

For vertical polarization in the 10-centimeter band, the basic idea shown in Fig. 3 has proved to be satisfactory. This employs three-wire lines exciting radiating cylinders. These cylinders may be either a half-wavelength long and fed from both ends, as shown in Fig. 4, or less than a quarter-wavelength and fed centrally in pairs, as shown in Fig. 5. It has been found that each of these schemes has its own advantages. For a six-element antenna having solid cylinders the standing-wave ratio  $< 1.2$  from 9.6 to 11 centimeters. Fig. 6 gives the elevation patterns for this antenna over a band of frequen-

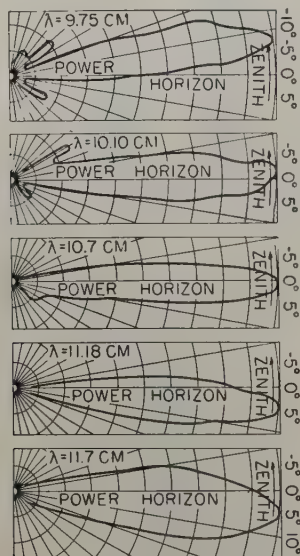


Fig. 6—Elevation patterns of a six-element antenna.

cies. The split elements were found to enjoy the advantage that a single element could be used in six-element arrays at all frequencies over a 20 per cent band centered at 10 centimeters, with a standing-wave ratio less than 1.4 over a 5 per cent band. This, of course, is not true for the solid cylinders, where the length of the cylinders varies of necessity with the frequency. A satisfactory arrangement for low-gain antenna design is shown in Fig. 7. Its elevation patterns, taken at 9, 10, and 11 centimeters, show a half width of approximately 50 degrees. The standing-wave ratio of this antenna, over a 20 per cent frequency band, is less than 1.25, while its azimuthal patterns have a three-fold symmetry with a ratio of maximum to minimum power of 1.6 to 1. Elevation patterns for the higher-gain antennas are determined by the ratio of over-all length to wavelength,

and thus are independent of polarization and element geometry.

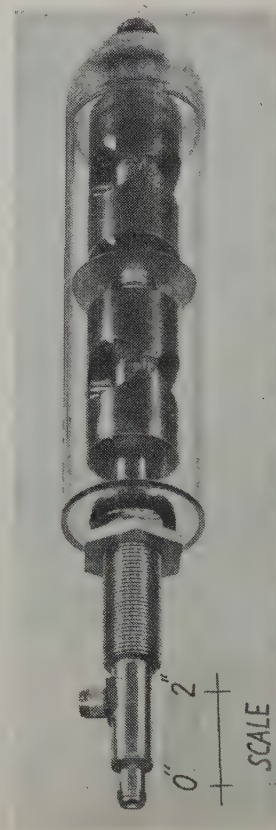


Fig. 7—Low-gain vertically polarized antenna.

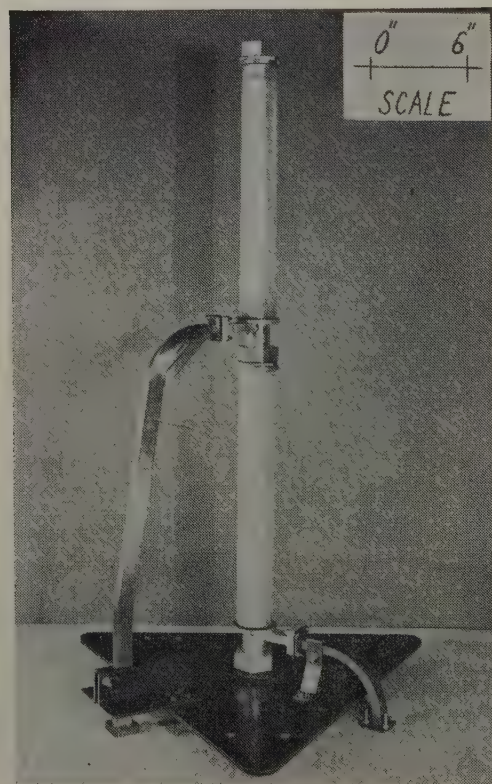


Fig. 8— $E_0$  wave-guide omnidirectional antenna.

Omnidirectional antennas at 3 centimeters have been horizontally polarized almost exclusively. At this frequency, tolerances are so close and feeding lines of practical size are relatively so large that attempts to build this type of antenna with dipoles have on several occasions in the past led to considerable difficulty.

Fig. 8 shows a completed 3-centimeter-band omnidirectional antenna. The upper of the two antennas acts as

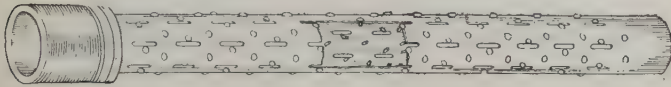


Fig. 9—Slotted  $E_0$  wave guide.

a receiving antenna, while the lower acts as a transmitting antenna. Fig. 9 is a sketch of the radiating portion of the antenna. It consists of twelve bays or elements, each composed of seven vertical slots cut symmetrically in the circumference of 1- to 1/4-inch wave guide. These slots are approximately one-half wavelength long and are excited by pins on one edge projecting radially into the guide. The elements are reversed every half guide wavelength by alternately placing the pins on different sides of the slots. An additional set of

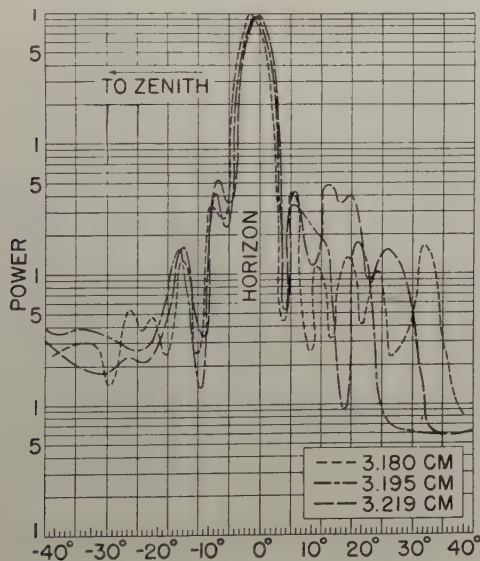


Fig. 10—Elevation patterns of twelve-bay 3-centimeter antenna.

matching pins is placed midway between the radiating elements for the purpose of increasing the bandwidth of the antenna. Not shown is a purely reactive terminating plug at the end of the antenna. The  $TM_{(0,1)}$  mode is excited by a special converter which in addition acts as a transition from rectangular to round wave guide. This mode is used, of course, because of its symmetric radial electric field.

Antennas using the principle just described have been built in 3-, 6-, and 12-bay sizes. The azimuth pattern of a typical 12-bay antenna is essentially a circle. Fig. 10

gives the elevation patterns of a 12-bay antenna over the operating frequency band. Fig. 11 shows the standing-wave ratio as a function of frequency for both a 12- and 3-bay antenna. To avoid the need for an external feeding line, a series of antennas using a double coaxial feeding line have been designed. These use either five or six slots on a 1-inch coaxial line with alternate elements staggered.

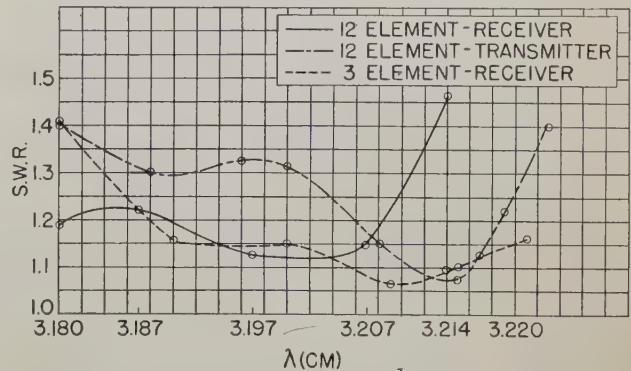


Fig. 11—Standing-wave ratio of slotted  $E_0$  wave-guide antennas.

It should be stated that the designer of a high-gain microwave omnidirectional antenna of the array type should have no difficulty in obtaining at least 90 per cent of the maximum theoretical gain available from a uniform line-current source of length  $L$ , where  $L$  equals the number of radiating elements times their spacing. This gain  $G$  is given by the expression

$$G = \frac{2L}{\lambda}$$

# AIRBORNE ANTENNAS

The antennas described thus far have been used, principally, either in ground or ship installations. For fast aircraft the problem is different. Here, an enormous

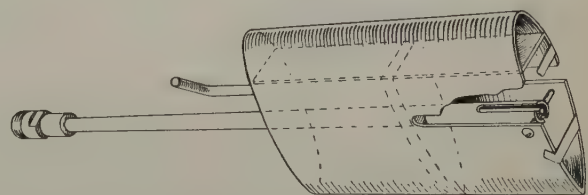


Fig. 12—Horizontally polarized half-slot antenna.

premium is placed on designs which lend themselves to streamlining. This requires an antenna which has one very narrow dimension. Both horizontally and vertically polarized streamlined antennas have been designed to meet this requirement.

Fig. 12 shows one version of a scheme which will give satisfactory performance for a horizontally polarized omnidirectional antenna at 10 centimeters. It is an experimental fact that a pair of slots centrally cut in the opposite sides of a thin wave guide, when excited 180 de-

grees out of phase, has a rather uniform azimuth pattern. These slots may be excited either by probes extending into the guides or by means of the slotted dipole shown in Fig. 12. Another antenna similar to that shown in Fig. 12 has been carefully tested. It has a full-length slot, approximately 0.7 wavelength long, centrally excited by means of the slotted dipole. The azimuthal patterns of these antennas are oval-shaped, with equal power in the fore and aft directions which exceeds the power at the sides by about 2.3 to 1. The elevation pattern for the full slot arrangement had a half width of 50 degrees, while the elevation pattern of the antenna of Fig. 12 had an 80-degree half width. For both of them the standing-wave ratio  $< 2$  over a 16 per cent band.

Fig. 13 is a picture of a two-bay antenna which is usable for vertical polarization at 10 centimeters. A thin protecting radome is not shown. These elements can be die cast. The azimuth pattern of this antenna is oval in

shape with a minimum in the plane of the dipoles which differs from the maximum by a factor of about 1 to 1.6.

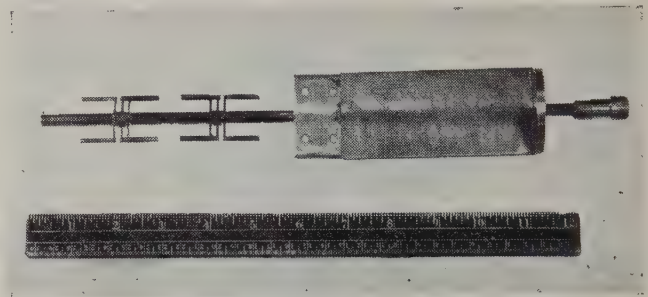


Fig. 13—Vertically polarized streamlined antenna.

Its elevation half-power width is 35 degrees and its standing-wave ratio  $< 1.3$  over a 7 per cent frequency band.

## Q Circles—A Means of Analysis of Resonant Microwave Systems\*

WILLIAM ALTAR†

### Part II

#### 3. MICROWAVE SYSTEMS AND CIRCUIT ANALYSIS

Proof of the relations exploited in Part I rests on the validity of representing our system by the circuit of Fig. 1, which must now be justified on the basis of electromagnetic field theory. So far as the matching transformer is concerned, the  $Q$  circle is merely a particularly convenient form of what might be termed the reactance circle of the transducer  $M.T.$ , meaning the locus of reflection coefficients as seen at  $T$  while the load at  $T'$  goes through a sequence of purely reactive values. An alternative method, which has occasionally proved useful for measuring a matching transformer, consists in replacing the magnetron proper by a transmission line equipped with a movable end plug free of loss, and to measure the standing-wave ratios at terminals  $T$  for, say, four consecutive plug positions at eighth-wave spacings. The four chart points thus obtained can then be used in much the same manner as the four points  $A, B, C$  and  $O$  of Section II (Part I), for an evaluation of the transducer properties. This case offers a more clean-cut situation and will be discussed first, thus

\* Decimal classification: R119.3. Original manuscript received by the Institute, April 29, 1946; revised manuscript received, August 8, 1946. Part I of this paper was published in Proc. I.R.E., vol. 35, pp. 355-361; April, 1947.

This method was first developed in the early spring of 1943 and, under then existing security restrictions, received only limited discussion at that time.

† Westinghouse Electric Corporation, East Pittsburgh, Pennsylvania.

avoiding certain complications arising from cavity coupling.

Let the system be completely surrounded by a closed surface  $S$  so that all field vectors vanish everywhere on  $S$  except at the two reference planes  $T$  and  $T'$  where  $S$  cuts through the two transmission lines. It will suffice to treat the case where each of these lines is a wave guide (not necessarily of identical type) operating in their respective lowest transverse electric modes. The system is passive in the sense that no radio-frequency power is generated inside  $S$ , nor are there internal space charges or currents with which the radio-frequency fields could interact.

Then let  $E_1, H_1$  and  $E_2, H_2$  be two independent steady-state solutions of the field equations for the boundary conditions imposed by the nature of the system, and oscillating at the same frequency. Combining a well-known vector identity with the fact that the fields satisfy Maxwell's equations, one has:

$$\begin{aligned} \operatorname{div} (E_1 \times H_2) &= H_2 \cdot \operatorname{curl} E_1 - E_1 \cdot \operatorname{curl} H_2 \\ &= -jk(H_1 \cdot H_2 + E_1 \cdot E_2). \end{aligned} \quad (14)$$

Interchanging the subscripts and combining with (14), one has:

$$\operatorname{div} (E_1 \times H_2 - E_2 \times H_1) = 0 \quad (15)$$

which, by integration over the volume  $V$  inside  $S$ , reduces to a surface integral over  $S$ . The only contribu-

tions not zero come from the two wave-guide cross sections at  $T$  and  $T'$ , where the fields are well-defined mode functions and can be formally integrated. Changing the definition of the positive normal of  $S$  so that, by definition, the positive surface normals at both  $T$  and  $T'$  point in the assumed direction of transmission, one finds the integrated form of (15):

$$\begin{aligned} J_{12} &= \iint_T (E_1 \times H_2 - E_2 \times H_1) da \\ &= \iint_{T'} (E_1 \times H_2 - E_2 \times H_1)' da' \times J_{12}'. \end{aligned} \quad (16)$$

It is well known that wave-guide modes fall into the two classes of  $TE$  and  $TM$  modes. It will suffice here to carry the argument through for the case of a  $TE$  mode without, however, restricting its generality with regard to the cross-sectional shape of the wave guide. It is only necessary that the guide geometry be such as to exclude all other modes.

The guide field at  $T$  is then formally representable as follows:<sup>2</sup>

$$\begin{aligned} E_s &= k(n \times \nabla) v (e^{ik_z z} + r e^{-ik_z z}) \\ E_z &= 0 \\ H_s &= k_z \nabla v (e^{ik_z z} - r e^{-ik_z z}) \\ H_z &= -j k_s^2 v (e^{ik_z z} + r e^{-ik_z z}) \end{aligned} \quad (17)$$

where the subscripts  $s$  and  $z$  refer to the sectional ( $x, y$ ) and, respectively, to the normal ( $z$ ) components of the field vectors. The propagation constant  $k_z$  is found from  $k = 2\pi/\lambda_0$  by means of  $k_z = \sqrt{k^2 - k_s^2}$ . The function  $v(x, y)$  is independent of  $z$  and is a solution of the two-dimensional characteristic-value problem

$$(\Delta + k_s^2)v = 0, \quad (18)$$

subject to the condition that the normal derivative of  $v$  at the boundary of the section area be zero.

Substituting two solutions  $E_1, H_1$  and  $E_2, H_2$  of the type (17) into the left side of integral relation (16) gives the following:

$$\begin{aligned} J_{12} &= k k_s \iint_T (\Delta v)^2 da [(e^{ik_z z} + r_1 e^{-ik_z z})(e^{jk_z z} - r_2 e^{-jk_z z}) \\ &\quad - (e^{ik_z z} + r_2 e^{-ik_z z})(e^{jk_z z} - r_1 e^{-jk_z z})] \end{aligned} \quad (19)$$

which, with the help of the vector identity

$$(\nabla v)^2 = \text{div}(v \text{ grad } v) - v \Delta v = \text{div}(v \text{ grad } v) + k_s^2 v^2 \quad (20)$$

together with the boundary conditions for  $v$ , may be simplified to

$$J_{12} = 2k k_s k_s^2 V(r_1 - r_2) \quad (21)$$

where

$$V = \iint_T v^2 dx dy.$$

<sup>2</sup> E. U. Condon, "Microwave radio," *Rev. Modern Phys.* vol. 14, p. 355; October, 1942.

The case treated concerns cylindrical resonators and is closely analogous to that encountered in wave guides which interests us here.

A similar relation can be derived for the reference plane  $T'$  at the receiver, where a form analogous to (17) can be written down for the field, except that complex alternating-current vectors  $t_1$  and  $t_2$ , representing transmission coefficients of the transducer, must be applied to the two solutions. We write

$$\begin{aligned} E_s' &= k(n \times \nabla) w t (e^{ik_z' z'} + r' e^{-ik_z' z'}) \\ E_z' &= 0 \\ H_s' &= k_s' \nabla w t (e^{ik_z' z'} - r' e^{-ik_z' z'}) \\ H_z' &= -j k_s'^2 w t (e^{ik_z' z'} + r' e^{-ik_z' z'}) \end{aligned} \quad (22)$$

where  $k_s'$  and  $w'$  are eigenvalue and eigenfunction of the characteristic-value problem associated with cross section  $T'$ , in analogy to (18). One has, therefore,

$$J_{12}' = 2t_1 t_2 k k_s' k_s'^2 W(r_1' - r_2') w = \iint_{T'} w^2 da'$$

and equation (16) reduces to

$$k_s k_s^2 V(r_1 - r_2) = t_1 t_2 k_s' k_s'^2 W(r_1' - r_2'). \quad (23)$$

If we want to eliminate all design parameters of the transmission line and such operating parameters as  $t_i$ , which are not usually measured along with the reflection coefficients, it will be easily seen that at least four pair of coexisting values ( $r_i, r_i'$ ) are needed so that an identical relation can be established between them. One then writes:

$$\frac{J_{13} J_{24}}{J_{14} J_{23}} = \frac{J_{13}' J_{24}'}{J_{14}' J_{23}'}$$

which, by (23), leads to

$$\frac{(r_1 - r_3)(r_2 - r_4)}{(r_1 - r_4)(r_2 - r_3)} = \frac{(r_1' - r_3')(r_2' - r_4')}{(r_1' - r_4')(r_2' - r_3')}. \quad (24)$$

With the help of (24) one is in a position, from three known pairs ( $r_i, r_i'$ ) and with no reference to line constants, to compute all other pairs. This, it will be seen, leads to a linear relation of the form

$$r = \frac{Er' + F}{Gr' + H} \quad (25)$$

where the general circuit constants  $E, F, G, H$  are characteristics of the transducer and may be computed from the given complex values  $r_1, r_2, \dots, r_3'$ . Relation (25) is the exact mathematical equivalent of a well-known relation between load and looking-in impedance, which is used in the theory of four-terminal networks and which can in fact be derived from (25), seeing that  $Z$  and  $r$  are related to each other by a linear transformation. The existence of this linear so-called network transformation (25) in the complex plane is basic in the derivation of all circuit theorems; for example, in proving the existence of an equivalent pi or tee circuit.

Expressions of the type found in (24) play an important role in the theory of complex variables, being

the so-called anharmonic cross ratio of two pairs ( $r_1, r_2$ ) and ( $r_3, r_4$ ) of complex numbers. Using a notation common in that theory, we re-write (24):

$$R(r_1, r_2; r_3, r_4) = R(r'_1, r'_2; r'_3, r'_4). \quad (24a)$$

This expresses the fundamental theorem that the cross ratio of four numbers  $r_i$  remains invariant under any linear transformation of the type (25) simultaneously applied to the  $r_i$ 's.

We shall make extensive use of (24), frequently in a more general form which is not limited to one direction ( $TT'$ ) of transducer operation but which permits one to correlate freely results of both network transformations respectively associated with the two possible directions of transmission.

To this end we shall make a distinction between load and image points in the  $T$  chart, referring to points measured in the operating direction ( $T'-T$ ) as load points, and to measurements in the direction ( $T-T'$ ) as image points. In the  $T'$  chart, similarly, load and image points are respectively associated with the operating direction ( $T-T'$ ) and ( $T'-T$ ). Reference will also be made to inverted points, meaning the conjugate of the reciprocal of a load or image point. Thus an inversion is the same as a geometrical transformation by reciprocal radii.

Since a standing-wave pattern at  $T$  (or at  $T'$ ) can be represented either by  $r$ , or alternatively by  $1/r$  if the direction assumed to be positive is arbitrarily reversed, the following formulation of equation (24) is valid:

**THEOREM.** Let each of four complex numbers  $a, b, c$ , and  $d$  represent either a load point or an inverted image point in the  $T$  chart of a given passive transducer, and let  $a', b', c'$ , and  $d'$  be respectively the inverted image points, or the load points, depicting coexisting load conditions in the  $T'$ -chart. Then the two cross ratios are complex conjugates:

$$R(a, b; c, d) = \widehat{R}(a', b'; c', d'). \quad (24b)$$

By writing the network transformation in any of the forms (24) one opens the way for application of a number of useful geometrical rules dealing with anharmonic cross ratios of complex numbers and which are compiled here without proof for later reference:

A. Four points in the complex plane lie on a circle if, and only if, their cross ratio has a real value.

B. Two pairs of points ( $a, b$ ) and ( $c, d$ ) in the complex plane form a harmonic sequence if their cross ratio has the real value  $R(a, b; c, d) = -1$ . It follows that four harmonic points lie on a circle; also, that the two lines tangent to the circle at points forming one pair intersect at a point colinear (in a straight line) with the points forming the other pair.

C. If each of four complex numbers is subjected to the same linear transformation, the numerical value of the cross ratio remains the same. This is true, particu-

larly, for four complex quantities (four admittances, four impedances, or four reflection coefficients) simultaneously subjected to a network transformation.

D. Let  $P, A, B, C$ , and  $D$  be points on a circle. Four rays through  $P$  and intersecting the circle at the respective points  $A, B, C$ , and  $D$  will intersect an arbitrary straight line  $g$  in four points  $a, b, c, d$  so that the cross ratios are equal:

$$R(A, B; C, D) = R(a, b; c, d)$$

where the real quantities  $a, b, c, d$  are distances measured from an assumed zero on the straight line.

E. If the equation of a circle (general point  $r$ ) can be written in parameter form

$$r = \frac{P\alpha + Q}{R\alpha + S} \quad \begin{array}{l} P, Q, R, S \text{ complex constants, } \alpha \text{ a real} \\ \text{variable} \end{array}$$

we shall refer to  $\alpha$  as a linear parameter for points on the circle. If so, the cross ratio of any four points on the circle is equal to that of the respective parameter values

$$R(r_1, r_2; r_3, r_4) = R(\alpha_1, \alpha_2; \alpha_3, \alpha_4)$$

as a direct consequence of Rule C.

F. Three or more circles are called confocal if by a linear transformation they can be transformed into concentric circles. Confocal circles thus remain confocal under linear transformations. In analytic form, a family of confocal circles in the  $XY$  plane is represented by the equation:

$$\begin{aligned} (X - x)^2 + (Y - y_0)^2 &= (x^2 - d^2) \quad \text{or:} \\ X^2 - 2Xx + (Y - y_0)^2 + d^2 &= 0 \end{aligned} \quad (26)$$

where the parameter  $x$  is constant for each circle but varies from one circle to the next.

G. If a circle is orthogonal to two given circles, it is also orthogonal to all circles confocal with the given two circles. All circles which are orthogonal to a family of confocal circles pass through two points which are called the foci of the family.

H. Let  $a$  and  $b$  be the segments into which the chord of a circle is separated by one of its points,  $P$ . The product of these lengths,  $a \cdot b$ , a function only of the position of  $P$  and the circle but not of the particular chord chosen, is called the "power" of point  $P$  relative to the circle.

The locus of all points in the plane having a given constant ratio of "powers" relative to two given circles is itself a circle confocal with the given circles.

It remains for us to extend the procedure to the case where only one transmission line leads into the system, while the transducer at its other end is terminated in a resonant cavity of arbitrary shape. Instead of varying the load through a sequence of purely reactive values by means of a variable end plug, we can then accomplish the same purpose by the simple expedient of varying the frequency.

Retracing our steps through equations (14) to (16) but now assigning different values  $k_1$  and  $k_2$  to the two solutions, we obtain the following relation in place of (15):

$$j \operatorname{div} (E_1 \times H_2 - E_2 \times H_1) = (k_1 - k_2)(H_1 \cdot H_2 - E_1 \cdot E_2) \quad (27)$$

which, by integration over the volume  $V$  inside  $S$ , reduces to

$$\begin{aligned} j \iint_T da (E_1 \times H_2 - E_2 \times H_1) \\ = (k_1 - k_2) \iiint_V (H_1 \cdot H_2 - E_1 \cdot E_2) dv. \end{aligned} \quad (28)$$

Again we may carry out the surface integration by substituting the expressions (17), with the complication that the quantity  $\rho = k_z/k$  is no longer the same for solutions belonging to different frequencies. Unless these frequencies are close to cut-off, however, we may consider the ratios as constant, which entails the negligible error of omitting a term of the order  $(k_1 k_{2s} - k_2 k_{1s}) = 1/2(\alpha_1 - \alpha_2)k_s^2$ , and we arrive then at the same equation (21) as before.

To carry out the volume integration in (28), we must assume all frequencies to be so close to resonance that the actual fields in the system may be expanded in terms of orthogonal mode functions and that all but one expansion term may be discarded because the overwhelmingly strong effect of the near mode drowns out all the others. In this approximation one has

$$\begin{aligned} E &= \frac{jE_0}{k_0^2 - k^2} (k_0 A - k B) \\ H &= \frac{jH_0}{k_0^2 - k^2} (-k A + k_0 B) \end{aligned} \quad (29)$$

where the "excitation integrals"  $A$  and  $B$  are to be taken over the boundary inside which the unperturbed mode  $E_0, H_0$  is defined:

$$A = \iint_S E \times H_0 \cdot da \quad \text{and} \quad B = \iint_S E_0 \times H \cdot da.$$

It will usually be possible and profitable to locate inside  $S$  a second closed surface  $S'$  which contains essentially all the stored energy associated with the operating mode but which excludes such amounts of stored energy as may be associated with modes other than the operating mode and which, therefore, do not vary appreciably throughout the frequency band studied. In view of (15) or (28), the surface integral of (28) should give the same result whether we integrate over  $S'$  or over  $S$ . For the volume integration, on the other hand, there may be considerable advantage in restricting the range of integration to within  $S'$ , where the approximation (29) is better justified.

Substituting the expressions (29) into (28) and using the notation

$$W_0 = \iiint_{V'} E_0^2 dv = - \iiint_{V'} H_0^2 dv$$

we find for the volume integral:

$$\begin{aligned} \iiint_{V'} (H_1 \cdot H_2 - E_1 \cdot E_2) dv \\ = \frac{W_0}{(k_0^2 - k_1^2)(k_0^2 - k_2^2)} [(k_0^2 + k_1 k_2)(A_1 A_2 + B_1 B_2) \\ - k_0(k_1 + k_2)(A_1 B_2 + A_2 B_1)]. \end{aligned}$$

For the unperturbed mode, the electric field  $E_0$  is orthogonal and the magnetic field  $H_0$  parallel to the boundary  $S'$  so that the integrals  $B$  of (29) are always zero. Thus the last expression, when introduced in (28), gives

$$2j\rho k_1 k_2 k_s^2 (r_1 - r_2) V = W_0 A_1 A_2 \frac{(k_0^2 + k_1 k_2)(k_1 - k_2)}{(k_0^2 - k_1^2)(k_0^2 - k_2^2)}. \quad (30)$$

To simplify (30) we introduce the frequency parameter  $\alpha$ , in terms of which

$$\alpha_1 - \alpha_2 = \frac{(k_0^2 + k_1 k_2)(k_1 - k_2)}{k_0 k_1 k_2}$$

so that equation (28) assumes the final form

$$\begin{aligned} r_1 - r_2 &= j(\alpha_1 - \alpha_2) F_1 F_2 \\ F_i &= \frac{A_i}{(k_0^2 - k_i^2) k_s} \sqrt{\frac{k_0 W_0}{2\rho V}}. \end{aligned} \quad (31)$$

Next one eliminates the  $F_i$  factors much as in the case previously discussed, provided that four  $r$  values at terminals  $T$ , measured at known frequencies, are available. Forming again the anharmonic cross ratio of the  $r$ 's, one obtains, in view of (31),

$$R(r_1, r_2; r_3, r_4) = R(\alpha_1, \alpha_2; \alpha_3, \alpha_4) \cong R(f_1, f_2; f_3, f_4) \quad (32)$$

showing that  $r$  is a linear function of the frequency parameter; exactly as if the resonant region inside  $S'$  were a simple  $LC$  circuit accessible through a frequency-insensitive transducer:

$$r = \frac{K\alpha + L}{M\alpha + N}. \quad (33)$$

From the values  $K, L, M$ , and  $N$  which are known functions of  $r_1, r_2, r_3, \alpha_1, \alpha_2, \alpha_3$ , one can in fact compute the lumped elements of a completely equivalent circuit, except for the unknown ratio of an ideal transformer at terminals  $T'$ . This leaves the characteristic impedance of the resonant element undetermined.

#### 4. INTERPRETATION OF Q-CIRCLE DATA— OUTLINE OF PROOFS

##### a. $Q$ and $\alpha$ Contours

If the reflection coefficients are measured at four arbitrary frequencies at terminals  $T$  looking to the left

toward the transducer and resonant element, the anharmonic cross ratio of their complex values is real by virtue of (32), and the frequency locus is a circle in the Smith chart by virtue of Rule A. The  $Q$  circle is determined by the conjugates of any three  $r$  values (points  $A, B, C$  in Section 2).

If these three points belong to equidistant  $\alpha$  values, the fourth harmonic point, as defined by Rule B, is the off-resonance point  $O$ , because the cross ratio of these four points is:

$$R(O, B; A, C) = \frac{\alpha_2 - \alpha_3}{\alpha_2 - \alpha_1} = -1.$$

The geometrical relation which Rule B establishes between four points of a harmonic sequence makes it then possible to locate point  $O$  graphically from the given  $A, B$ , and  $C$  essentially as described earlier.

From (32) one also deduces that  $\alpha$  is a linear parameter for the  $Q$  circle within the meaning of Rule E, thus establishing the existence of a linear  $\alpha$  scale. If one makes line  $g$  of Rule D parallel to the line joining points  $P$  and  $O$ , projection of points on the circle from the projection center  $P$  on the line  $g$  will assign the correct parameter value  $\alpha = \infty$  to the infinite point on  $g$ ; it then suffices to correctly assign their  $\alpha$  values to two

Points  $O, S$ , and  $F$  are image points in the  $T$  chart corresponding to the respective frequencies  $\infty, f_0$ , and an arbitrary frequency  $f$ . The corresponding load points in the  $T'$  chart lie on the chart rim, being unit vectors  $r' = 1, r' = -1$  and  $r' = e^{j\theta'}$ . For the fourth point to be used in (27b) we choose the load point  $r = \text{zero}$ , i.e., a match at terminals  $T$ . Its corresponding image  $r_m'$  in the  $T'$  chart is found by substituting the four point pairs into (27a):

$$\begin{aligned} R(S, O; F, \infty) &= \widehat{R}(-1, 1; e^{j\theta'}, \hat{r}_m') \\ &= R(-1, 1; e^{-j\theta'}, 1/r_m') \end{aligned} \quad (34)$$

or, explicitly,

$$\frac{S - F}{O - F} = \frac{1 - e^{-j\theta'}}{1 + e^{-j\theta'}} \frac{1 + r_m'}{1 - r_m'}. \quad (35)$$

This equality between purely imaginary quantities can be rewritten after dropping a  $j$  factor:

$$\tan \psi/2 = \frac{1}{Q_m} \tan \psi'/2 \quad (36)$$

where  $\psi$  and  $\psi' = 180 - \theta'$  (Fig. 6) are the angles through which the point on the  $Q$  circle and the corresponding point on the rim of the  $T'$  chart, travel as the frequency

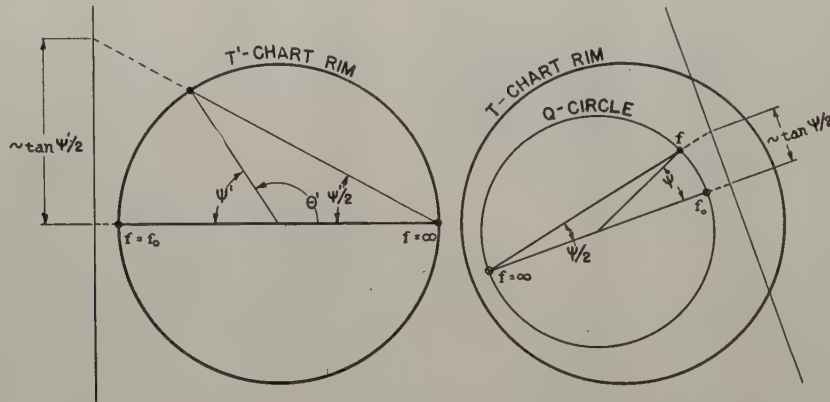


Fig. 6

more points on line  $g$  in order that all other points on  $g$  shall read correctly by linear interpolation. In particular, point  $P$  was chosen coincident with  $O$  in Section 2, thus making line  $g$  parallel to the tangent at  $O$ .

In order to determine  $Q_{\text{match}}$ , use is made of (24b) with a suitable choice of points; three load points and their images referring to transducer operation in the direction  $T-T'$  and the fourth to the opposite direction. It has been said earlier that the dividing line at  $T'$  is to be drawn in a manner so as to make the resonant frequency  $f_0$  of the resonant element the same as the resonant frequency  $f_m$  of the system when terminated in a matched load. Later it will be shown that point  $S$ , corresponding to  $f_m$  on the  $Q$  circle, lies diametrically opposite to the off-resonance point  $O$ . Points  $S$  and  $O$  are the respective short-circuit and open-circuit points of the transducer  $M.T$ .

shifts from  $f_0$  to  $f$ . Indeed, since the two vectors  $(O-F)$  and  $(S-F)$  are perpendicular, the left side of (36) is  $j \tan \psi/2$ . In the  $T'$  chart, where the load is purely reactive, the load point travels through the angular interval from  $\theta' = 180$  degrees (for  $f_0$ ) to  $\theta'$  (for  $f$ ). The second factor at the right of (35) is simply related to the loaded  $Q$  of the resonant system. Generally, the impedance which an arbitrary load presents at  $T'$  is

$$\frac{Z'}{Z_0'} = \frac{1 + r'}{1 - r'} = \frac{1}{Q} - j\alpha \quad (37)$$

where  $\alpha$  is the frequency parameter measuring the extent of frequency pulling by the external load. Indeed, the frequency for which the reactance  $Z_0'\alpha$  just neutralizes the image reactance presented becomes the resonant frequency for the loaded system.

For a matched load at  $T$ , because of our choice of refer-

ence zero for  $\alpha$ , (37) becomes simply  $1/Q_{\text{match}}$ , thus verifying equation (36). Now, in the  $T'$  chart, the frequency parameter  $\alpha$  and the angle  $\psi'$  are related:

$$\tan \psi'/2 = \frac{f}{f_0} - \frac{f_0}{f} = \alpha \quad (38)$$

so that the final formula is obtained:

$$Q_m = \frac{1}{\alpha} \tan \psi/2 \approx \frac{f_0}{2\Delta f} \tan \psi/2. \quad (39)$$

Proof of the formulas for mapping  $Q$  and  $\alpha$  contours proceeds along similar lines, using equation (24a) again with a slightly different choice of points. We identify the load conditions, associated with indexes 1 and 2, with open- and short-circuit at terminals  $T'$  while the indexes 3 and 4 will be identified with one match and one arbitrary load, both at terminals  $T$ . If we use a bar to denote the inverted image of a load, equation (24a) assumes the specialized form

$$R(1, -1; \bar{r}', \bar{r}_m') = R(\bar{O}, \bar{S}; r, \text{zero}) \quad (40)$$

or, explicitly,

$$\frac{1-r'}{1+r'} \frac{1+r_m'}{1-r_m'} = \frac{\bar{O}-r}{\bar{S}-r} \frac{\bar{S}}{\bar{O}}$$

which, by (37), may be written

$$\frac{1}{Q} - j\alpha = \frac{1}{Q_m} \frac{\bar{O}}{\bar{S}} \frac{\bar{S}-r}{\bar{O}-r}. \quad (41)$$

We must forego details of the geometrical interpretation of the vector relation (41) by which one verifies the description given in Section 3, for mapping the  $Q$  and  $\alpha$  contours.

It remains to show that points  $S$  and  $O$  are diametrically opposite each other on the  $Q$  circle. The  $\alpha$  contours meet the inverted  $Q$  circle at points obtained by inverting the points on the  $Q$  circle which belong to the respective  $\alpha$  values. In particular, the  $\alpha$  contour passing through the center of the chart will, when subjected to an inversion, become that circle which passes through  $\bar{O}$  and through the infinite point, and will have to be perpendicular to the inverted  $Q$  circle. Hence, it must be the diameter of the  $Q$  circle at the off-resonance point  $q.e.d.$

### b. Efficiency Contours

Space would not permit us to derive here the functional relation between input and output power and the load for a given transducer by an independent method. It is more expedient to make use of known<sup>3</sup> formulas for the contour lines of circuit efficiency in the receiver chart of complex load admittance, and to translate them into

contour lines in our chart. In the plane of admittance  $Y=G-jB$ ,

$$\begin{aligned} \left[ G + \frac{1}{2k}(p - 1/e) \right]^2 + \left[ B + \frac{q}{2k} \right]^2 \\ = \left[ \frac{1}{2k}(p - 1/e) \right]^2 + \left( \frac{q}{2k} \right)^2 - \frac{h}{k} \end{aligned} \quad (42)$$

where the constants  $p, q, h, k$  are known functions of the general transducer constants and of no particular concern in the following.

Equation (42) must be transformed into an equivalent relation in the complex  $r$  plane, using the relation

$$Y = Y_0 \frac{1-r}{1+r}. \quad (43)$$

Note first that (42) is of the form which, by Rule F (Section 3), is characteristic of a family of confocal circles.

$$\begin{aligned} (G - g)^2 + (B - b_\infty)^2 &= g^2 - d^2 \\ g &= -g_\infty + \frac{R_\infty}{e} \quad d = \sqrt{g_\infty^2 - R_\infty^2} \end{aligned} \quad (44)$$

where the center of the contour line  $e = \infty$  has the coordinates  $(-g_\infty, b_\infty)$  with  $g_\infty$  a positive number, and its radius is  $R_\infty$ . According to Rule F, the efficiency contours remain a family of confocal circles after the linear transformation (43) which transforms the admittance plane to the  $r$  plane. Moreover, they will be confocal in the  $r$  plane with the two known contour lines  $e = \text{zero}$ , which is the chart rim, and  $e = \infty$ , which is the inverted  $Q$  circles.

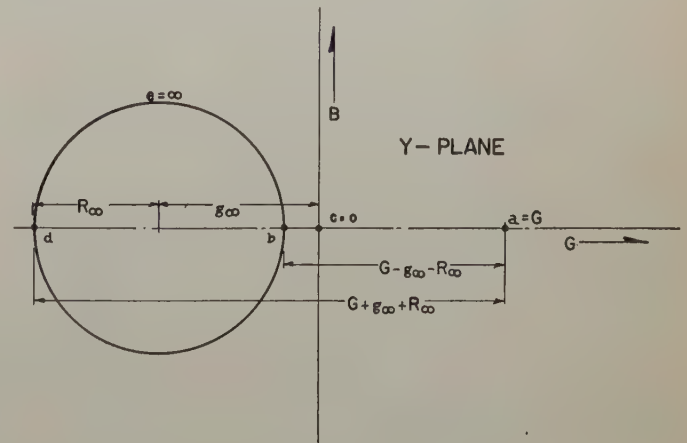


Fig. 7

Thus, we have reduced the mapping of the  $e$  contours to the purely geometrical problem of drawing circles confocal with two given circles: a straightforward geometrical construction which has been given in Section 2(c). It remains to locate the centers for given  $e$  values.

To this end, we solve the admittance equation (44) for  $e$ . Since circuit efficiency involves only active powers

<sup>3</sup>J. G. Tarboux, "Introduction to Electrical Power Systems," 1944. pp. 113-115, International Textbook Co., Scranton, Pa.

which do not vary along the loss-free output wave guide, it is permissible to temporarily shift the reference plane  $T$  of the guide until the centers of all  $e$  circles fall on the  $G$  axis, thus making  $b_\infty$  of (44) zero (Fig. 7). In the Smith chart, this is tantamount to a rotation of the diagram without affecting the relative configuration, and after completing the argument we shall restore the original reference terminals  $T$  simply by tilting the diagram back to its proper angular position.

With  $b_\infty = 0$ , one has from (44)

$$e = \frac{2R_\infty G}{(G + g_\infty + R_\infty)(G + g_\infty - R_\infty) + B^2} \quad (45)$$

Since the line  $B=0$  will be transformed into the real axis of the Smith chart, we ask for the points of intersection of the  $e$  contours with this line, first in the admittance plane and then in the Smith chart. With  $B=0$ , equation (45) assumes a simpler form which is entirely expressible in terms of anharmonic cross ratios, an important advantage for executing the transformation. One has in the admittance plane (Fig. 7)

$$\begin{aligned} (e)_{B=0} &= \frac{2RG}{G + g_\infty + R_\infty(G + g_\infty - R_\infty)} \\ &= \frac{(a - c)(b - d)}{(a - b)(a - d)} \\ &= R(a, b; c, d) \times R(a, c; \infty, b) \end{aligned} \quad (46)$$

where the quantities  $a, b, c, d$  are real, representing points along the  $G$  axis. Now because of the invariance of cross ratios under linear transformations (see Rule C), (46) can be transformed from admittance to Smith chart quantities, simply by replacing each admittance symbol  $a, b, c, d$  by the corresponding reflection coefficient. For instance, the infinite point of the admittance plane corresponds to a reflection coefficient  $r = -1$ , while point  $c$ , the zero of the admittance plane, becomes  $r = 1$  in the Smith chart. The circle with center  $(-g_\infty, 0)$  and radius  $R_\infty$  is the contour line  $e = \infty$ ; it is transformed into the inverted  $Q$  circle in the Smith chart, so that points  $b$  and  $d$  are transformed into the intersections  $B$  and  $D$  of the inverted  $Q$  circle with the real axis. Finally, the load admittance  $a$  is transformed into the load point  $r$  in the Smith chart. With these substitutions, (46), transformed to the  $r$  plane, can be written down directly, thus:

$$\begin{aligned} (e)_{B=0} &= R(r, B; -1, D) \times R(r, -1; 1, B) \\ &= \frac{(1+r)(D-B)}{(D-r)(B+1)} \frac{(1-r)(B+1)}{2(r-B)} \\ &= \frac{(D-B)}{2} \times \frac{(1-r^2)}{(D-r)(r-B)} \\ &= R_0 \frac{1-r^2}{(D-r)(r-B)} \end{aligned} \quad (47)$$

Using the terminology of Rule H, we express (47) in the form

$$e = R_0 \frac{\text{"power" of load point relative to the chart rim}}{\text{"power" of load point relative to inv. } Q \text{ circle}} \quad (48)$$

So far, we have proved (48) only for a purely resistive load, i.e., for the point of intersection of an  $e$  contour with the real axis, but we can remove this restriction, using the second statement of Rule H. Hence, (48) applies to any point in the load chart.

The "power" of the load point relative to the  $T$  chart rim bears a simple relation to the active power passing through the  $T$  reference plane. It is not difficult to show that the voltage, current, and complex power vectors at that reference plane when the outgoing wave has unity amplitude are respectively represented by  $(1+r)$ ,  $(1-r)$ , and  $(1+r')(1-r)$ . (Complex quantities!!) It follows that the active power output at  $T$  is given by

$$P = P_0(1 - |r|^2) \quad (49)$$

where  $P_0$  is the power output per unit amplitude supplied to a matched load. Comparison with (48) then leads to the following formula for the active input power  $P'$  at terminals  $T'$  under these conditions:

$$P' = P_0 \frac{\text{"power" of load point } r \text{ relative to inv. } Q \text{ circle}}{\text{radius of inverted } Q \text{ circle}}$$

Equation (47) assumes a particularly simple form for  $r=0$ , showing that the circuit efficiency of the system when terminated in a matched load is

$$e_{\text{match}} = \frac{R_0}{-BD} = \frac{D-B}{2BD} \quad (50)$$

Thus, one finds that  $e_{\text{match}}$  is numerically equal to the radius of the  $Q$  circle, verifying a previous statement to this effect.

To find the center of a given  $e$  contour, we rewrite equation (47) as a quadratic equation for  $r$ :

$$r^2(e - R_0) = 2rx_0e = \text{constant} \quad (51)$$

where  $x_0 = 1/2(D+B)$  is the abscissa of the center of the inverted  $Q$  circle. The center of each contour has an abscissa (measured along our temporary real axis)

$$x = 1/2(r_1 + r_2) = \frac{x_0e}{e - R_0} \quad (52)$$

This shows that  $e$  is a linear parameter for the point sequence of centers along the  $x$  axis; hence the existence of a linear  $e$  scale follows in accordance with Rule E, and the procedure for setting up the  $e$  scale which was outlined in Section 2(c) need only be justified for three points, preferably for the centers of the contours  $e=0$  (chart rim),  $e=\infty$  (inverted  $Q$  circle), and for the  $e$  value which is numerically equal to  $R_0$  (contour a straight line; contours center at infinity).

# Characteristics of Certain Voltage-Regulator Tubes\*

GEORGE M. KIRKPATRICK†, MEMBER, I.R.E.

**Summary**—A quantitative presentation is made of the data from a large number of tests on three types of gaseous voltage-regulator tubes, VR75, VR105, and VR150, from several manufacturers. The effect of several factors, such as time and temperature, on the voltage stability has been investigated, and an equivalent circuit of the tube was experimentally determined. These results are especially applicable where the VR tubes are used as a source of reference voltage, such as in a degenerative electronic voltage stabilizer.

## INTRODUCTION

**G**ASEOUS voltage-regulator tubes are extensively used as a means of obtaining a reference voltage in the degenerative-type electronic voltage stabilizer. When this circuit is analyzed to determine the internal resistance and the regulating factor,<sup>1,2</sup> the voltage-regulator tubes are commonly considered to be the equivalent of a constant voltage in series with a linear resistance.

In the course of development work to obtain a regulated power supply of unusually high stability, a large number of tests were made on three types of voltage-regulator tubes, VR75, VR105, and VR150, from several manufacturers. A comparatively wide range of operating characteristics was found even in tubes of the same type, but enough quantitative data has been accumulated to enable a user to predict the order of stability to be expected.

If they are to be used as a means of obtaining a reference voltage, such tubes should have the following characteristics:

1. They should be free from spontaneous changes in voltage. This includes sudden "jumps," as well as random drifting of the voltage.
2. They should have a low temperature coefficient of voltage drift.
3. The tube voltage should be the same after each firing when the operation is at the same tube current.
4. The dynamic resistance should be as small as possible, though a negative dynamic resistance is not desirable.

The VR75 tubes tested were found to be better than the VR105 or VR150 tubes, as judged by the performance on the first three characteristics. However, the dynamic resistance of VR75 tubes has values intermediate to that of VR105 and VR150 tubes when the operation is at 20 milliamperes. The VR150 tube has the highest dynamic resistance, although there is considerable

variation among tubes of the same type, especially at low currents.

The data for the three types of VR tubes investigated are presented under headings which segregate the various effects as much as possible, although it must be realized that sometimes more than one factor is involved. This is intended to be a report of the results of a number of tests, and no attempt is made to explain the observed effects. For an explanation of the operation of voltage-regulator tubes, readers are referred to books and papers on the subject by several writers.<sup>3-5</sup>

## INITIAL DRIFT

The highest rates of drift of tube voltage are found to occur during the first few minutes of each operation. The percentage change in the tube voltage in an arbitrary period of five minutes, immediately after the tube is fired, is called the initial drift, for purposes of comparison. In the tests, for which the results are tabulated in Table I, the tubes were allowed to cool to room temperature before refiring.

The drift in the initial five minutes' operation of ten tubes of each of the three types was measured five times, and the average of the fifty readings for each type was then taken to obtain the average initial drift. The total change in the tube voltage in the five-minute period was used in computing the average, without regard to whether the voltage increased or decreased. While the tube voltage of the VR75 tubes consistently decreased during warmup, the direction of the initial drift of the VR105 and VR150 tubes was random.

TABLE I  
Drift in Initial 5 Minutes Operation

Tube Type	Average Initial Drift (per cent)	Maximum Initial Drift (per cent)	Tube Current (milliamperes)
VR75	0.03	-0.05	5.5
VR105	0.12	+0.80	20.0
VR150	0.18	-1.50	20.0

## OPERATING VOLTAGE

It is pointed out in the introduction that it is desirable that the tube voltage return to the same value after each firing when the operation is at the same tube current. A number of readings of the tube voltage following firing must be examined to determine the constancy of the operating voltage. As a measure of how well individual

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† General Engineering and Consulting Laboratory, General Electric Company, Schenectady, N. Y.

<sup>1</sup> F. V. Hunt and R. W. Hickman, "On electronic voltage stabilizers," *Rev. Sci. Instr.*, vol. 10, p. 6; January, 1939.

<sup>2</sup> W. R. Hill, Jr., "Analysis of voltage-regulator operation," *Proc. I.R.E.*, vol. 33, pp. 38-45; January, 1945.

<sup>3</sup> J. D. Cobine, "Gaseous Conductors," chap. VIII, McGraw-Hill Book Co., New York, N. Y., 1941.

<sup>4</sup> L. R. Koller, "Physics of Electron Tubes," chap. X, McGraw-Hill Book Co., New York, N. Y., 1937.

<sup>5</sup> L. B. Loeb, "Fundamental Processes of Electrical Discharge in Gases," chap. XI, John Wiley and Sons, Inc., New York, N. Y., 1939.

tubes meet this requirement, ten tubes of each type were fired five times each, and the readings of the tube voltage were taken after five minutes of operation. These five readings of the tube voltage are tabulated in Table II for five tubes of each type.

TABLE II  
Tube Voltage after 5 Minutes of Operation

Tube Type	Tube No.	Tube Voltage				
		Test No. 1	Test No. 2	Test No. 3	Test No. 4	Test No. 5
VR75	22	72.57	72.57	72.57	72.58	72.59
VR74	13	71.94	71.94	71.93	71.93	71.94
VR74	S1	71.63	71.63	71.63	71.63	71.61
VR75	S2	72.12	72.11	72.09	72.11	72.09
VR75	S3	71.77	71.76	71.72	71.76	71.75
VR105	H	108.41	108.42	108.59	108.52	108.53
VR105	D	105.31	105.83	106.19	106.58	106.77
VR105	K	102.83	103.50	103.90	104.03	104.40
VR105	M2	107.72	107.71	107.72	107.73	107.89
VR105	EE	107.79	107.77	107.79	107.77	107.72
VR150	1	149.31	149.22	149.19	149.19	149.17
VR150	5	151.43	151.28	151.16	151.10	151.03
VR150	K2	151.97	152.35	151.80	151.67	151.83
VR150	DD	152.04	151.22	150.89	150.64	150.58
VR150	K7	152.95	152.80	152.74	152.74	152.84

The average voltage and extremes as tabulated in Table III are for all the fifty readings taken on each type. An estimate of how much the voltage across a tube might change in the first five minutes of operation can be made on the basis of the data in Table I. An examination of Table II shows that the tube voltage of the VR75 varies but little between successive operations, and is much more consistent than either the VR105 or VR150.

TABLE III  
Average, Maximum, and Minimum Tube Voltages (10 tubes)

Tube Type	Average Tube Voltage	Maximum Tube Voltage	Minimum Tube Voltage	Tube Current (milli-amperes)
VR75	71.90	72.59	71.13	5.5
VR105	106.68	108.53	102.83	20.0
VR150	151.25	152.95	149.40	20.0

#### OPERATING DRIFT

In order to obtain data on the amount of drift in the VR-tube voltage with time, continuous recordings were made of the tube voltage for several hours. As in all voltage measurements, a fraction of the VR-tube voltage was compared with that of a saturated-type standard cell, and a self-balancing potentiometer circuit operated a recording microammeter to indicate the difference voltage. The VR tubes were operated in a vertical position on a testboard, and were subject to fluctuations in the ambient room temperature. The direct tube current was supplied by a regulated power supply.

The data available on the record rolls is difficult to present in a condensed form; however, some interpretations of this data can be made. The voltage of the three types tested drifted in a random manner, and it is probable that the variations in ambient room temperature accounted for most of the drifting. Changes in the ambient temperature of two or three degrees centigrade can produce voltage changes of the same order of magnitude as those recorded. The VR75 voltage, when the tube current is 5 milliamperes, drifts somewhat less than the VR105 or VR150 voltage. If the latter are operated at the same or higher tube currents, however, it is not easy to present data leading to this conclusion. The consistent low temperature coefficient of voltage change (see Table V) of the VR75 tube, as contrasted to the nearly seven-times-greater temperature coefficient of voltage change of the VR150, and the higher, unstable one for the VR105, leads to the conclusion that the VR75 tube would be more stable.

Some data from a few of the record rolls are presented in a condensed form in Table IV on operating-voltage drift, and are the greatest variation in an hour interval on each record. The greatest variation in a one-hour interval on a single record is termed the "maximum drift rate." This is the maximum drift in per cent per hour for that test, and would likely have another value if the test were repeated. The first hour of operation is excluded to avoid the high initial drift, and by taking the average over a one-hour period, exceedingly high rates of drift due to sudden fluctuations are excluded.

TABLE IV  
Operating Voltage Drift  
(Percentages are based on operating voltages)

Type	No.	Maximum Drift Rate*	Duration of Test	Total Change**	Tube Current
		per cent per hour	hours	per cent	milli-amperes
VR75	13	0.01	72	0.075	5.0
VR75	12	0.02	65	0.23	5.0
VR75	13	0.03	17	0.25	20.0
VR105	R	0.02	140	0.21	5.0***
VR105	R	0.03	53	0.06	20.0
VR105	M1	0.04	36	0.30	20.0***
VR150	10	0.04	25	0.22	20.0
VR150	K1	0.28	19	0.14	20.0

\* The first hour of operation was excluded in finding the maximum drift rate.

\*\* The total change is the difference between the voltage after one hour of operation and the voltage at the conclusion of that test.

\*\*\* Abrupt changes in the tube voltage, which are illustrated by Fig. 1, occurred during these tests.

#### VOLTAGE JUMPS

Abrupt changes in the VR-tube voltage which are of a spontaneous nature have been termed voltage jumps. For the amplitudes of importance here, these have been found only in the voltage across the VR105 tube, and are illustrated by the recording of a VR105-tube voltage

reproduced in Fig. 1. Voltage jumps initiating spontaneously within the tube are found to occur in a large percentage of VR105 tubes and at all current levels tested. These voltage jumps may be as great as 0.2 per cent of the operating voltage, and occur in a random manner.

It has been found that, if the tube current is varied through a range of 2 milliamperes or more on each side of the chosen operating point at a frequency of 60 cycles per second, abrupt changes in the output voltage can be eliminated. Any sudden breaks in the current-voltage characteristic which are passed over in the current

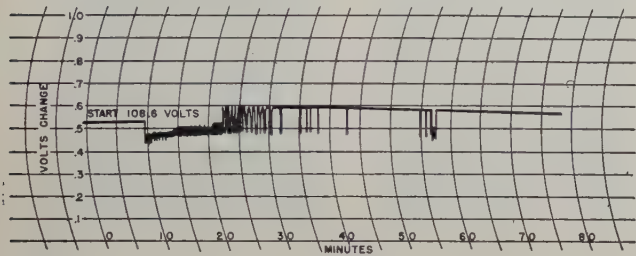


Fig. 1—Section of chart showing spontaneous changes in VR105-tube voltage.

sweeping give rise to voltages which are harmonics of 60 cycles and can be filtered out to give nearly pure direct voltage. The 60-cycle voltage will be about a tenth of a volt root-mean-square per VR105 tube, and can be reduced by a factor of 14 by using a single-section low-pass filter consisting of a 10-henry choke and a 10-microfarad capacitor.

If two VR tubes are to be used in series, then the alternating current can be introduced where the anode of one tube is connected to the cathode of the other, as shown in Fig. 2. This is found to reduce the alternating voltage across the tubes to about one third the value for one tube alone.

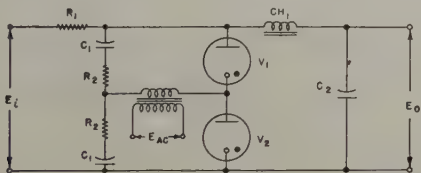


Fig. 2—Balanced circuit for use with two VR105 tubes.

TEMPERATURE COEFFICIENT OF VOLTAGE CHANGE

In order to obtain measurements on the voltage change with temperature, five tubes of each type, VR75, VR105, and VR150, were placed in an insulated box, and the temperature in the box was raised to 100 degrees centigrade. The voltage across one tube of each type was recorded continuously during the test, and measurements were made on the other tubes at the beginning at room temperature, at 100 degrees centigrade, and then again at room temperature. From these data an average temperature coefficient of voltage change was computed and tabulated in Table V for the VR75

and VR150. The change in the tube voltage of the VR105 with temperature was so erratic that no average coefficient was computed. A positive temperature coefficient of voltage change is one in which the tube voltage increases with increase of tube temperature.

TABLE V Temperature Coefficient of Voltage Change		
Type	Average Temperature Coefficient (per cent per degree centigrade)	Tube Current (milliamperes)
VR75	-0.004	5.5
VR105	See text	20.0
VR150	+0.026	20.0

EQUIVALENT CIRCUIT OF VR TUBES

It has been determined experimentally that the equivalent circuit shown in Fig. 3 is valid over the frequency range investigated. The magnitude of the resistance is

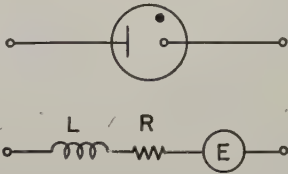
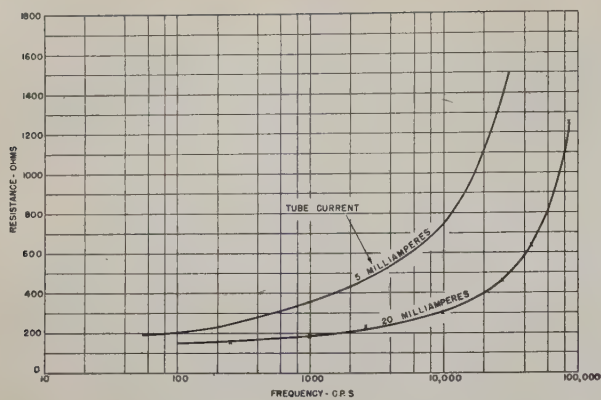


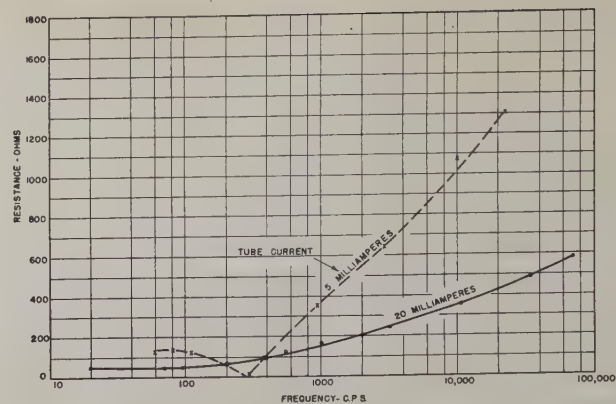
Fig. 3—Equivalent circuit for VR105 tube.

defined as the slope of the voltage-current curve at the operating current and is therefore a dynamic resistance, which is not constant when the tube current is varied (as can be seen by examination of Figs. 5 and 6). The ion inertia effects cause the dynamic resistance to change with frequency, as shown on Figs. 4(a), 4(b), and 4(c), and also introduces a quadrature component of tube current with the tube voltage leading the current. At a particular frequency the quadrature component of the tube current could be caused by an inductance, and the values of this inductance are plotted as a function of frequency on Figs. 4(d), 4(e), and 4(f). If the alternating-current amplitude is less than 10 per cent of the direct current, a voltage-regulator tube can be represented approximately for the purposes of equivalent circuit analysis by linear circuit elements in series with a constant voltage at a single frequency.

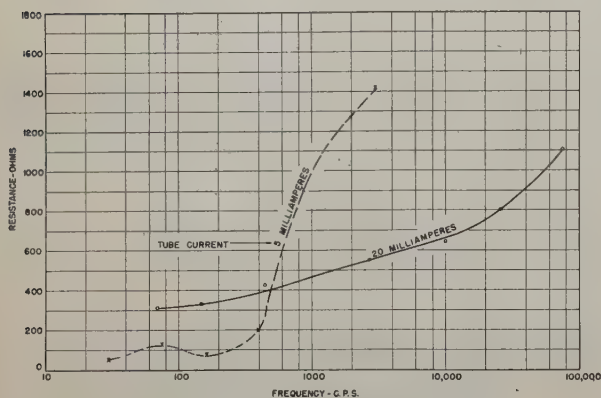
The values of the circuit elements were measured on a number of tubes of the three types investigated, and the values for one tube of each type are shown in Fig. 4(a) through 4(f). There is considerable variation of dynamic resistance and inductance values from tube to tube of the same type, particularly at low currents. The values of dynamic resistance as shown in Figs. 4(a), 4(b), and 4(c) are positive for most tube currents and frequencies; however, at a frequency of approximately 170 cycles per second and a current of 5 milliamperes the dynamic resistance of a VR105 is indicated as being zero on Fig. 4(b). Actually the dynamic resistance must have been



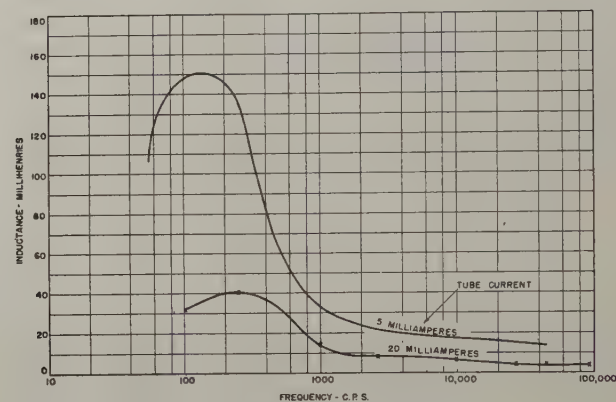
(a) Resistance of a VR75 tube.



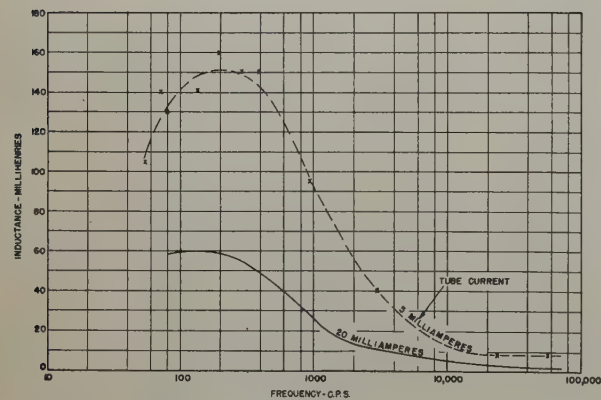
(b) Resistance of a VR105 tube.



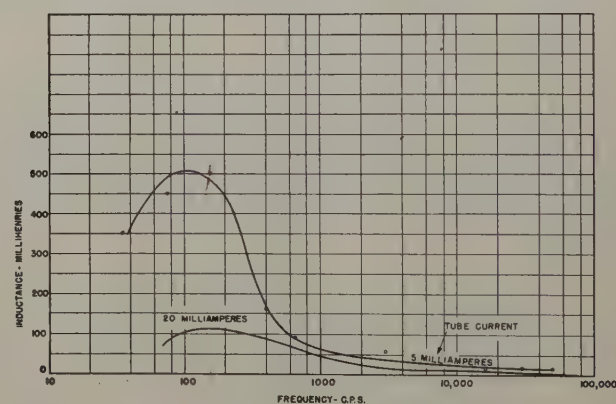
(c) Resistance of a VR150 tube.



(d) Inductance of a VR75 tube.



(e) Inductance of a VR105 tube.



(f) Inductance of a VR150 tube.

Fig. 4—Measured values for circuit elements for the equivalent circuit.

negative at this point, as oscillations<sup>6</sup> of approximately sinusoidal wave form would take place in the circuit when the signal voltage source was shorted out. The magnitude of the negative dynamic resistance was not measured because of the tendency of the circuit to oscillate in an uncontrollable manner.

The dynamic current-voltage characteristics of several VR105 and VR150 tubes were obtained by causing

the current through the tubes to increase and decrease linearly and photographing a cathode-ray-oscillograph screen with the current on one axis and the voltage on the other axis. Figs. 5 and 6 show the regulation curves for a repetition rate of 4 per second, and a current range of 5 to 30 milliamperes. The voltage scale shows the change in voltage across the tube, and not the total tube voltage. The following facts about the curves are of interest:

1. The path followed for increasing current is not the same as for decreasing current, giving a "hysteresis loop" effect to the picture.

<sup>6</sup> F. A. Maxfield and R. R. Benedict, "Theory of Gaseous Conduction and Electronics," pp. 350-351; McGraw-Hill Book Co., New York, N. Y., 1941.

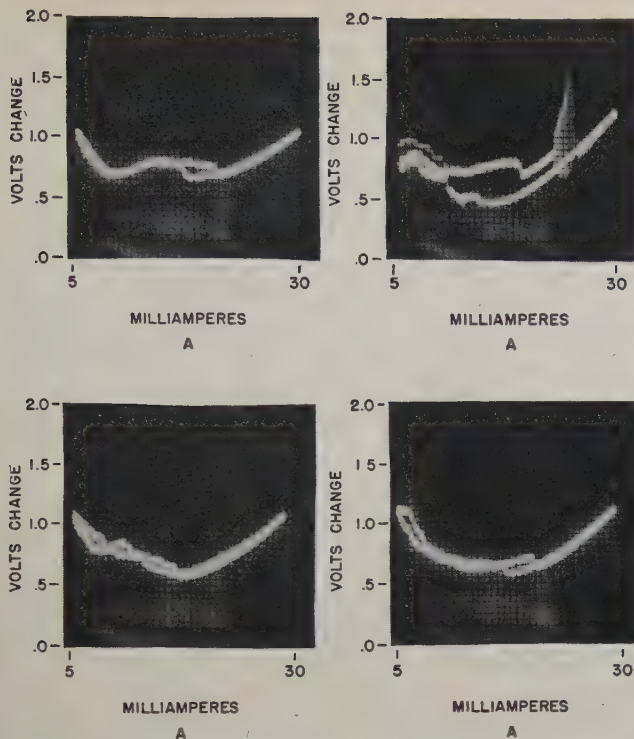


Fig. 5—Regulation characteristics of four VR105 tubes.

2. There are many abrupt changes of voltage over the operating current range. These are observed to occur simultaneously with sudden changes in cathode glow area.

3. Practically every curve has a negative slope over some current range, usually in the low-current region. One VR105-tube regulation curve shows an instability

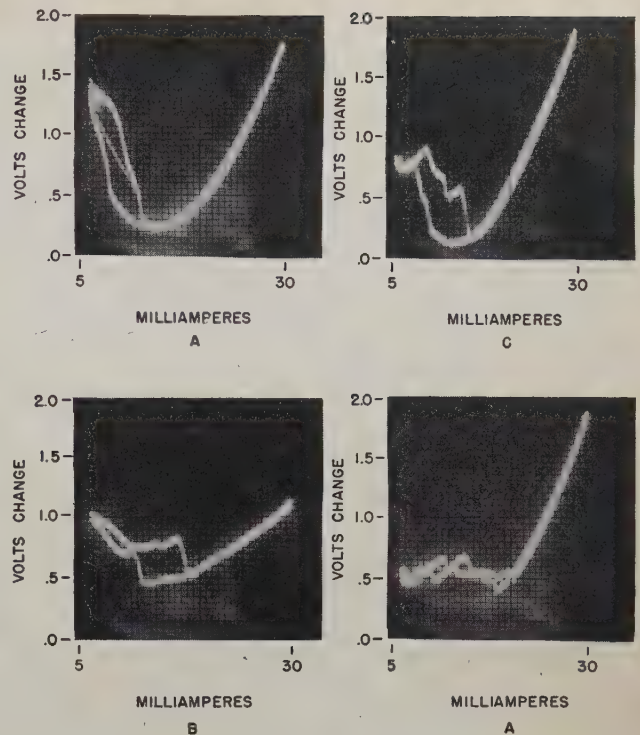


Fig. 6—Regulation characteristics of four VR150 tubes.

and oscillation of some type taking place at a current of around 20 milliamperes.

4. The slope of the regulation curve is higher for VR150 tubes than for VR105 tubes. At 20 milliamperes the slope in the case of most VR150 tubes exceeds 100 ohms, whereas in the case of most VR105 tubes the slope is approximately 50 ohms.

## Discussion on

# “A Current Distribution for Broadside Arrays Which Optimizes the Relationship Between Beam Width and Side-Lobe Level”\*

C. L. DOLPH

Henry J. Riblet:<sup>1</sup> C. L. Dolph, in a recent paper, has applied the properties of the Tchebyscheff polynomials to the problem of determining the current distribution in an equispaced broadside array which optimizes the relationship between beam width and side-lobe level. The following limitations may be removed by easy extensions of Dolph's method: (a) the spacing between radiators  $\geq \lambda/2$ ; (b) the beam width is characterized by the existence of a first null; (c) the current distribution is in phase; and (d) the current distribution is symmetrical about the center position of the array.

By the arguments of pages 336 and 337 of this paper, the voltage pattern of a symmetric, in-phase, equispaced linear array of  $n$  isotropic radiators is a polynomial in  $x$ ,  $x = \cos(d\pi/\lambda) \sin \theta$ . It is even and of  $n-1^{\text{st}}$  degree if  $n$  is odd, and odd of  $n-1^{\text{st}}$  degree, if  $n$  is even. The crux of the matter is that the pattern of the linear array is determined completely from a limited portion of the graph of an even or odd polynomial in  $x$ . Specifically, as  $\theta$  varies from  $-\pi/2$  through zero to  $+\pi/2$ ,  $x$  varies from  $\cos-(d\pi/\lambda)$  to one and then back to  $\cos(d\pi/\lambda)$ . It was Dolph's idea to select the Tchebyscheff polynomial of correct degree and use its properties to obtain the desired pattern. To make the method clear, consider

\* PROC. I.R.E., vol. 34, pp. 335-348; June, 1946.

<sup>1</sup> Submarine Signal Company, Boston, Massachusetts.

the graph of  $T_8(Z)$  shown in Fig. 1. All the roots of  $T_8(Z)$  occur between  $\pm 1$  and the maximum and minimum values lying between  $\pm 1$  are alternately  $\pm 1$ . It can easily be argued that  $T_8(Z)$  increases as  $Z^8$  for  $|Z| > 1$ . The observable pattern is given by points on this curve which lie between  $a = \cos(\pi d/\lambda)$  and one. It has a number of lobes depending on  $d$  of equal size. For his purpose, Dolph has considered the polynomial

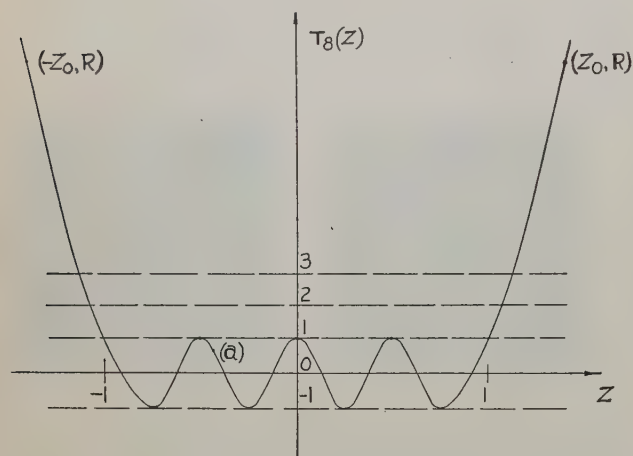


Fig. 1

graphed in Fig. 2 differing from the graph of Fig. 1 only by a change in the scale of the abscissa. It is clear that as  $x = Z$  ranges from  $\cos(\pi d/\lambda)$  to one, we first trace the side lobes and then reach the peak of  $R$  and retrace the pattern as  $x$  returns to  $\cos(\pi d/\lambda)$ . The side lobes are down by a factor of  $1/R$  and are all equal. The entire pattern is easily determined from Figs. 1 or 2. An antenna pattern obtained in this way may conveniently be called a Tchebyscheff pattern.

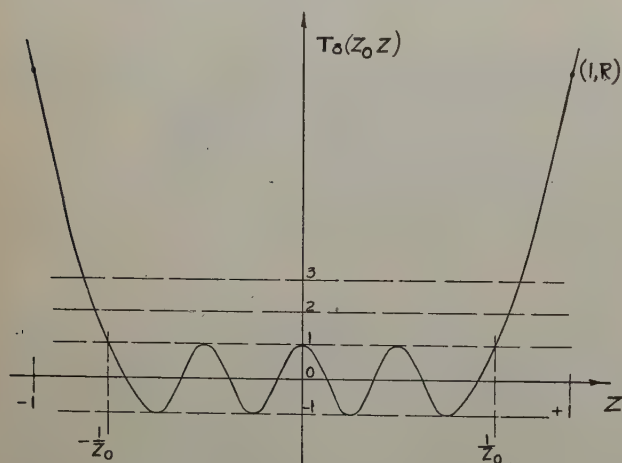


Fig. 2

The theorem that Dolph has used for his results is that any even polynomial of eighth degree which passes through  $(1, R)$  and the maximum null of  $T_8(Z_0 Z)$  and for all smaller positive values of  $Z$  is less than or equal to 1 in absolute value, must coincide with  $T_8(Z_0 Z)$ . It will be clear to the reader that such a curve would inter-

sect the curve of Fig. 2 in at least five points on the right side of the vertical axis and thus in ten points altogether. The fact that any two eighth-degree polynomials which have more than eight points in common must coincide, gives the desired result.

We are now in a position to extend the theorem of Dolph<sup>2</sup> in the following manner:

**THEOREM I.** For  $d \geq \lambda/2$ , any pattern from  $n$  symmetric, in-phase, equispaced, isotropic current sources which falls anywhere inside of or on the main beam of the Tchebyscheff pattern, which has the same peak, and which remains less than or equal to one in absolute value after the first side lobe of this pattern, coincides with the Tchebyscheff pattern.

*Proof* If  $d \geq \lambda/2$ , then the argument given above applies even though the hypothetical pattern has no nulls at all.

The reason for the condition on  $d$  is clear. If  $d < \lambda/2$ , then point  $a$  of Fig. 1 will fall on the right of the vertical

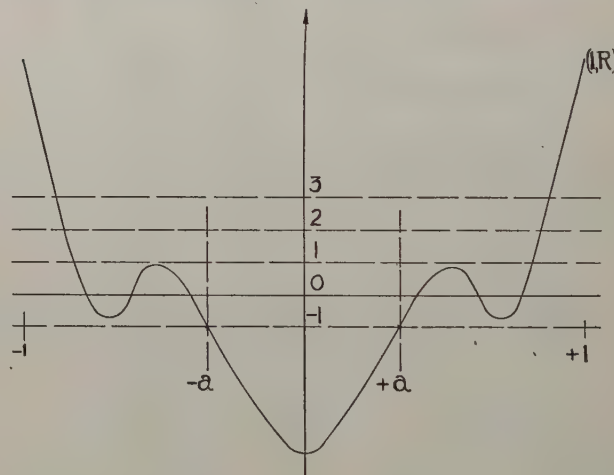


Fig. 3

axis and it is perfectly possible for an observable pattern to have side lobes which satisfy the conditions of Theorem I without getting any contradiction, since the corresponding polynomial is not restricted between  $\pm a$ . For example if  $d = \lambda/3$ , and  $a$  is equal to  $1/2$  polynomials of the appearance shown in Fig. 3 may be constructed having better side-lobe and beam-width characteristics than are obtainable from Dolph's procedure. A specific example is discussed later on.

**THEOREM II.** For  $d \geq \lambda/2$ , any pattern from  $n$  symmetric, equispaced, isotropic current sources which falls anywhere inside of or on the main beam of the Tchebyscheff pattern which has the same peak, and which remains less

<sup>2</sup> It is the theorem on antenna patterns implied by pp. 338-340 of the Dolph paper and is not to be confused with the theorem on polynomials referred to above.

<sup>3</sup> The proof given is probably not adequate when the hypothetical pattern merely touches the main beam of the Tchebyscheff pattern. A modification of the argument of the next theorem will cover this case.

than or equal to one in absolute value after the first side lobe of this pattern, coincides with the Tchebyscheff pattern.

*Proof:* The change in assumptions simply means that the polynomial  $P(x)$  in  $x$ , which defines the pattern of the linear array, will have complex coefficients in general. Let us assume that it is normalized so that its value at unity is the real number  $R$  and write it

$$P(x) = P_1(x) + jP_2(x)$$

where  $P_1(x)$  and  $P_2(x)$  have real coefficients. Now

$$|P(x)|^2 = P_1(x)^2 + P_2(x)^2$$

so that the general level of  $P(x)$  is greater than that of  $P_1(x)$  but  $P_1(x)$  is even or odd according as  $P(x)$  is even or odd and  $P_1(1) = R$ . Since it satisfies the assumptions of Theorem I it is  $T_{n-1}(Z_0x)$ , but  $P(x)$  falls inside or on this pattern and thus  $P_2(x) = 0$ .

**THEOREM III.** For  $d \geq \lambda/2$ , any pattern from  $n$  equispaced, isotropic current sources which falls anywhere symmetrically inside or on the main beam of the Tchebyscheff pattern, having the same peak, and which remains less than or equal to one in absolute value after the first side lobe of this pattern, coincides with the Tchebyscheff pattern.

*Proof:* It is clear that the most general pattern which a linear array can have is given by a limited range of values of  $P(x) + Q$  where  $Q$  has the form of (1) and (2) (page 336) except that cosine is replaced by sine. Since these functions are odd functions of  $\theta$  which contain  $y = \sin(\pi d/\lambda) \sin \theta$  as an odd factor, the remaining factor must be an even function of  $\theta$  and hence a polynomial in  $x$ . For example,

$$\sin 3 \frac{\pi d}{\lambda} \sin \theta = \sin \frac{\pi d}{\lambda} \sin \theta \{4x^2 - 1\}.$$

The most general linear array pattern then is obtained from a limited range of the expression

$$P = P(x) + yQ(x),$$

where the degree of  $Q(x)$  is one less than the degree of  $P(x)$  and where  $Q(x)$  is odd or even in  $x$  accordingly as  $P(x)$  is even or odd. Now it is clear from the evenness of  $x$  and oddness of  $y$  in  $\theta$  that if  $|P(x) + yQ(x)| \leq |P(x)|$  for  $\theta$ , that  $|P(x) + yQ(x)| \geq |P(x)|$  for  $-\theta$ . Thus  $|P(x) + yQ(x)| \leq 1$  for a range of plus and minus values of  $\theta$  only if  $|P(x)| \leq 1$  for the same values of  $\theta$ . A similar argument holds for any two points  $\theta_1$  and  $-\theta_1$  symmetrical about the peak of the Tchebyscheff pattern. Thus  $P(x)$  satisfies the hypothesis of Theorem II and so is  $T_{n-1}(Z_0x)$ ; but  $|P(x) + yQ(x)| \leq T_{n-1}(Z_0x)$  at  $\pm\theta_1$  and is less than or equal to 1 for plus and minus values of  $\theta$  after the first side lobe of the Tchebyscheff pattern only if  $y(\theta_k)Q(\theta_k) = 0$ , where  $\theta_k$  includes those values of  $\theta$  corresponding to values of  $x$  where  $|T_{n-1}(Z_0x)| = 1$  and  $\theta_1$ . Since  $y(\theta) \neq 0$  for any of these values it follows that

$Q(\theta)$  must have  $n$  roots. This, of course, is impossible unless  $Q(x) \equiv 0$ . It is to be observed that Theorems I, II, and III lose their meaning if  $d$  approaches  $\lambda$  too closely, since then  $a$  will fall to the left of  $-(1/Z_0)$ .

The condition that  $d$  must be greater than or equal to  $\lambda/2$  seems to have been overlooked by Dolph. If  $d < \lambda/2$  and the linear array has an odd number of radiators, it is possible to exhibit patterns with narrower beam widths and lower side lobes than those given by Dolph. In fact an optimum for which Theorems I, II, and III still hold can be shown for the following construction: We choose  $x = \cos(2\pi d/\lambda \sin \theta)$  so that the polynomial determining the pattern is general and of degree  $n$ , if there are  $2n+1$  isotropic radiators. Then, if we use  $T_n(x)$  and subject the abscissa to a translation and change of scale which places all of the small ripples of  $T_k(x)$  in the range from  $a$  to 1 so as to have the proper side-lobe level, we obtain an optimum pattern. Incidentally, this uses up all the Tchebyscheff polynomials so that there appears to be no equivalent procedure for linear arrays with an even number of elements spaced less than a half-wavelength apart.

By way of an example, consider seven radiators spaced  $\lambda/3$  apart. The most general pattern which they may have is obtainable from the expression

$$a_0 + a_1 \cos \frac{2\pi}{3} \sin \theta + a_2 \cos \frac{4\pi}{3} \sin \theta + a_3 \cos \frac{6\pi}{3} \sin \theta.$$

If we put

$$\frac{2\pi}{3} \sin \theta = \cos^{-1} x,$$

this may be written

$$a_0 + a_1x + a_2(2x^2 - 1) + a_3(4x^3 - 3x).$$

As  $\theta$  ranges from  $-\pi/2$  through zero to  $+\pi/2$ ,  $x$  varies from  $-1/2$  to 1 and back to  $-1/2$ . In order to locate all of the ripples of  $T_3(x)$  in this region and also obtain a side-lobe level  $1/9$  of the main lobe, we must subject the abscissa to a linear transformation  $ax + b$  so that

$$a(-1/2) + b = -1$$

$$a(1) + b = 3/2.$$

The number  $3/2$  is chosen because  $T_3(3/2) = 9$ . It is easily found that

$$a = 5/3 \quad \text{and} \quad b = -1/6.$$

So that

$$\begin{aligned} a_0 - a_2 + (a_1 - 3a_3)x + 2a_2x^2 + 4a_3x^3 &= T_3(5/3x - 1/6) \\ &= 18.5x^3 - 5.55x^2 - 4.45x + .481. \end{aligned} \quad (1)$$

From which we immediately conclude that

$$a_0 = -2.32; a_1 = 9.45; a_2 = -2.77; a_3 = 4.63. \quad (2)$$

To return to the variable  $\cos(2\pi d/\lambda) \sin \theta$ , we replace  $x$  in (1) by  $2x^2 - 1$ .

We then have an even polynomial in  $x$  whose graph would be similar to that given in Fig. 3. By increasing the number of elements for any fixed over-all length, we can construct a linear array antenna of arbitrary beam width and side-lobe level. The current distribution given in (2) shows the rapid phase-reversal characteristic of "super-gain" antennas.

**C. L. Dolph:**<sup>4</sup> I am indebted to H. J. Riblet for his observation that the "optimum relationship between beam width and side-lobe level" which I derived in a recent paper is valid only if the spacing between the elements is greater than or equal to one-half wavelength and for his generalization of the method which removes this limitation. It should be remarked, however, that the absence of the optimum property for spacings less than one-half wavelength in no way affects the other characteristics of the Tchebyscheff distribution and which, therefore, still offers many advantages from the viewpoint of pattern control, even in this case.

I would like to take this opportunity to indicate how some of the calculations necessary for the determination of the Tchebyscheff distribution can be further simplified. In order to determine the Tchebyscheff distribution for a particular side-lobe level it is first necessary to solve the equation

$$T(z_0) = r \quad (24)$$

for  $z_0$ , where  $r/1$  represents the desired side-lobe level. Now it is known that<sup>5</sup>

$$T_M(z) = \frac{(z + \sqrt{z^2 - 1})^M + (z - \sqrt{z^2 - 1})^M}{2} \quad (a)$$

for all values of  $z$ . (To see that the expression on the right-hand side of the above equation agrees with  $\cos(M \arccos z)$  for  $|z| \leq 1$ , it is only necessary to set  $z = \cos \phi$  and use De Moivre's theorem.) Thus (24) can be replaced by

$$(z_0 + \sqrt{z_0^2 - 1})^M + (z_0 - \sqrt{z_0^2 - 1})^M = 2r. \quad (24.1)$$

Equation (24.1) can be solved explicitly for  $z_0$  by setting

$$(w)^{1/M} = z_0 + \sqrt{z_0^2 - 1} \quad (b)$$

and observing that

$$\left(\frac{1}{w}\right)^{1/M} = z_0 - \sqrt{z_0^2 - 1} \quad (c)$$

so that (24.1) becomes

$$w + \frac{1}{w} = 2r \quad (24.2)$$

or

$$w^2 - 2rw + 1 = 0 \quad (24.3)$$

which has the roots

$$w_1 = r + \sqrt{r^2 - 1}$$

and

$$w_2 = r - \sqrt{r^2 - 1}$$

with the property that

$$w_2 = \frac{1}{w_1}.$$

From (b) it therefore follows that

$$\begin{aligned} z_0 &= \frac{1}{2} \left[ (w_1)^{1/M} + \left(\frac{1}{w_1}\right)^{1/M} \right] \\ &= \frac{1}{2} \left[ \left(\frac{1}{w_2}\right)^{1/M} + (w_2)^{1/M} \right] \end{aligned}$$

or, substituting the value of  $w_1$ , and rationalizing,

$$z_0 = \frac{1}{2} \{ (r + \sqrt{r^2 - 1})^{1/M} + (r - \sqrt{r^2 - 1})^{1/M} \}. \quad (24.4)$$

Thus one finds the remarkable result that to solve (24) or (24.1) for  $z_0$ , it is only necessary to interchange  $z_0$  and  $r$  and replace  $M$  by  $1/M$  in (24.1).

The determination of the currents  $I_k$  as functions of  $z_0$  can also be accomplished without the necessity of using the expansions of the pattern and  $T_M(z_0 x)$  as polynomials in  $x$  which lead to (22) and (23). One merely observes that the problem of determining the  $I_k$ 's is equivalent to that of finding the coefficients in a finite Fourier series, which for the  $(2N+1)$  cases is explicitly

$$\sum_{k=0}^N T_k \cos 2ku = T_{2N}(z_0 \cos u).$$

This problem is, however, a standard one in interpolation theory, and it can be treated by standard techniques since by (a), with  $M=2N$ , the right-hand side can be simply computed at any desired point. Since a complete and immediately applicable discussion of this problem can be found in many books, including Whittaker and Robinson,<sup>6</sup> the explicit formulas for the  $I_k$ 's will not be repeated here. It is clear that, for arrays containing a great number of elements, this method possesses certain computational advantages.

<sup>6</sup> E. T. Whittaker and G. Robinson, "The Calculus of Observations," pp. 260-267; D. Van Nostrand Co., New York, N. Y., 1924.

<sup>4</sup> University of Michigan, Ann Arbor, Michigan.

<sup>5</sup> Courant-Hilbert, "Methoden der mathematischen physik," vol. 1, p. 439; Julius Springer, Berlin, 1931, and Interscience Publishers, Inc., New York, N. Y., 1943.

# Correspondence

## Empirical Formula for Amplification Factor

The amplification factor  $\mu$  of vacuum tubes is ordinarily computed by the tube-design engineer using formulas developed from electrostatics. A formula derived from Maxwell was possibly the earliest to be proposed. Later, the improved formula of Vogdes and Elder<sup>1</sup> became commonly used, although its validity was questionable for ratios of grid-wire diameter to pitch greater than about 0.3. In 1944, Herne<sup>2</sup> supplied much more accurate calculations which apply up to ratios as high as 0.7. His results were obtained by involved calculation and are useful chiefly in tabular or curve form, for which Herne supplied numerical values.

A simple empirical formula of sufficient validity to be useful is often a great asset since it can be treated analytically, differentiated, integrated, etc. Van der Bijl, in 1914, found such a formula experimentally, but its results are not in good agreement with any of the more recent and theoretically accurate formulas. Herne stated his results in the following form:

$$\mu = (2\pi NS + c)b$$

where  $N$  is the turns per unit length of the grid,  $S$  is the grid-to-anode spacing factor,  $d$  is the grid-wire diameter, and  $b$  and  $c$  are very complicated functions of  $Nd$ . Ordinarily  $c$  (which has a maximum value of 0.7) can be neglected in comparison with  $2\pi NS$  and only the  $b$  function is of importance. Herne's table for the  $b$  values was used by the writer to obtain a simple empirical formula:

$$b = 0.2 + 6.8Nd + 680(Nd)^5$$

which gives agreement to better than 8 per cent with Herne's values between  $Nd=0.002$  and  $Nd=0.50$ . Since grids of larger or smaller  $Nd$  are extremely uncommon, the agreement is sufficient for practical use. For  $Nd$  less than 0.15, only the first two terms need be used. Thus it is seen that:

$$\mu \approx 2\pi NS[0.2 + 6.8Nd + 680(Nd)^5]$$

A plot of the error of the empirical formula, assuming Herne's table to be correct, is given on Fig. 1. The errors of the Maxwell and Vogdes and Elder formulas are also plotted. It is seen that the empirical formula is sufficiently close for most calculations needed in tube design, whereas even the Vogdes and Elder form is seriously in error above  $Nd=0.3$ . In using an empirical formula in analysis, the disagreement from correctness, which may be small in the function and its integral, may become much greater in the derivative. The derivative of the above formula was compared with that of Herne's calculated curve and found to agree within 15 per cent between  $Nd=0.01$  and  $Nd=0.45$ . This is again believed to be sufficiently accurate for most practical purposes.

It is customary to use such  $\mu$  formulas

with mesh grids by using the reasoning advanced by Kusunose.<sup>3</sup> As he pointed out,  $Nd$  is the fraction of the total area covered by the grid and  $N$  is the total lineal length of

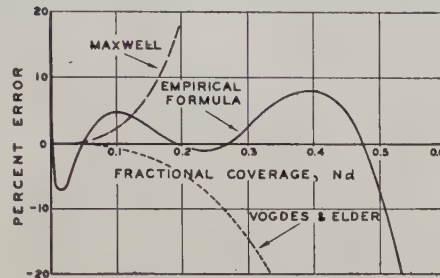


Fig. 1—Curve showing error of proposed empirical formula compared with the inexact theoretical formulas of Maxwell and Vogdes and Elder for amplification factor.

exposed grid conductor per unit area of the grid. When the grid-to-cathode distance is small compared with the grid pitch, any formulation of amplification factor must be corrected for variations along the cathode surface as suggested by Liebmann.<sup>4</sup>

E. W. HEROLD  
RCA Laboratories  
Princeton, N. J.

<sup>3</sup> Y. Kusunose, "Calculation of characteristics and the design of triodes," *Proc. I.R.E.*, vol. 17, pp. 1706-1749; October, 1929.

<sup>4</sup> G. Liebmann, "The calculation of amplifier valve characteristics," *Jour. I.E.E.*, vol. 43, Part III, pp. 138-152; May, 1946.

## Note on the Sporadic-E Layer

The analysis of the sporadic-E region at the Watheroo Magnetic Observatory by H. W. Wells<sup>1</sup> reintroduces the subject of the formation of the *intense Es cloud*. It has been established by Mimno and Pierce<sup>2</sup> that the vertical extent of the high-density *Es cloud* is less than 5 kilometers. Recently,<sup>3</sup> certain investigators of the Department of Scientific and Industrial Research in Great Britain have shown experimentally that the vertical thickness of the *Es cloud* is between 50 meters and 500 meters. The limited lateral extent of the *Es cloud* has been mentioned in the observations of Wilson,<sup>4</sup> Eyfrig,<sup>5</sup> and Conklin.<sup>6</sup> Much of the study of *Es* formation must be interpreted in terms of the relationship of radio wave propagation and maximum usable frequencies above 40 megacycles. These data may be obtained through the use of automatic ionospheric

<sup>1</sup> H. W. Wells, "Sporadic E-region ionization at Watheroo Magnetic Observatory 1938-1944," *Proc. I.R.E.*, vol. 34, pp. 950-955; December, 1946.

<sup>2</sup> H. R. Mimno and J. A. Pierce, "The apparent motion of clouds of abnormal E region ionization," *Phys. Rev.*, vol. 55, p. 1128; June 1, 1939.

<sup>3</sup> Discussion before the Royal Astronomical Society, London, England; May 31, 1946.

<sup>4</sup> M. S. Wilson, "Five-meter wave paths," *QST*, vol. 25, pp. 23-29; August, 1941.

<sup>5</sup> R. Eyfrig, "Echo measurements in long-distance transmission and their relation to zenithal reflections," *Hochfrequenz Tech. und Elektroakustik*, vol. 56, pp. 161-174; December, 1940.

<sup>6</sup> E. H. Conklin, "56 megacycle reception via sporadic-E-layer reflections," *Proc. I.R.E.*, vol. 27, pp. 36-41; January, 1939.

recorders<sup>1,2,3,5</sup> and vertical-incidence frequency measurements, or by numerous observations of point-to-point communication systems employing radio frequencies in excess of 40 megacycles.

With the exception of Wilson,<sup>4</sup> Fendler,<sup>7</sup> and Conklin,<sup>6</sup> there has been little work in the correlation of vertical-incidence frequency measurements and single-hop radio communication at the very-high frequencies. An initial examination by the writer of periods of time when *Es*-supported communication in the 56- to 60-megacycle radio amateur band indicates that the extent and density of the *Es cloud* may also be found by this method. An important result of this analysis is the spatial distribution of the points of incidence with the *Es cloud* for single-hop radio transmission, which appears to confirm the prevalence of large-scale air circulation at the height of the *E* region.

Air-mass circulation of high velocities at the *E*-region height has been particularly demonstrated in the rapid distortion of long-enduring meteor trains.<sup>8</sup> Rawer<sup>9</sup> has suggested that this mass transport of ions and free electrons within the *E* region to anticyclonic areas may create the *Es cloudlike* patches of very high density. The observations of Eyfrig<sup>5</sup> in Germany have, however, tentatively established the drift of *Es clouds* to be from 15 to 125 kilometers per hour. These were taken from vertical-incidence measurements at Kochel and Berlin which were made in 1939.

The limit of *Es*-supported very-high-frequency communication is approximately 1500 miles for a single-hop transmission. This has been correlated with amateur two-way contacts by Conklin.<sup>6</sup> By simple optics the midpoint in the great-circle path between any two radio amateur stations in two-way contact must represent the area of the *Es cloud*, assuming a constancy of height of about 110 kilometers. When over 95 two-way amateur contacts were examined in the 56- to 60-megacycle band on June 5, 1938, there was an apparent spatial and time distribution of the midpoints. Between 1730 and 1800 hours Eastern Standard time, approximately 80 per cent of the reported two-way contacts had points of incidence at the height of the *E* region centered directly over the Commonwealth of Pennsylvania. From 1815 to 1845 hours Eastern Standard time, approximately 75 per cent of the midpoints during this period were located within the area bounded by the northwest shore of Lake Erie and the southern border of the State of Ohio. From 1930 to 2000 hours Eastern Standard time, approximately 85 per cent of the points of incidence were calculated to be directly over the center of the State of Indiana. This corresponds to a drift of from 290 to 380 kilometers per hour. Al-

<sup>7</sup> E. Fendler, "The transmission of ultra-short waves through ionospheric action," *Hochfrequenz und Elektroakustik*, vol. 56, pp. 41-47; August, 1940.

<sup>8</sup> C. P. Olivier, "Long enduring meteor trains," *Proc. Amer. Phil. Soc.*, vol. 85, pp. 93-135; January, 1942.

<sup>9</sup> K. Rawer, "The formation of the abnormal E layer," *Naturwiss.*, vol. 28, p. 577; August, 1940.

<sup>1</sup> Vogdes and Elder, *Phys. Rev.*, vol. 24, p. 683; 1924; vol. 25, p. 255; 1925.

<sup>2</sup> H. Herne, *Wireless Eng.*, vol. 21, p. 59; February, 1944.

though these velocities are considerably higher than those obtained by Eyfrig, it is too early in this study to ascertain if they are abnormal.

Although Wells holds the C. T. R. Wilson<sup>10</sup> hypothesis of possible *Es* formation as the direct result of high electric fields to be untenable in view of his observations at Huancayo, Peru, it may become necessary to take into consideration certain factors of air circulation and subsidence at the *E*-region height before we may discard this suggestion. I should like to point out that the recent observations of Stoffregen<sup>11</sup> in Upsala, Sweden, have again indicated a very close relationship between intense *Es* and thunderstorms. Whether the *Es* is the result of the runaway electron or is formed in the pocket above the heart of the thunderstorm remains to be determined.

OLIVER P. FERRELL  
CQ Magazine

342 Madison Ave., New York 17, N. Y.

<sup>10</sup> C. T. R. Wilson, "The electric field of a thundercloud and some of its effects," *Proc. Phys. Soc.*, vol. 37, pp. 32D-37D; February, 1925.

<sup>11</sup> W. Stoffregen, "Ionospheric measurements in connection with thunderstorm research," *Arkiv. für Mat., Astr. and Fysik.*, Bd. 30, Hft. 4; Summer, 1944.

## A Vacuum Heating Element

The vacuum heating element to be described is a vacuum tube with an envelope temperature well over 1000 degrees centigrade.<sup>1</sup> Although a simple device, this unusual feature may render its description interesting to designers of electronic tubes in connection with problems which can be solved by increased envelope temperature.

This element is the continuation of a development initiated by Paul Schwarzkopf.<sup>2</sup> He designed a high-temperature heating element consisting of a molybdenum rod in a tightly fitting sillimanite tube with gas-tight terminals at both ends. Further improvements, such as higher resistance and location of both terminals at the same end, were desirable and resulted in the present construction, shown in Fig. 1.

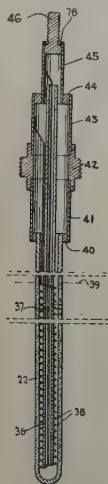


Fig. 1

The gas-tight ceramic tube 22, which is closed at the lower end, contains the heater

<sup>1</sup> This heating element was developed at the Research Laboratories of the American Electro Metal Corporation in Yonkers, N. Y.

<sup>2</sup> P. Schwarzkopf, "Electrical resistance furnaces for high temperatures," *Metals and Alloys*, vol. 13, pp. 45-49; January, 1941.

coil 38. The lead wire 37 goes back to the terminal along the axis of the coil insulated by the ceramic tube 36. The heating portion of the element extends from the lower end of tube 22 to the plane 39. The section between plane 39 and the other end of the tube 22 is located within the furnace wall. Its length is determined by the thickness of the furnace wall. The end of the ceramic tube 22 is sealed vacuum tight to the metal tube 41 with the glass ring 40. Tubes 41, 42, and 43 are brazed together. Tube 45 is sealed to tube 43 with a glass ring 44, and the ends of the heater coil are welded to the tubes 43 and 45. The element is placed into a vacuum bell jar and evacuated through the gap between the tube 45 and the terminal screw 46. This gap is sealed in vacuum by melting down with high-frequency heating the ring 78 consisting of a brazing alloy. The supply wires are fastened to the screw 46 and the threaded part of tube 42.

The selection of materials depends upon the maximum temperature desired. The heating element was designed to operate at surface temperatures up to 1500 degrees centigrade. Under these conditions, mullite ( $3\text{Al}_2\text{O}_3\text{-SiO}_2$ ) proved to be most suitable for the tube 22. Mullite is still fairly rigid at 1500 degrees centigrade, and is not reduced by the molybdenum heater wire. It also remains sufficiently vacuum tight. The diffusion of gas through high-refractory ceramic tubes has been investigated by William F. Roeser.<sup>3</sup> The basic mechanism does not seem to be very well understood. His measurements indicate that the leakage is very small up to 1000 degrees centigrade, and increases at a more rapid rate above this temperature. It varies considerably among different products and even among tubes of the same manufacture. An estimate based on his values indicates that the amount of air diffusing into the element is small enough to be taken up by an ample supply of the getter described below. It is not known whether the rate of diffusion changes during operation.

Mullite has approximately the same thermal expansion as tungsten. Hence kovar was used for the tube 41 and Corning 775 for the glass ring 40. The inner ceramic tube 36 consists of alumina. The temperature in the axis of the element may be considerably higher than the surface temperature. The difference depends upon the heat dissipation. The inner tube has to withstand the full supply voltage. One particular construction permitted a maximum alternating-current supply voltage of 75 volts at the maximum operating temperature of 1500 degrees centigrade.

It is common practice to connect a vacuum tube to an exhaust system and to degassify all elements by heating them for some time to a temperature well above operating temperature. This treatment cannot be applied to the heating element, as it will operate at the highest temperature the inner and outer ceramic will withstand. Elements which were not thoroughly degassed and did not contain a getter had a maximum life of several hours only. The molybdenum was transported from the coil

to the inner wall and caused the short-circuiting of adjacent turns. This phenomenon, which is well known in poorly pumped incandescent lamps, is caused by traces of water vapor. It has been quantitatively treated for tungsten wire, which behaves similarly to molybdenum wire, by H. Alterthum and F. Koref.<sup>4</sup> The hot wire is oxidized; the oxide sublimates on the wall, and is there reduced.

It was possible to bind the oxygen released from the parts and diffusing through the wall during operation of the element by the employment of a getter. Various getters were tried, and zirconium proved to be the most suitable. Zirconium powder was mixed with a refractory oxide such as beryllium oxide or aluminum oxide and deposited between the turns of the coil. The zirconium is gradually converted into oxide (and possibly nitride) during operation. The oxide is not reduced by hydrogen or molybdenum at the operating temperatures.

A moderate vacuum was produced in the element in order to prevent the creation of a pressure in excess of one atmosphere within the element. This would occur since zirconium does not bind the nitrogen fast enough, especially at temperatures which are lower than the maximum operating temperature. But it may be possible to dispense with the evacuation.

At the maximum operating temperature of 1500 degrees centigrade, and with large heat dissipation, the mullite tube develops porous spots after approximately 500 to 800 hours. This is probably caused by the large temperature gradient in its wall, and could possibly be avoided by reducing the heat dissipation. Life tests were conducted at 1300 degrees centigrade with the heating elements operated continuously 16 hours a day. The elements were still in perfect operating condition after one and one-half years.

GERHARD S. LEWIN

692 Valley Street

Maplewood, New Jersey

<sup>4</sup> H. Alterthum and F. Koref, "Heterogeneous equilibria between tungsten and oxygen and tungsten and water vapor at elevated temperatures," *Zeit. für Elek.*, vol. 31, pp. 508-511; September, 1925.

## Radar Reflections from the Lower Atmosphere

Radar reflections arising, apparently, from stratifications in the lower atmosphere at heights of from 100 to 1200 yards were observed recently in an exploratory experiment conducted at the Holmdel Laboratory.<sup>1</sup>

The existence of abrupt discontinuities in the dielectric constant of the atmosphere had been postulated in order to account for some of the multiple-path transmission phenomena observed during our microwave propagation experiments.<sup>2</sup> To see if more direct evidence of such reflecting layers could be obtained, the radar experiment was set up using apparatus which was immediately available, although perhaps not in first-

<sup>1</sup> Earlier evidence of radar echoes from the atmosphere has been reported by M. W. Baldwin, Jr., of these Laboratories: "Radar echoes from the nearby atmosphere," presented at the joint U.S.R.I.-I.R.E. Meeting, Washington, D. C., May 2-4, 1946.

<sup>2</sup> A. B. Crawford and W. M. Sharpless, "Further observations of the angle of arrival of microwaves," *Proc. I.R.E.*, vol. 34, pp. 845-848; November, 1946.

<sup>3</sup> W. F. Roeser, "The passage of gas through the walls of pyrometer protection tubes at high temperature," *Jour. Res. Nat. Bur. Stand.*, vol. 7, pp. 485-494; September, 1931.

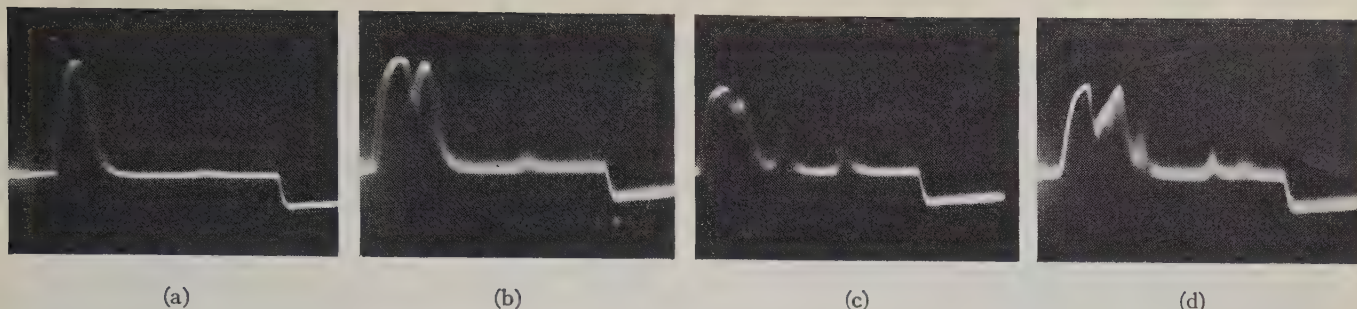


Fig. 1—(a) Taken under normal transmission conditions. Outgoing pulse at left, range mark (1100 yards) at right. (b) Strong echo at 150 yards. (c) Echoes at 100 yards, 325 yards, and 650 yards. (d) Multiple echoes.

class operating condition. This apparatus included a 3-centimeter radar transmitter, a double-detection receiver, a radar indicator, and two shielded-lens antennas with beams pointed upward. Separate antennas were used for transmitting and receiving to eliminate the radar transmit-receive switch with its relatively long recovery time.

During the day and on windy nights when a "standard atmosphere" and normal transmission conditions might be expected to exist, the radar oscilloscope usually appeared as shown in Fig. 1 (a), in which only the outgoing pulse is seen. On clear, calm nights, such as those favorable for anomalous propagation, echoes were observed as shown in Figs. 1 (b), (c), and (d). The echo activity varied greatly from instant to instant as well as from day to day. A particular echo usually did not persist for a long period of time; it might disappear after a few seconds or a minute only to reappear at approximately the same range a short time later.

It is rather surprising that atmospheric stratifications apparently can exist which are sufficiently abrupt as to cause reflections at normal incidence of waves as short as 3 centimeters. Also, these reflections may or may not be closely associated with the phenomenon of multiple-path transmission. However, the preliminary data we have obtained strongly suggest that here is a tool which might well be used along with the standard meteorological-balloon technique to investigate the role of the lower atmosphere in microwave propagation.

The experimental work described was carried out by W. E. Kock, A. B. Crawford, and V. C. Rideout of the Radio Research Department of Bell Telephone Laboratories.

H. T. FRIIS

Bell Telephone Laboratories, Inc.  
Holmdel, New Jersey

## Distortion and Acoustic Preferences

I am inclined to believe that the contributions to the discussion on "Broadcast Listener Acoustic Preferences" have hardly done justice to Messrs. Eisenberg and Chinn.

Accepting the fact that amplitude distortion will result in listener preference for a restricted range, the whole problem divides very neatly into three sections:

- What amount of distortion is just recognizable?
- What frequency range is preferred if this limit is not exceeded?
- Would the preference be the same if electroacoustic equipment were not in-

terposed between the orchestra and the listener?

Data are available on all these points and, while not sufficient to "prove" the point, they do give considerable support to Messrs. Eisenberg and Chinn.

Regarding (a), Massa<sup>1</sup> published the results of an investigation in 1933, which indicated that 3 to 5 per cent was a lower limit of "just recognizable" distortion, even when comparison with the undistorted (?) was possible.

These results are now about 14 years old and were somewhat restricted in scope, but a much more detailed investigation was made by Braunmuhl and Weber<sup>2</sup> in 1937. This was particularly thorough in that second- or third-harmonic distortion (and appropriate intermodulation) could be introduced over the whole band or over restricted portions of the band.

Broadly the results serve to confirm Massa's figures of about 3 per cent as the "just recognizable" point. It will be remembered that Messrs. Eisenberg and Chinn quote figures below 1 per cent as their system distortion.

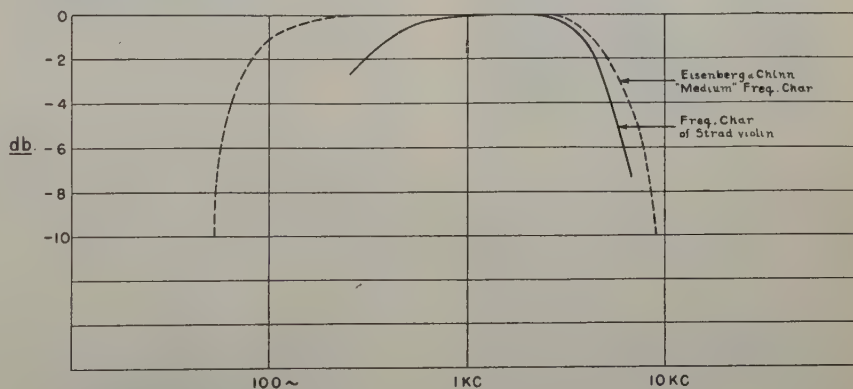


Fig. 1

Point (c) is really the whole crux of the question, as it would serve to settle the whole problem. Strong evidence is contained in a series of papers by Professor Saunders<sup>3</sup> on the factors that determine acceptable quality of tone in the violin. A wide range of suggestions was explored, but as the present discussion concerns frequency range, we will confine attention to this point. We may summarize the series of papers in stating

<sup>1</sup> F. Massa, "Permissible amplitude distortion of speech in an audio reproducing system," *Proc. I.R.E.*, vol. 21, pp. 682-690; May, 1933.

<sup>2</sup> H. J. Braunmuhl and W. Weber, "Recognition of nonlinear distortion," *Akus. Zeits.*, pp. 135-147; 1937.

<sup>3</sup> F. A. Saunders, "Mechanical action of instrument of the violin family," *Jour. Acous. Soc. Amer.*, vol. 17, pp. 169-187; January 1946.

that the chief difference between violins of the Stradivarius class and 30/— (\$6) models is that the energy output of the former is largely confined to the band below 6 kilocycles. For comparison the "frequency characteristic" of a typical Strad and the medium range used by Eisenberg and Chinn is plotted in Fig. 1.

The evidence is so strong that it is worth while examining the reasons for the belief that "wide band" should be preferred. These appear to be as follows:

- (1) Analysis of the energy output of orchestral instruments indicates that there are components up to 20 or 30 kilocycles.
- (2) Analysis of the performance of the human ear indicates that there is response up to 10 to 20 kilocycles, depending on age.
- (3) An intuitive feeling that we ought not to restrict the range because the ear having wide range will appreciate wide-range noise.
- (4) A lot of unco-ordinated evidence of the type, "I have a wide-range radio set and everybody says it's wonderful," etc.

Our intuition may be right, but to the

best of the writer's knowledge every attempt to confirm point (4) (Messrs. Eisenberg and Chinn's experiment is the latest and most thorough attempt) has resulted in the rejection of wide range unless the audience was composed of engineers presumably aware of (1) and (2). There can be no doubt that there is something wrong either with our intuition or our experimental techniques, and that quibbling about minor points is not the right attitude. This thesis is elaborated in a contribution to appear shortly in *Electronic Engineering*.

JAMES MOIR

The British Thomson-Houston Co., Ltd.  
Rugby, England

## CORRECTION

Dr. M. J. O. Strutt has called to the attention of the editor the following errors which appeared in his paper, "Noise Reduction in Mixer Stages," PROCEEDINGS OF THE I.R.E., vol. 34, pp. 942-950, December, 1946.

1. Page 943, footnote references 7 and 15: "Mautner" should read "Malter."

2. Page 943, footnote reference 11: "Alcademischer" should read "Akademischer."

3. Page 944, first column, first line below equation (9): the expression  $8kT_e T_c \Delta f$  should read  $8kT_c \Delta f$ .

4. Page 945, equations (10) and (11): one symbol  $T_c$  should be canceled. The same cancellation should be carried out on page 946, the eighth line below equation (21), and the fifteenth line below equation (21) (in both

expressions of the latter line), as well as on the nineteenth line from the bottom of the first column of page 948.

5. Page 946, first column, third line above the section III heading: the symbol  $F_2$  should read  $F_r$ . In the fifth line above the section III heading: the symbol  $P_{in}$  should read  $P$ .

6. Page 946, second line below equation (21):  $I_{f_{in}}^2$  should be  $I_{f_{in}}^2$ .

7. Page 949: in the last sentence of the caption of Fig. 6, the indications "a" and "b" should be reversed.

8. Page 950, second column, eighth line from top: the word "No" should start a new paragraph.

## Contributors to Proceedings of the I.R.E.



ANATOLE M. GUREWITSCH

Anatole M. Gurewitsch (A'38-SM'45) was born in Russia on June 12, 1911. He was graduated from the Swiss Federal Institute of Technology in Zürich, Switzerland, with the degree of Diploma Engineer in 1935. In 1937 he entered the employ of the General Electric Company, and in 1940 he was graduated from the three-year program of the advanced course in engineering of the General Electric Company. He is now a member of the General Electric Company Research Laboratory Staff, Schenectady, N. Y.



George M. Kirkpatrick (S'41-M'45) was born on August 18, 1919, at Roseville, Illinois. He received the B.S. degree in electrical engineering in 1941 from the University of Illinois, and immediately entered the employ of the General Electric Company. He is at the present time associated with the advanced engineering training program of the General Electric Company in Schenectady,

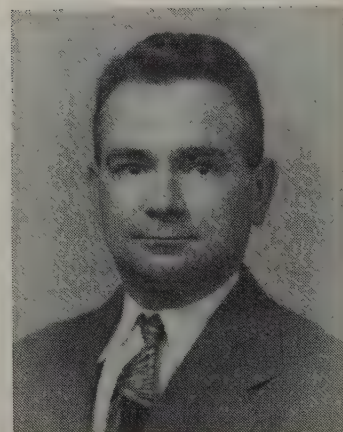
New York. He is a member of Sigma Xi, Tau Beta Pi, and Synton.



Chaim L. Pekeris was born in Alytus, Lithuania, in 1908. He received the B.S. degree in physics from the Massachusetts Institute of Technology in 1929, and the D.Sc. degree in meteorology in 1934. From 1934 to 1936 he carried on geophysical research at M.I.T. and Cambridge University, Cambridge, England, as a Fellow of the Rockefeller Foundation. From 1937 to the present time he has been research associate in geophysics at M.I.T. and honorary research associate in astrophysics at the Harvard College Observatory. During the war he was engaged in research on underwater wave propagation at Columbia University as a member of Division 6 of the Office of Scientific Research and Development. In 1945 he was appointed Director of



GEORGE M. KIRKPATRICK



CHAIM L. PEKERIS

the Columbia University Mathematical Physics Group. The Columbia University Mathematical Physics Group has been engaged in extending the theory of microwave propagation under standard and nonstandard conditions of atmospheric refraction.

Dr. Pekeris' principal contributions have been in the field of wave propagation, including atmospheric waves, seismic waves, underwater acoustics, and microwaves. In 1946 he was awarded a Guggenheim Fellowship to write a book on Wave Propagation.

Dr. Pekeris is a Fellow of the American Physical Society. He is a member of the American Mathematical Society, the American Geophysical Union, and the Society of Exploration Geophysicists of the Seismological Society of America.



For a photograph and biography of WILLIAM ALTAR, see page 379 of the April, 1947, issue of the PROCEEDINGS OF THE I.R.E.



JOHN R. RAGAZZINI

John R. Ragazzini was born in 1912 and received the degrees of B.S. and E.E. at the City College of New York in 1932 and 1933. He then received the degrees of A.M. and Ph.D. at Columbia University in 1939 and 1941. After a brief association with one of the City Departments he became an instructor in the School of Technology at the City College, and in 1941 joined the faculty of the School of Engineering of Columbia University. During the war he was in charge of two contracts with the Office of Scientific Research and Development, one of which involved electronic simulators and computers. He is a member of Phi Beta Kappa, Sigma Xi, and Tau Beta Pi, and is an associate member of the American Institute of Electrical Engineers.



Robert H. Randall was born in Grand Rapids, Michigan, in 1903. He undertook graduate work at Columbia College in New York, where he received the A.B. degree and his Ph.D. degree in 1930. In 1930 he was appointed instructor in physics at the City College of New York, where he has remained as teacher, now with the rank of assistant professor.

During the years 1930 to 1940, Dr. Randall performed research in several phases of



ROBERT H. RANDALL

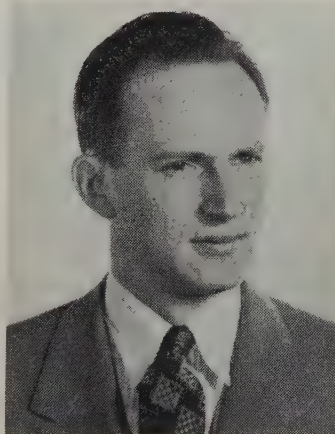
experimental physics. One such study was the investigation of electron temperatures in mercury vapor, in conjunction with H. W. Webb, at Columbia University. Later a special grant made possible a series of studies of internal friction in metals, the work being performed at the City College by Dr. Randall and Dr. Clarence Zener. The results of these studies were published in the *Physical Review*. Shortly after the outbreak of the war he joined the technical staff at Columbia University and worked on war projects.

Dr. Randall is a member of Phi Beta Kappa, Sigma Xi, and the American Physical Society.



Henry J. Riblet (A'45) was born at Calgary, Canada, on July 21, 1913. He received the B.S. degree in 1935, the M.S. degree in 1937, and the Ph.D. degree in mathematics in 1939, from Yale University.

From 1939 to 1941 Dr. Riblet was instructor in mathematics at Adelphi College



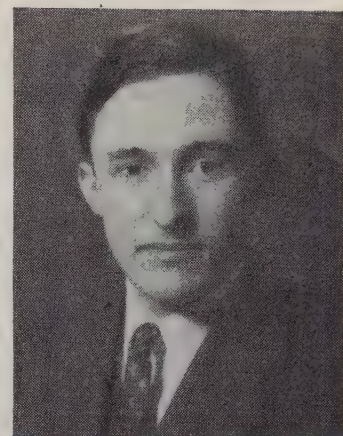
HENRY J. RIBLET

and from 1941 to 1942 he was assistant professor of mathematics at Hofstra College. From 1942 to 1945 he was at the Radiation Laboratory, where he was in charge of that section of the antenna group which devoted its time to the design of omnidirectional, linear-array, broad-band, and circularly polarized microwave antennas. He is now employed in the radar engineering group at the Submarine Signal Company.

He is a member of Sigma Xi, the American Mathematical Society, and the American Physical Society.



Frederick A. Russell was born in New York City on April 18, 1915. He received the B.S. and E.E. degrees from Newark College of Engineering in 1935 and 1939, respectively, and the M.S. degree from Stevens Institute of Technology in 1941. He was employed for two years as a vacuum-tube development engineer with Arcturus Radio Tube Corporation, and then entered the teaching profession at Newark College of Engineering. During the war years he was employed as a senior research engineer by



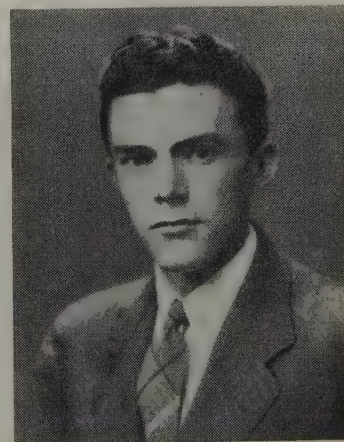
FREDERICK A. RUSSELL

the Columbia University Division of War Research, working on a contract under the Office of Scientific Research and Development. At the conclusion of the war he returned to the Newark College of Engineering, where he is now assistant professor in electrical engineering and executive associate of the electrical engineering department.

Mr. Russell is a professional Engineer in the State of New Jersey, and a member of the American Institute of Electrical Engineers, the Society for American Engineering Education, and Tau Beta Pi.



J. R. Whinnery (A'41-SM'44) was born on July 26, 1916, at Read, Colorado. He received the B.S. degree in electrical engineering from the University of California in 1937, at which time he was employed by the General Electric Company at Schenectady, N.Y. In 1940 he was a graduate of the three-year program of the advanced course in engineering of the General Electric Company, and during the following two years supervised the high-frequency section of that program. From 1942 to January of 1946, he was with the electronics laboratory and the research laboratory of the General Electric Company. Mr. Whinnery is now with the division of electrical engineering of the University of California, in Berkeley, California.



J. R. WHINNERY

# Institute News and Radio Notes

## THE PAST AND FUTURE OF RADIO

Analyzing the history of radio and thoughtfully predicting its future possibilities in an optimistic vein, Brigadier-General David Sarnoff (F'17), president of the Radio Corporation of America, addressed a large audience last fall at a dinner given in New York to commemorate his forty years of service in the radio field.

In relation to the utilization of modern radio methods in the field of meteorology, General Sarnoff stated:

"Automatic radio weather stations in remote places in the polar regions, in deserts, in jungles, and on the seas can collect and broadcast weather data. Already radar spots a hurricane, peers into its vortex, plots its movement, and photographs it from minute to minute. Radio-controlled and electronically equipped rockets will permit exploration of the upper atmosphere. Within minutes, new electronic computing devices can analyze such information on a global basis."

He stressed that "the evolution of radio is unending" as indicated clearly by the evolution of television, radar, and the great assembly of other electronic devices and services. He felt that the future offered bright promise for further development.

"Our descendants," he suggested, "will look back upon the radio services of this era and compare them, as a candle to the electric light, the horse and buggy to the automobile, and the ocean liner to the stratoliner."

"Already, the electron tube responds to our sense of touch, sound, and sight. We shall learn how to make it respond also to our sense of taste and smell. The tireless workers of radio science will produce a radio-mail system that will be inexpensive, secret, and faster than any mail-carrying plane can travel."

"Portable communication instruments will be developed that will enable an individual to communicate directly and promptly with anyone, anywhere in the world. As we learn more about the secrets of space, we shall increase immeasurably the number of usable frequencies until we are able to assign a separate frequency to an individual as a separate telephone number is assigned to each instrument."

The great social value of radio affords justification, were that needed, for the establishment of generally accepted freedoms. In this relation, General Sarnoff presented the following bases for public policy:

"In America, radio has grown rapidly as a great public servant—not only because of freedom to speak and freedom to listen but because of the freedom of science to advance. Science must be free. We can permit no restrictions to be placed upon the scientists' right to question, to experiment, and to think. Because America has held liberty above all else, distinguished men of science have come here to live, to work, and to seek new knowledge. The world has been the benefactor and science has moved forward."

In war, science dares the impossible; it must continue to dare the impossible in peace if a fuller life is to permeate society.

"Radio has never ceased to stir the imagination; it has continually inspired research. That is why radio is always new. It has met the challenges of two world wars and of the 20 years of peace that intervened."

"Radio has become one of the world's great social forces; it educates, informs, and entertains. Distance has been annihilated. All people have been brought within the sound of a single voice. A nine-word message has encircled the earth in nine seconds. The face of the moon has felt the ping of a radar pulse and echoed it back in two seconds to revive predictions of interplanetary communications."

He concluded on a note of optimism:

"As we look ahead through the vista of science with its tremendous possibilities for progress in peacetime, let us not feel that we are looking beyond the horizon of hope. The outlook is not discouraging, for there is no limit to man's ingenuity and no end to the opportunities for progress."

## Note from the Executive Secretary

Fellow-Members:

We here at Headquarters always like to hear from you. If something has gone wrong, it helps all of us here to know about it. If information of the sort that the Institute can properly furnish is needed, we would like to supply it. And if you happened to be pleased about something that the Institute, its officers, or its staff have done, all of us find the day more pleasant when you write us along that line.

The Institute has grown to over 20,000 members. Its mail is heavy. But every effort is made to acknowledge all letters promptly. We may not have the complete answer, but we always want you to know that your letter has reached us, and that we shall try to find the final answer as soon as we can.

If each one of you could be here at Headquarters for a day, we believe you would be pleased at the spirit you would find here. Everyone feels that the Institute belongs to *you*, and we are proud of having the responsibility of conducting the daily operations of so fine an organization.

GEORGE W. BAILEY

## SACRAMENTO SECTION

The petition for the formation of the Sacramento Section has been approved by the Board of Directors of the Institute at its March 4, 1947, meeting.

## DELEGATES TO INTERNATIONAL TELECOMMUNICATIONS CONFERENCE

Dr. W. R. G. Baker was designated, at the April 1, 1947, meeting of the Executive Committee, as the official delegate of The Institute of Radio Engineers to the International Telecommunications Conference at Atlantic City, New Jersey in May. Executive Secretary Bailey was designated as alternate.

## I.R.E. REPRESENTATION ON A.S.A. SECTIONAL COMMITTEE ON OPTICS

The Institute of Radio Engineers has accepted the invitation of the American Standards Association to be represented on the Section Committee for the proposed Standards for Optics.

## WIDENED SCOPE OF INSTITUTE ACTIVITIES

The scope of the Institute has been widened to include phonograph recording and reproduction, special electromagnetic-wave devices (e.g., X-ray equipment), and nuclear engineering involving electronic instruments and controls, with the result that the personnel of the Papers Procurement Committee and Papers Review Committee, as well as the Board of Editors, has been enlarged.

## ELECTRON-TUBE CONFERENCE

An Electron-Tube Conference will be held at Syracuse University on Monday and Tuesday, June 9 and 10, 1947. This Conference, sponsored by the Electron-Tube Committee of The Institute of Radio Engineers, is held annually for active workers in the field of electron-tube research.

Notices have been mailed to those who attended the Conference held at Yale University last year.

Topics for this year's Conference have not definitely been selected; however, the traveling-wave tube undoubtedly will be included. A copy will be mailed to each of those who have registered.

Tube research workers who have not received notice in the mail and who feel that they could contribute to the success of the Conference are asked to communicate with the chairman of the invitations committee, Dr. L. S. Nergaard, RCA Laboratories Division, Princeton, New Jersey.

Advance registration will be required for attendance at this Conference.

# 1947 I.R.E. National Convention

FULLY exemplifying its basic theme, "Electronics at Peace," the 1947 National Convention and Radio Engineering Show of The Institute of Radio Engineers was an outstanding and memorable success, with a record registered attendance of 11,895 and 170 exhibitors. It was not only the largest radio engineering event in history, but probably enjoyed the most numerous attendance of any kind of engineers' gathering ever held. The 11,895 registration figure compared with a registered attendance of 7020 at the 1946 I.R.E. Winter Technical Meeting and an estimated 3000 the previous year.

Viewed in the light of I.R.E. President W. R. G. Baker's characterization of the meeting as "the annual audit of the technical phases of the radio and electronics industry," showing "to what extent the 20,000 scientists and engineers comprising the Institute of Radio Engineers have fulfilled their responsibility to the industry and the public," the Convention fully demonstrated the extension of wartime-developed research and production techniques into the nation's peacetime economy.

Under the competent planning and execution of Professor Ernst Weber and his technical program committee, approximately 120 technical papers (previously summarized in these pages)<sup>1</sup> were presented in twenty-five major sessions, each carefully planned and carried out meticulously on schedule.

All papers presented were selected as exceptionally significant contributions to the art. With so many major

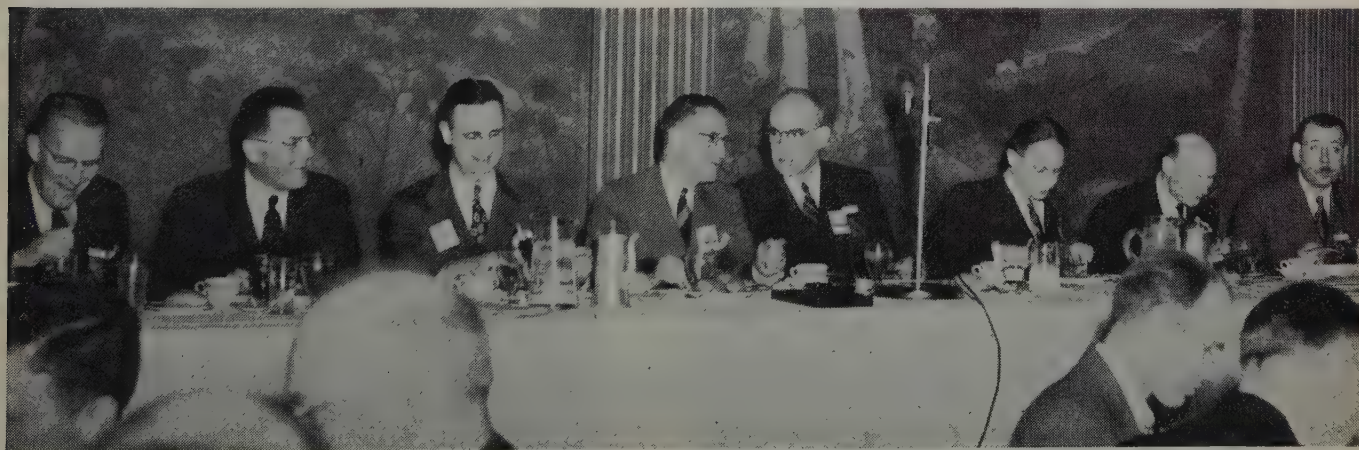
<sup>1</sup> "Summaries of Technical Papers," PROC. I.R.E., vol. 35, pp. 173-182; February, 1947.

fields represented, it is difficult to select any as of outstanding interest. However, while all sessions had capacity attendance, four were noteworthy in that they had to be repeated for the benefit of crowds who were crowding the anterooms and unable to hear the first presentation. These were the sessions on "Microwave Components and Test Equipment" and "Television A" on Tuesday morning at Grand Central Palace, "Electronic Digital Computers" and "Television B" on Tuesday afternoon, and one paper on printed circuits on Wednesday morning at the Hotel Commodore. Also delivered to an overflow audience was the Monday afternoon session on "Particle Accelerators and Nuclear Studies."

The special Wednesday afternoon session on the engineering profession also was very well attended. Emphasizing the growing recognition of the engineering profession as a dominant factor in the social life of the nation, bringing with it added responsibility and the need for reorientation of engineering education, this symposium presented the respective viewpoints of the industrialists, represented by Dr. C. B. Jolliffe of the Radio Corporation of America, and the educators, as expressed by Dr. H. S. Rogers, Polytechnic Institute of Brooklyn.

The "get-together" party, held at the cocktail hour on March 3 in the Main Ballroom at the Hotel Commodore, was attended by over 1200 members of the Institute and their guests.

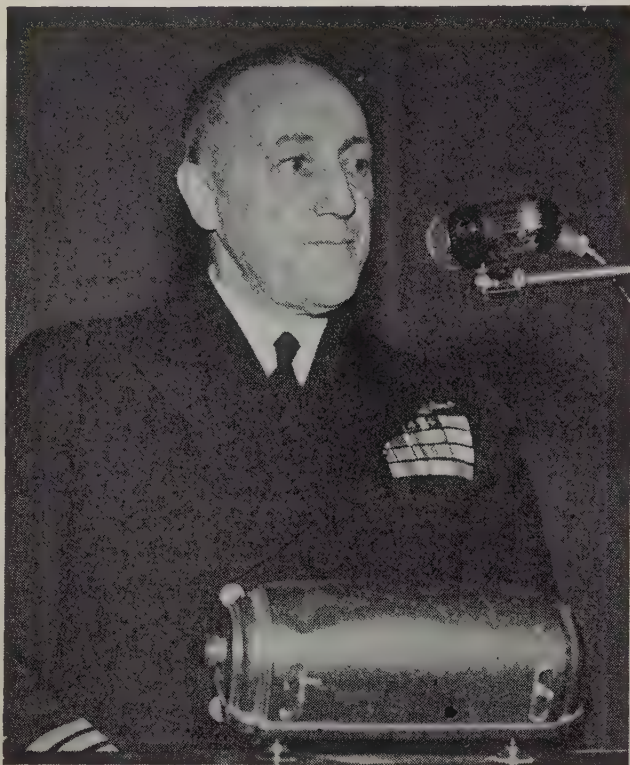
At the President's Luncheon on Tuesday, March 4, the incoming 1947 President of the Institute, Dr. W. R. G. Baker, was honored. Following brief talks by



Head Table at Press Luncheon, March 3, 1947.

Left to right: John M. Moorhead; Virgil M. Graham, Chairman, Publicity Committee; James E. Shepherd, Chairman, Convention Committee; W. R. G. Baker, President, I.R.E. 1947; George W. Bailey, Executive Secretary, I.R.E.; Frederick B. Llewellyn, President I.R.E., 1946; Ernst Weber, Chairman, Papers Committee, 1947 Winter Technical Meeting; and Clinton B. De Soto, Technical Editor, PROCEEDINGS OF THE I.R.E.

# 1947 National



Vice-Admiral Charles Andrew Lockwood, Jr., U. S. Navy, addressing the guests at the President's Luncheon, March 4, 1947.



Dr. Walter R. G. Baker, President of the I.R.E., presents the Morris Liebmann Memorial Prize for 1946 to Dr. Albert Rose and for 1947 to Dr. John R. Pierce.

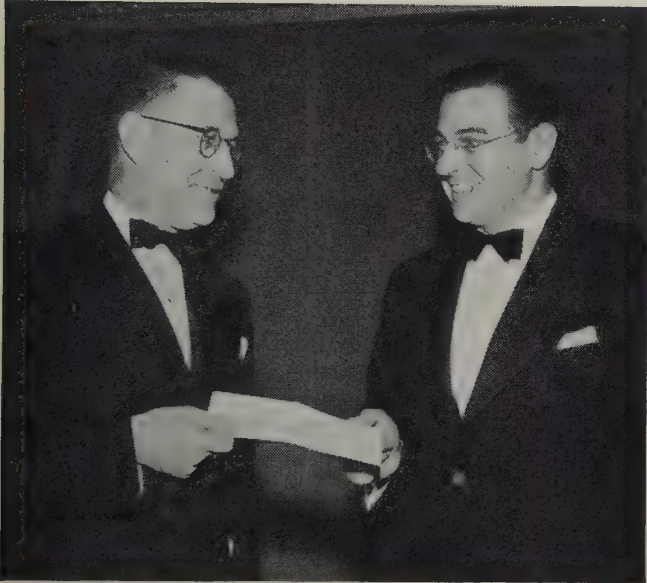


Mr. Frederick R. Lack, Toastmaster at the Banquet.



Mr. William O. Swinyard, Chairman, Sections Committee.

# I.R.E. Convention



Dr. Baker presents the Browder J. Thompson Memorial Prize for 1946 to Dr. Charles L. Dolph.



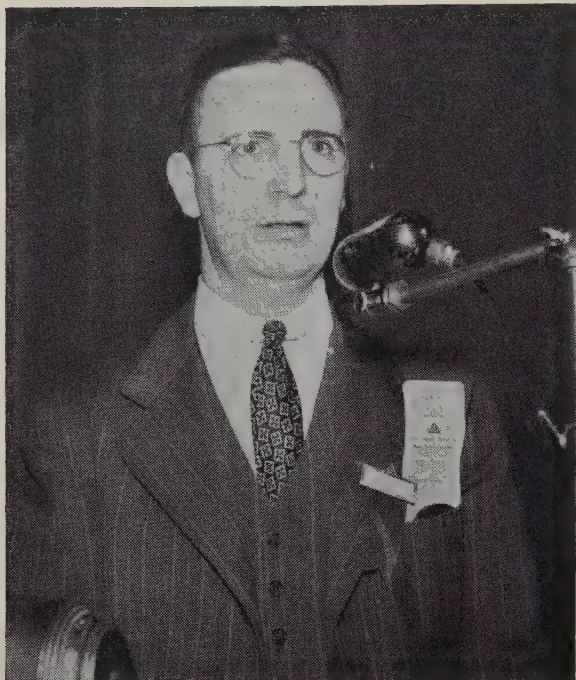
Mr. Charles R. Denny, Chairman, Federal Communications Commission, addressing the Annual Banquet, March 5, 1947.



Dr. Frederick B. Llewellyn, President, 1946, and Dr. Walter R. G. Baker, president, 1947.



Mr. Donald G. Fink makes speech of acceptance for recipients of Fellow Awards.



Dr. James E. Shepherd, Chairman 1947 I.R.E. National Convention.

Dr. Baker and by the junior past president, Dr. F. B. Llewellyn, the principal address was delivered by Vice-Admiral Charles A. Lockwood, formerly Commander of the submarine fleet in the Southwest Pacific and now Inspector-General of the U. S. Navy, who spoke on the subject, "Electronics and Submarine Warfare." Admiral Lockwood revealed, for the first time, many instances of the use and importance of electronic weapons in the undersea warfare conducted against the Japanese in the Pacific.

At the 35th Anniversary banquet on Wednesday evening, March 5, which was televised over WNBC, New York, and WTTG, Washington, D. C., the featured speaker was Charles R. Denny, chairman of the Federal Communications Commission. Mr. Denny, choosing as his topic "The Job Ahead," followed a tribute in light vein to the radio engineering profession with a discussion of current allocation problems, and concluded with a challenge to radio engineers to find means to prevent present limited radio facilities from "imposing a ceiling on the expanding communications and commerce of the world."

Dr. W. R. G. Baker also delivered a challenging, forward-looking address, stating, "It is the business of the scientist to find the fundamental means for the mastery of nature. It is the responsibility of the engineer to make these truths of use to mankind."

Dr. F. B. Llewellyn discussed analytically the topic "A Test for the Success of an I.R.E. Section."

Three special awards and twenty-five fellowships were presented at the annual banquet.

Two Morris Liebmann Memorial Prizes were pre-

sented, one for 1946 and one for 1947. The first award went to Albert Rose, for his contributions to the art of converting optical images to electrical signals, particularly the image orthicon; and the second to John R. Pierce, for his development of a traveling-wave amplifier tube having both high gain and very great bandwidth.

Twenty-five fellowships were awarded to:

George P. Adair, Benjamin de F. Bayly, George L. Beers, Lloyd V. Berkner, Edward L. Bowles, Robert S. Burnap, Robert F. Field, Donald G. Fink, William W. Hansen, David R. Hull, Fred V. Hunt, Karl G. Jansky, Ray D. Kell, Charles V. Litton, James W. McRae, Ilia E. Mouromtseff, Daniel Earl Noble, Pedro J. Noizeux, Robert M. Page, John A. Pierce, Frank H. R. Pounsett, Conan A. Priest, Winfield W. Salisbury, Robert Watson-Watt, and Edward N. Wendell.

Women's activities under the chairmanship of Mrs. F. B. Llewellyn were conspicuously successful. Of the 275 ladies registered at the Convention, over 200 attended the Institute Tea at the new I.R.E. Headquarters building. The Tea was very festive, with WOR providing an accordion player, wives of past presidents receiving, wives of Section Chairmen assisting, and two members of the Institute's Editorial Department staff conducting guided tours through the building. Many of the women whose homes are away from New York enjoyed the sightseeing trip of New York, a trip to the Cloisters, Empire State Building, and a "Behind the Scenes" expedition at R. H. Macy's (which included a fashion show). The broadcast studios were most generous in giving out tickets for their most special broadcasts. Tickets for the theater party were sold out well in



Mrs. Frederick B. Llewellyn, Chairman Women's Committee, 1947 I.R.E. National Convention.

advance, with luncheon gatherings at various hotels first. Three busses carried the group to the United Nations, where a welcome and talk were given to the ladies followed by attendance at one of the sessions. Luncheon at the Aviation Terrace Room at La Guardia Airport ended the women's activities of the four-day Convention.

The unexpected size of the registration—nearly double that of early estimates—imposed special problems on all concerned with the Convention, particularly on the registration and hospitality committees. Efficient preliminary planning by General Chairman James E. Shepherd and his committeemen had established procedures which carried the unanticipated load, however, and registration and other facilities functioned efficiently and smoothly. A novel aspect of the handling of this problem was the use for the first time in such an application of special International Business Machines tabulating equipment to record names and salient data concerning all registrants. So promptly did this system function that up-to-the-minute alphabetical lists of all those in attendance and their local addresses were constantly posted at both the Hotel Commodore and Grand Central Palace for the benefit of other members.

The greatest Radio Engineering Show in history filled the two principal floors at Grand Central Palace with the displays of 170 exhibitors, including practically every major industrial and manufacturing firm in the communication and electronics fields, the principal technical publications, and several government agencies. Presented at an investment by exhibitors of several hundred thousand dollars, according to Exhibits Manager W. C. Copp, radio and electronic equipment ranging from microscopic thermistors and other ultra-miniature

components to 10-kilowatt broadcast transmitters and full-scale frequency-modulation broadcast antennas and towers was on display. The largest exhibits in terms of space were those of the United States Army and Navy. Occupying a total of 2400 feet of floor space, these exhibits featured a variety of radar and radio systems and equipment, much only recently released from security classifications, including particularly an array of guided missiles of both automatic-radar and radio-controlled types.

To survey the Show as a whole would be to attempt to describe the entire field of current electronic engineering and production.

Essentially the exhibits divided into four basic groups: (1) new components; (2) new materials, such as metals, alloys, ceramics, and similar materials; (3) new techniques, notably in measuring equipment; (4) typical examples of operating equipment in the form of complete assemblies—transmitters, receivers, etc. Especially noticeable was the extraordinary variety of electron tubes of all sizes, types and applications, and the amount and variety of microwave (wave-guide, and the like) components. An especially strong impression was created by the multiplicity and scope of the measuring and testing devices on display, ranging from audio-frequency recording and reproduction to nucleonics instrumentation.

A number of interesting exhibits were presented in special demonstration rooms, including component units of the EDVAC electronic digital computer, declassified only the week before the show; the transformation through electronic means of the color in minerals and gems; and tiny "printed-circuit" miniature transmitters and receivers employing subminiature components.

## I.R.E. People

### FRED MULLER, SR.

Fred Muller, Sr., (M'30-SM'43) United States Naval Reserve, electronics officer of the Atlantic Reserve Fleet, was awarded the Marconi Memorial Medal of Achievement for "forty years of progressive achievement in the radio art" by the Veteran Wireless Operators Association at their 1947 annual meeting. Born in Paterson, N. J., Captain Muller joined the United States Navy in 1908, and after being graduated from the Navy Electrical and Wireless School, served in the Atlantic Fleet as general electrician in the installation and operation of the Navy's first advanced fire control and intercommunication systems. His enlistment terminated in 1912 when he was the senior wireless operator on board the flagship *USS Minnesota*, but during World War I, he rejoined the navy as assistant for communication personnel and materiel in the Third Naval Dis-

trict. While a civilian, he was associated with the Tropical Radio Telegraph Company as an inspector and then division superintendent, and with the Collins Radio Company as assistant to the vice-president and general manager.

In 1940 Captain Muller again was called to active duty with the Navy where he had the supervision of radio, radar, and sonar countermeasures and special devices, shipboard intercommunication, fire-control, sound-motion-picture, and entertainment systems. He was commended for meritorious service by the Chief of the Bureau of Ships for his work in the planning and production of electronics equipment for the amphibious force headquarters communication ships. In 1944 Captain Muller was selected by the Navy as electronics officer of the Tenth Naval District, and in 1945 was assigned to the position of electronics officer for the U. S. Atlantic Reserve Fleet.

### ROBERT E. MCCOY

Robert E. McCoy (A'41-M'46) has left the Radio Direction Finding Branch of the Signal Corps Engineering Laboratories to become an independent consulting engineer with headquarters in Gresham, Oregon.

Mr. McCoy received the B.S. degree in electrical engineering from the Oregon Institute of Technology in 1937. He was employed by Northwestern Electric Company from 1937 until 1942, when he entered the United States Army. After a brief tour of duty as a second lieutenant in the Signal Corps, he entered the Signal Corps General Development Laboratory, predecessor of SCEL, as a radio engineer. He remained with the RDF Branch nearly five years, first at Eatontown Signal Laboratory and then at Evans Signal Laboratory.

Mr. McCoy is a member of the American Institute of Electrical Engineers.

# I.R.E. People



Official Coast Guard Photo

## EDWARD M. WEBSTER

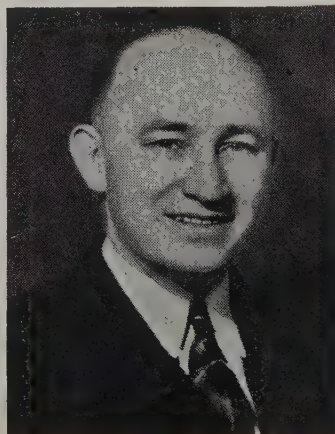
Edward M. Webster (A'30-M'38-SM'43-F'44) was nominated on March 7, 1947, by President Truman as one of the Federal Communications Commissioners. A retired commissioned officer of the United States Coast Guard, Commodore Webster was born in Washington, D. C., in 1889, graduated from the United States Coast Guard Academy in 1912, and saw service with the Navy during World War I. In 1923 he was made chief communications officer of the Coast Guard and established an efficient communications system for that branch of the Service. Retiring from active duty in 1934, he accepted a position with the Federal Communications Commission, later becoming assistant chief engineer. In this position, he administered in particular communication matters relating to marine, aviation, experimental, point-to-point, emergency and amateur services, and radio-operator problems.

Recalled to active duty in the Coast Guard in 1942, he was reassigned as chief communications officer with the rank of captain, and was promoted to rank of commodore in 1945. During this time he further increased the development of the Coast Guard communications system and was also active in other allied war communication activities. He was awarded the Legion of Merit by the President "for exceptionally meritorious conduct in the performance of outstanding services to the Government of the United States." Relieved from active duty in November, 1946, he was appointed director of Telecommunications of the National Federation of American Shipping, Inc.

Commodore Webster has been prominent for many years in co-ordinating communication activities within the government, acting on advisory and research committees, and has also attended international communications conferences as a government representative. He is a former director of The Institute of Radio Engineers, a member of the Propellor Club, the Veterans Wireless Operators Association, and the Army-Navy Club.

## I. J. YOUNGBLOOD

I. J. Youngblood (A'45) was recently appointed vice-president in charge of sales at Clarostat Manufacturing Company, Inc., Brooklyn, N. Y. Following his education at Drexel Institute of Technology, Mr. Youngblood served in the United States Navy from 1917 to 1923, where his work was concerned with radio compass and underwater detection. After a year with Atwater Kent, he went to Philco as production supervisor and was later given charge of components parts engineering and the development of sources for these. He spent a year with the Farnsworth engineering staff, and in 1940 joined Clarostat as sales engineer.



## WILLIAM C. SIMON

William C. Simon (M'32-SM'43) was awarded the Marconi Memorial Medal of Merit by the Veteran Wireless Operators Association, of which he is a Life Member, in recognition of his many years of service to the organization both as an officer and member of the Board of Directors. The presentation was made at the "United Nations Radio Dinner" of the Association on their twenty-second anniversary. Mr. Simon is General Manager of Tropical Radio Service Corporation.



## HAROLD O. BISHOP

Harold O. Bishop (M'45) has been elected to the board of directors of Stavid Engineering Corporation of Plainfield, N. J. During the war he served as a Lieutenant Commander on the Radar Staff of the Postgraduate School at Annapolis and at present is directing the research and development of guided missiles countermeasures for the Navy Department. Mr. Bishop owns the Electrical Service Company, Radio Station WABX in Harrisburg, Pennsylvania and Station KOAX in Albuquerque, New Mexico.

## WILLIAM G. ELLIS

Appointment of William G. Ellis (A'19-M'25-SM'43) as sales manager of industrial electronics sales was announced recently by T. A. Smith (J'25-A'26-SM'45) general sales manager of the Radio Corporation of America's Engineering Products Department. A native of Philadelphia, Mr. Ellis was graduated from the Drexel Institute of Technology as a mechanical engineer. Pioneering in radio as an amateur operator some forty years ago, he began his professional career in 1917 when he engaged in research in radio direction finding for the United States Navy.

In 1922 Mr. Ellis went into consulting industrial engineering and in 1927 entered the field of engineering sales of industrial equipment where he remained for 15 years. During the war, he served with a joint agency responsible for electronic production for both the Armed Services. Mr. Ellis is a member of the Franklin Institute, the American Society of Mechanical Engineers, and the American Institute of Electrical Engineers.



## WALTER A. WEISS

Walter A. Weiss (S'40-A'41-M'44) recently was appointed supervisor of quality quality control for the radio tube division of Sylvania Electric Products, Inc. Mr. Weiss, a graduate of Pennsylvania State College with a B.S. degree in electrical engineering, joined Sylvania in 1941 as a student engineer, later serving as a test-equipment engineer. In September, 1942, he was appointed supervisor of quality control for the Emporium plant.

Mr. Weiss is an associate member of the American Institute of Electrical Engineers and a member of the Society of Quality Control Engineers.



WALTER A. WEISS

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# Books

## Introduction to Electron Optics, by V. E. Cosslett

Published (1946) by Oxford University Press, 114 Fifth Ave., New York 11, N. Y. 269 pages + 3-page index + x pages. 155 figures + 8 plates.  $6\frac{1}{2} \times 9\frac{1}{2}$  inches. Price, \$6.50.

This new book is intended as a text for advanced undergraduates in physics. As the subtitle—"The Production, Propagation, and Focusing of Electron Beams"—implies, it covers, in addition to the motion of charged particles in electric and magnetic fields, the various known methods of producing electron currents. Furthermore, the principal applications of electron optics—cathode-ray tubes, electron multipliers, image tubes, television pickup tubes, electron-diffraction apparatus, electron microscopes, apparatus for separating and accelerating charged particles, and beam amplifiers—make up about a third of the subject matter. A brief chapter on velocity-modulated beams, an appendix setting forth the use of the iconals in the derivation of the aberration expressions, and some useful tables complete the book. The mathematical requirements are the usual ones, i.e., a knowledge of calculus and elementary partial differential equations.

The most fully covered and most adequate sections of the book are, properly, those dealing with the determination of field distributions, ray tracing, and the focusing properties of various types of electron lenses. While practically all methods of ray tracing are considered, the reviewer feels that a fuller treatment of the numerical integration of the ray equation and the introduction of Scherzer's form as well as a first-order form could well have replaced the

detailed treatment of the trigonometric method which is not suited to the aspheric shape of the equipotentials in electron optics. The employment of Scherzer's ray equation (eventually introduced on page 111) would also have prevented the introduction of an inaccurate expression for the weak-lens focal length. A ray equation for magnetic fields regarded by the author as more accurate than the paraxial-ray equation includes some higher-order terms, omits others of equal or lower order. It is unfortunate that the reader is not made aware of the universality of the rules regarding crossed principal planes and the non-existence of negative lenses.

The chapter on aberrations describes the character of the several geometric aberrations and presents Scherzer's convenient explicit formulas; axial chromatic aberration and Gabor's proposal for an aberration-free space-charge lens are also discussed. The discussion of the aberrations contains a number of errors which may prove disturbing; such as that the spherical aberration is corrected if the sine condition is fulfilled; that limitations introduced to reduce spherical and chromatic aberration generally largely eliminate distortion; that the dependence on voltage of the chromatic aberration of a cathode lens is the same as for other lenses; and that the spherical aberration of magnetic lenses may be reduced by increasing the accelerating potential. The author also has not escaped the unfortunately common confusion between particle and wave velocity in the statement of Fermat's theorem.

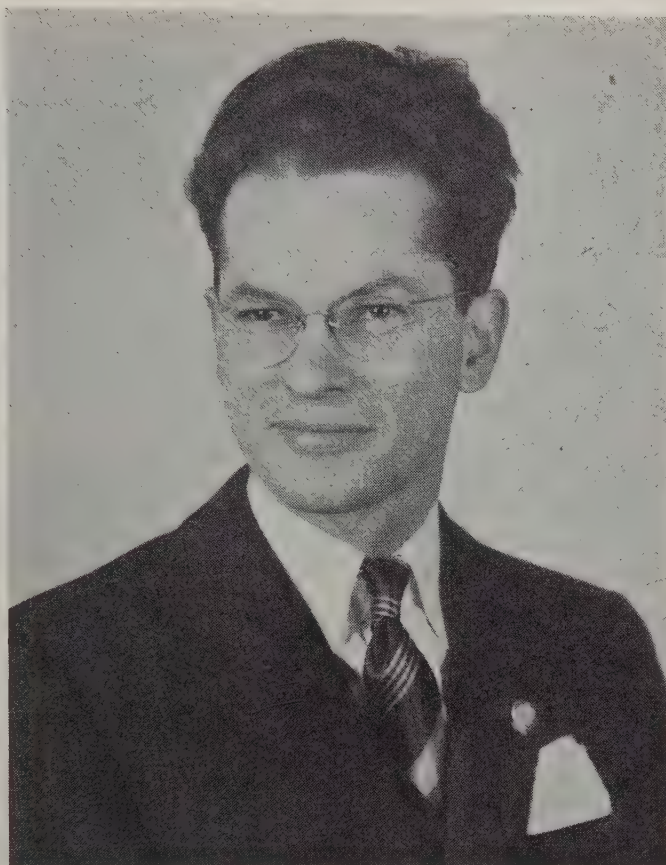
The discussion on the production of electron beams gives an interesting survey over thermionic emission, electron guns, field emission, the point-emission microscope, photoemission, image tubes, secondary emission, and electron multipliers. The treat-

ment is necessarily sketchy and not free of defects; examples of the latter are to be found in the discussion of the requirements for the ejection of secondary electrons by electrons and ions, an overestimate of the Schottky effect by 3 orders of magnitude, the statement of an exponential temperature dependence of the photoeffect, and the assumption that Pierce neglected space charge in determining electrode shapes for his high-current guns.

The several chapters on applications contain good descriptive sections giving an idea of the broad scope of electron optics. The weakest point here is the discussion of the principles of electron diffraction. The derivation of the diffraction limit of a microscope and the discussion of image formation in the electron microscope are also faulty. The author does not appear to distinguish clearly between the chromatic aberration of the electrostatic univoltage lens, which is large, and its very small sensitivity to voltage changes. The descriptive portion dealing with the several types of microscopes again is good, if brief and uninclusive.

Considering the book as a whole, it is well arranged, not unnecessarily repetitive, and quite readable. In a few cases terms are not defined and there are occasional inconsistencies, corresponding to some of the errors already mentioned. The plates are excellent, the drawings somewhat uneven—the reviewer noted seven of the latter which were misleading. Most of the errors in the equations are such that they will not lead a diligent student far astray. Interpreted by a capable teacher the book should quite satisfactorily fulfill its function as a class text on electron optics.

V. K. ZWORYKIN  
RCA Laboratories Division  
Princeton, N. J.



## David J. Knowles

Secretary-Treasurer, Emporium Section—1946

David J. Knowles was born on November 30, 1918, in San Francisco, California.

While studying pre-engineering and physics at San Antonio Junior College, from which he graduated in 1939, he acted as laboratory assistant in the physics and chemistry laboratories. During 1940 he was checking estimates and expediting for the Brandt Iron Works in San Antonio, Texas.

After receiving his B.S. degree in electrical engineering from the Rice Institute in 1942, Mr. Knowles went to Sylvania Electric Products, Inc., Emporium, Pennsylvania, as a vacuum-tube engineer engaged in the design, development, and production of a miniature thyratron for computers used by the armed forces, a subminiature glow tube for instrument use, and a special gas-filled voltage regulator. At present he is working on the design and development of rectifiers and power-output tubes, both standard receiving types and specialties such as the commercial versions of the T-3 proximity-fuze tubes.

Mr. Knowles joined The Institute of Radio Engineers in 1941 as a Student, became an Associate in 1943, and a Member in 1945. He is an Associate of the American Institute of Electrical Engineers and a member of Tau Beta Pi.

Unless terminology is clear and unequivocal in its import, discussions involving terms easily become confused and at cross purposes. The importance of definiteness and general agreement in nomenclature has been too often underrated, but is here suitably explained, and with appropriate recommendations, by a radio engineer of long and constructive experience who, for the last seven years, has edited the periodical *FM and Television*.—*The Editor*.

## The Need for Clear Terminology

MILTON B. SLEEPER

When the first Standardization Committee of The Institute of Radio Engineers was formed, its task of working out definitions for radio terms was relatively simple. I say relatively simple because there were so few terms used in the art.

I was a member of that first Committee, and blush to recall that I had at least an opinion to express on nearly every term we undertook to define. Since then the number of terms and the problems of defining them have multiplied manifold.

Today I am not even sure that it is still in order to speak of "radio terms," because it seems as if the word *radio* itself has given way to *electronics*, even among the members of The Institute of Radio Engineers.

Now I want to make a plea for re-establishing the use of *radio*, and setting up distinguishing definitions for radio and electronics. The reason is not nostalgic, but practical.

The need goes back to World War II practice of lumping everything related to the use of vacuum tubes under the heading of "electronics." The practice of combining electric-wave communications equipment with everything from tube-operated sound-ranging apparatus to calculating machines under the heading of "electronics" was further complicated by the inclusion of such mystery devices as radar and loran.

Resulting confusion, worse confounded by indiscriminate use of *electronics* in advertising and unscientific fiction, has now reached the point where, if a man says he is an "electronics engineer," it is necessary to question him to find out if he has been working on electrically warmed sleeping blankets, kidney-bean sorters, or printing-registry devices, if he really means that he's a radio engineer, or if he has been engaged in the development of tubes used for all those applications!

Certainly these conditions call for the revival of the noun and adjective *radio*. The Second Edition of Webster's International Dictionary defines the noun as "the transmission and reception of signals by means of electric waves without a connecting wire." The adjective is defined as

"of or pertaining to, employing, or operated by radiant energy, specifically that of electric waves; hence, pertaining to, or employed in radiotelegraphy or radiotelephony or other applications of radio waves."

These established definitions are so informative that the word to which they relate should not fall into disuse.

In fact, the definition of *electronics*, in the New Words section of Webster's Dictionary, shows clearly that it is not a generic term for all tube-operated devices, radio and otherwise, since the word denotes "that branch of physics which treats with the emission, behavior, and effects of electrons, especially in vacuum tubes, photoelectric cells, and the like."

It should be noted that, unlike *radio*, which is both a noun and an adjective, *electronics* is only a noun. The adjective form is *electronic*, defined merely as "of or pertaining to an electron or electrons."

Webster's Dictionary further disallows *electronic tube*, since it lists *electron tube*.

Thus, a radio engineer is properly one concerned with the applications of radio waves; and an electronics engineer is one who has to do with vacuum tubes, photoelectric cells, and the like. This indicates that an engineer who is concerned with the use of tubes for various nonradio electrical and mechanical devices and systems should be called a "tube-applications engineer."

Similarly, it appears that, since the application of the noun *electronics* is limited to use in connection with vacuum tubes, photoelectric cells, and the like, devices and systems operated or controlled by tubes should not be referred to as "electronic equipment," but as "tube-actuated equipment."

Finally, by established definition, radio equipment is that used for the transmission of signals by electric waves, without the use of a wire.

May I commend to Institute members the careful consideration of these definitions, to the end that we may plan to end the confusion now developing not only in our own ranks, but in the minds of the lay public which has such an intimate interest in so much of our work.

# Impedance Measurement on Transmission Lines\*

D. D. KING†, MEMBER, I.R.E.

**Summary**—A derivation of the formulas available for the measurement of terminal impedances on transmission lines is given in terms of hyperbolic functions. The accuracy and usefulness of a number of different methods are considered. Results are obtained in a form suitable for convenient application to practical measuring problems involving standing-wave and resonance-curve methods.

## INTRODUCTION

A STANDARD method of determining the impedance presented between the terminals of a given device consists in attaching these terminals to a properly designed transmission line; measurements are then carried out on the line. In the frequency range from 100 to several thousand megacycles this method is readily adaptable to a large variety of unknown load impedances ranging from antennas to vacuum tubes. It is the purpose of this paper to furnish a survey of various transmission-line methods, some more common than others, and to discuss their practical significance in impedance measurements.

Since the results are to be useful in experimental work, the subject is approached first from the point of view of the physically measurable quantities. From transmission-line theory formulas are then derived which conveniently relate the measured values to the desired unknown terminal resistance and reactance. The mathematical form of the relations involved has been chosen to make evident the basic similarities in different measuring procedures. Particular attention is devoted to the inherent accuracy of the various methods discussed and to their practical utility.

## PHYSICALLY MEASURABLE QUANTITIES

The impedance-measuring apparatus being considered consists essentially of a section of transmission line, parallel or coaxial, with right-hand terminals which permit attaching various load impedances, and the left end of which is coupled to a generator. The determination of current or voltage amplitudes on the line suffices for the measurement of terminal impedance; accordingly, a detector sensitive to amplitude but not to instantaneous phase is required. The measuring line on which this detector operates must permit at least one of the following quantities to be varied: (1) the position of the detector along the line; (2) the total length of line; or (3) the operating frequency of the generator.

The measurable quantities available correspond to the properties of the line. In the event the detector is moved along the line, a curve of voltage or current amplitude is obtained as a function of position along the

line. If the detector is kept at a fixed distance from the load, and the length of line between generator and load is varied, then a resonance curve is obtained giving the amplitude at the detector as a function of total line length. Finally, another type of resonance curve is available in case the frequency is altered with fixed line and detector adjustment. Here amplitude is plotted against frequency.

In order to utilize quantitatively the information contained in these plots of amplitude against a scale reading or against frequency, certain reference points are required. The first of these is taken as the position of the curve. This is defined as the value of the independent variable at the maximum or minimum detector amplitude. The independent variables which have been mentioned are the separation between detector and load, the total length of line, and the frequency. In addition to its position, a second property of each of these curves is its shape. This may be defined either by the width at an arbitrary level or by the ratio of maximum to minimum value. Thus, the width of resonance curves is commonly taken at a power level equal to half the peak power amplitude. In general, some other fraction of the maximum or a multiple of the minimum might prove useful in defining the level.

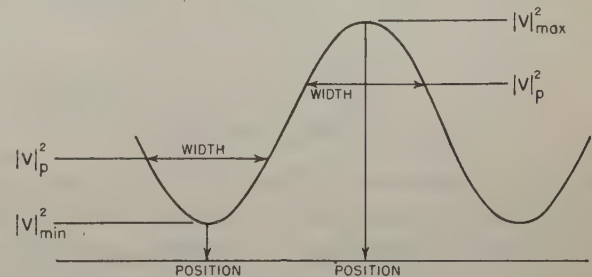


Fig. 1—Significant quantities in curves of detector output; the abscissa is either detector position or total line length.

The definitions of position and shape which have been made imply symmetrical curves; it will be shown later that only symmetrical curves need be considered. The quantities involved in the discussion above are illustrated in Fig. 1. The task of relating them to the terminal impedance of the line requires a general formula which may be derived from transmission-line theory.

## BASIC RELATIONS

A modified hyperbolic solution of the transmission-line equations forms the basis of the derivations to follow. This treatment of the solution has been discussed in the literature<sup>1,2</sup> but is not commonly used at present. However, it may be derived at once from standard forms

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† Cruft Laboratory and Research Laboratory of Physics, Harvard University, Cambridge, Massachusetts.

<sup>1</sup> R. King, "Transmission line theory and its application," *Jour. Appl. Phys.*, vol. 14, pp. 577-600; November, 1943.

<sup>2</sup> R. King, "General amplitude relations for transmission lines," *Proc. I.R.E.*, vol. 29, pp. 640-648; December, 1941.

to be found in texts on the subject. A diagram of the line is shown in Fig. 2. The load  $Z_r$  is located at a distance  $l$  from the sending end, the co-ordinate of which is 0. The voltage is measured at a distance  $z$  from the generator end which corresponds to a separation  $w$  from

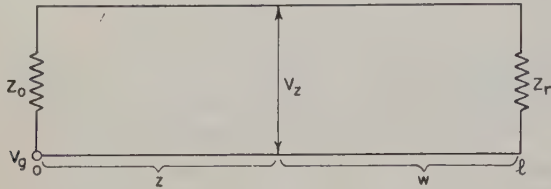


Fig. 2—Circuit of a measuring line; the unknown load is  $Z_r$  and the detector measures  $V_z$ .

the load; therefore  $w = l - z$ . A tabulation of the necessary quantities is written below.

$\gamma = \alpha + j\beta$ , the complex propagation constant, where  
 $\alpha$  = attenuation constant in nepers per meter,  
 $\beta$  = phase constant in radians per meter

$V_r$  = voltage across the load  $Z_r$

$V_z$  = voltage across the line at a distance  $z$  from the generator

$V_0$  = open-circuit voltage of the generator

$Z_0$  = equivalent series impedance of the generator

$Z_r$  = load impedance

$Z_c$  = characteristic impedance of the line.

Current amplitudes are omitted, since all of the final formulas obtained for measuring purposes apply equally to current detection. With the above notation the following standard equations may be written down:<sup>3</sup>

$$V_r = \frac{V_0 Z_c Z_r}{Z_c(Z_r + Z_0) \cosh \gamma l + (Z_c^2 + Z_r Z_0) \sinh \gamma l} \quad (1)$$

$$V_z = V_r \cosh \gamma w + (V_r/Z_r) Z_c \sinh \gamma w. \quad (2)$$

Combining these two equations yields the desired relation for the voltage across the line at the point  $z$ . This is the voltage measured by the detector.

$$V_z = V_0 Z_c \frac{\cosh \gamma w + (Z_c/Z_r) \sinh \gamma w}{(1 + Z_0/Z_r) \cosh \gamma l + (Z_c/Z_r + Z_0/Z_c) \sinh \gamma l} \quad (3)$$

A great simplification in form is accomplished by substituting in terms of hyperbolic terminal functions involving the complex angle  $\theta_2$ . The defining equations are written as follows:

$$Z_0/Z_c = \coth \theta_0. \quad (4a)$$

$$Z_r/Z_c = \coth \theta_r. \quad (4b)$$

Omitting the subscript  $z$ , (3) takes on a much more compact form with the aid of these definitions:

$$V = V_0 \frac{\sinh \theta_0 \cosh (\gamma w + \theta_r)}{\sinh (\gamma l + \theta_0 + \theta_r)}. \quad (5)$$

Most detectors give an indication proportional to the

power level, and hence the square of the magnitude of (5) is of practical interest. The real and imaginary parts of the hyperbolic angle  $\theta$  are defined as follows:

$$\theta = \rho + j\Phi. \quad (5a)$$

The desired formula involving these quantities is

$$|V|^2 = |V_0|^2 \frac{(\sinh^2 \rho_0 + \sin^2 \Phi_0)(\sinh^2 (\alpha l + \rho_0 + \rho_r) + (\alpha w + \rho_r) + \cos^2 (\beta w + \Phi_r))}{\sinh^2 (\alpha l + \rho_0 + \rho_r) + (\alpha w + \rho_r) + \cos^2 (\beta w + \Phi_r)} \cdot \frac{1}{\sin^2 (\beta l + \Phi_0 + \Phi_r)}. \quad (6)$$

At this point the reader may doubt the value of so much uncommon symbolism in the treatment of the practical problem of transmission-line measurements. The advantages of the method are twofold. In the first place, the resultant equations are of unusually simple form and permit making a clear comparison of the various measuring methods available. The remainder of the paper is devoted to this purpose. A second feature of the terminal functions is that they correspond to the physical variables of the problem. In effect, the use of terminal functions replaces the load impedance by a length  $s$  of a fictitious line, such that  $\rho = \alpha s$  and  $\Phi = \beta s$ . Thus, the termination damping  $\rho$  is a direct measure in nepers of the power absorbed in the termination. The value  $\rho = 0$  corresponds to a perfectly reactive load, while  $\rho = \infty$  applies to a matched or nonreflecting termination. Likewise, the termination phase shift  $\Phi$  measures the electrical length of the termination directly in radians. It has half the value of the reflection coefficient, as used on a Smith plot, for example. This representation in  $\rho$  and  $\Phi$  may be preferable to  $R$  and  $X$  in gauging the efficiency of a termination.<sup>4</sup> The relation between the terminal functions and the load resistance and reactance is given by the following:

$$R = \frac{R_c \sinh 2\rho}{\cosh 2\rho - \cos 2\Phi} \quad (7a)$$

$$X = \frac{-R_c \sin 2\Phi}{\cosh 2\rho - \cos 2\Phi}. \quad (7b)$$

Here it is assumed that the measuring line has low losses, i.e.,  $Z_c \approx R_c + j0$ ,  $\beta = 2\pi/\lambda$ . Computation from these relations is relatively simple, and complete curves are available which are to be published at a future date. Evidently the resistance and reactance of a given termination are completely specified by the terminal functions  $\rho$  and  $\Phi$ . It will be shown in the following sections that all the measurable quantities mentioned may be expressed conveniently in terms of  $\rho$  and  $\Phi$ .

Before proceeding, it is advisable to purchase some simplification of (6) at the price of additional notation. It will first be assumed that the generator voltage  $V_0$  and the contribution from the termination at the

<sup>3</sup> W. L. Everitt, "Communication Engineering," McGraw-Hill Book Co., New York, N. Y., 1937, pp. 157-160, equations (41) and (48).

<sup>4</sup> D. D. King and Ronald King, "Terminal functions for antennas," *Jour. Appl. Phys.*, vol. 15, pp. 186-192; February, 1944.

sending end,  $(\sin^2 \rho_0 + \sinh^2 \Phi_0)$ , both remain constant at all times. Also, define

$$f(\beta, w) = \sinh^2(\alpha w + \rho_r) + \cos^2(\beta w + \Phi_r) \quad (8a)$$

$$g(\beta, l) = \sinh^2(\alpha l + \rho_0 + \rho_r) + \sin^2(\beta l + \Phi_0 + \Phi_r). \quad (8b)$$

$$\left. \begin{aligned} A' &= \alpha w + \rho_r \\ A'' &= \alpha l + \rho_0 + \rho_r \\ F' &= \beta w + \Phi_r \\ F'' &= \beta l + \Phi_0 + \Phi_r \end{aligned} \right\}. \quad (9)$$

Then

$$|V|^2 = K \frac{f(\beta, w)}{g(\beta, l)} \quad (10)$$

where  $K$  is a constant depending on the sending end. This equation forms the basis for all the methods of terminal-impedance measurement to be considered. Data from the measurable curves mentioned previously are used to obtain values of  $\rho$  and  $\Phi$  for an unknown terminating impedance. Determination of the phase shift  $\Phi$  is considered first.

#### PHASE-SHIFT MEASUREMENTS

The phase shift  $\Phi$  of a given termination is obtained from the position of either the voltage-distribution curve or the resonance curve. Since only one measurement is required, the frequency may be held constant. In accordance with the definition of position mentioned previously, that value of  $w$  or  $l$  must be found which yields maximum or minimum amplitudes. Maximum and minimum values of (10) are found by equating to zero the derivative with respect to  $w$  or  $l$ . The choice of variable depends on whether a movable detector is used or the line length is varied to produce a resonance curve. In either case, the following result is obtained:

$$\cos 2F = \pm \sqrt{1 - (\alpha/\beta)^2 \sinh^2 2A}. \quad (11)$$

This result applies to both the distribution-curve method ( $A'$  and  $F'$  in (9)) and the resonance-curve method ( $A''$  and  $F''$  in (9)). Whenever a result applies equally to either method the primes will be omitted.

A very simple final form is obtained by restricting the values of  $A$ . The physical meaning of this approximation is considered later.

$$(\alpha/\beta)^2 \sinh^2 2A \ll 1. \quad (12)$$

Subject to this condition, the extreme values of  $|V|^2$  in (10) occur at  $F = m\pi/2$ , where  $m$  is an integer. Now let  $F_1$  denote the extremized condition for the system with some standard termination  $\rho_{r1}$ ,  $\Phi_{r1}$  substituted for the load at the receiving end. Likewise, let  $F_2$  denote the extremized condition with the unknown termination  $\rho_{r2}$ ,  $\Phi_{r2}$  as the load. It follows that

$$\left. \begin{aligned} F_1 &= \beta \left\{ \frac{l_1}{w_1} \right\} + \left\{ \frac{\Phi_0}{0} \right\} + \Phi_{r1} = m\pi/2 \\ F_2 &= \beta \left\{ \frac{l_2}{w_2} \right\} + \left\{ \frac{\Phi_0}{0} \right\} + \Phi_{r2} = n\pi/2. \end{aligned} \right\} \quad (13)$$

In these equations the upper term in the brackets applies to the resonance method, the lower term to the distribution-curve method with movable detector. The same result is obtained for either method by subtraction.

$$\beta \left( \left\{ \frac{l_1}{w_1} \right\} - \left\{ \frac{l_2}{w_2} \right\} \right) + (\Phi_{r1} - \Phi_{r2}) = (m - n)\pi/2. \quad (14)$$

If the same maximum or minimum is used for both standard and unknown terminations, then the two integers  $m$  and  $n$  are equal. In case extra half-wavelengths are inadvertently included in the reading for either termination, the contribution to the left side of (14) will be plus or minus some integral multiple of  $\pi$  radians. Therefore the value of  $(m - n)$  on the right side is either zero or some even integer. Inspection of (7a) and (7b) shows that the impedance is unchanged by the addition or subtraction of some multiple of  $\pi$  to the termination phase shift  $\Phi$ . Therefore no ambiguity in the impedance value is possible for any consistent procedure to determine minima or maxima along the line. Having once determined  $l_1$  or  $w_1$ , as the case may be, the unknown phase shift  $\Phi_{r2}$  is given in terms of the known  $\Phi_{r1}$ , the length  $l_2$  or  $w_2$ , and the propagation constant  $\beta$ .

This result presumes the inequality (12) to be satisfied. The restrictions implied in this condition are later shown to be extremely mild.

#### DETERMINATION OF DAMPING

The shape of the distribution or resonance curves is a measure of the damping in the circuit. The simplest and most useful formula is derived from the ratio of maximum to minimum of a given curve. The result may be deduced by inspection from (10).

$$\frac{|V|^2_{\max}}{|V|^2_{\min}} = \frac{\sinh^2 A + 1}{\sinh^2 A} = \coth^2 A. \quad (15a)$$

If the variable is  $w$ , corresponding to a moving detector on the measuring line, a standard result is obtained.

$$\coth^2 A' = S^2. \quad (15b)$$

Here  $S$  is the voltage-standing-wave ratio. Equation (15b) affords a convenient relation between the damping in nepers and the usual standing-wave ratio. An analogous equation applies to the ratio of maximum to minimum of the resonance curve, as obtained by changing the length  $l$  of the line. The equation for this ratio,  $S_{res}$ , is included below.

$$\coth^2 A'' = S_{res}^2. \quad (15c)$$

These results assume that the frequency is held constant. Varying the frequency, i.e., varying  $\beta$ , yields results which involve the actual magnitude of  $w$  and  $l$ . In addition, it must be assumed that the damping is not a function of frequency. The method of frequency variation is better suited to the type of procedure considered below.

The analysis in terms of the width of a curve is most easily set up by following the experimental procedure.

Consider the case of a resonance curve obtained by changing the length  $l$  of the measuring line. The detector is kept fixed, as is the frequency. Accordingly, (10) takes the form:

$$|V|^2 = \frac{\text{constant}}{g(l)}. \quad (16)$$

As the length  $l$  is varied, a maximum value of  $|V|^2$ , namely  $|V|_m^2$ , will occur for which the denominator of (16) has the corresponding value  $g_m(l)$ . If the curve is symmetrical about the maximum, a smaller value of  $|V|^2$ , say  $|V|_p^2$ , is obtained at  $l + \delta l$  and  $l - \delta l$ . At these two points  $g(l)$  has the value  $g_p(l)$ . The width of the curve shown in Fig. 1 is  $2\delta l$ , as measured between points of amplitude  $|V|_p^2$ . Now let

$$|V|_p^2 = |V|_m^2/p^2. \quad (17)$$

Here  $p$  is the ratio of the two voltages. Then, by (16),

$$p^2 g_m(l) - g_p(l) = 0. \quad (18)$$

Using (8) and (9), this may be written

$$p^2(\sinh^2 A'' + \sin^2 F'') - (\sinh^2(A'' \pm \alpha \delta l) + \sin^2(F'' \pm \beta \delta l)) = 0. \quad (19)$$

Since  $A''$  and  $F''$  are taken to maximize (16), it follows that  $\sin^2 F''$  vanishes. The inequality (12) assures this and is assumed to apply. In order to secure a suitable final result, an additional restriction must be placed on the damping terms before expanding (19).

$$2\alpha \delta l \ll A'' \quad (20a)$$

$$2(\alpha \delta l)^2 \ll 1. \quad (20b)$$

Under these conditions (19) takes a simple form.

$$(p^2 - 1) \sinh^2 A'' = \sin^2 \beta \delta l. \quad (21)$$

The width of resonance curves is commonly measured between half-power points. According to (17), this means that  $p^2 = 2$ . For small values of damping the functions in (21) may be replaced by their arguments. Under these conditions the total damping  $A''$  is given by the following:

$$A'' = \beta \delta l. \quad (22)$$

The shape of the voltage-distribution curve obtained with a movable detector may be studied in identical fashion. In this instance (16) becomes

$$|V|^2 = (\text{constant}) f(w). \quad (23)$$

Since  $f(w)$  is in the numerator while  $g(l)$  was in the denominator, the same arguments are now applied to the *minimum* of the voltage-distribution curve as were previously applied to the maximum of the resonance curve.

The amplitude  $|V|_p^2$  is now larger than the minimum value  $|V|_m^2$ ; hence  $p^2$  must be replaced by  $1/p^2$  in (17). Likewise,  $1/f(w)$  is substituted for  $g(l)$  by virtue of (23). This gives the same form to (18). Accordingly (19) through (22) now apply to the minimum of the distribution curve with  $A''$  and  $l$  replaced by  $A'$  and  $w$  respectively.

In order to complete the discussion of the resonance and distribution curves obtained at constant frequency, the minimum of the resonance curve and the maximum of the voltage-distribution curve must be examined. The analysis proceeds in the same fashion as above. Consider first the resonance curve at constant frequency. Equation (19) is a convenient starting point; it must be written with  $p^2$  replaced by  $1/p^2$  to denote the fact that a minimum instead of a maximum is treated.

$$\sinh^2 A'' + \sin^2 F'' - p^2(\sinh^2(A'' \pm \alpha \delta l) + \sin^2(F'' \pm \beta \delta l)) = 0. \quad (24)$$

Here  $\sin^2 F''$  has the value of unity, since (16) is minimized. Subject to the same conditions as before, this yields:

$$(p^2 - 1) \sinh^2 A'' = 1 - p^2 \cos^2 \beta \delta l. \quad (25)$$

This applies to the region about the minimum of the resonance curve. Upon replacing  $A''$  by  $A'$  and  $l$  by  $w$ , the same equation applies to the region about the peak of the voltage-distribution curve. The analogy in the application of (21) and (25) to both resonance and distribution curves is complete.

The independent variables in the preceding discussion were  $l$  and  $w$ , corresponding to adjustment in line length or detector position. It remains to examine the situation when these quantities are fixed and the frequency is varied. The analysis proceeds from (10) written in the following form:

$$|V|^2 = K \frac{\sinh^2 A' + \cos^2(\beta w + \Phi_r)}{\sinh^2 A'' + \sin^2(\beta l + \Phi_0 + \Phi_r)}. \quad (26)$$

The line is now adjusted for maximum detector deflection, in accordance with the inequality (12). The method requires that  $A'' \approx A' \approx \text{constant}$  over the frequency range used.

$$|V|_m^2 = K \frac{\sinh^2 A + 1}{\sinh^2 A}. \quad (27a)$$

Since  $\beta = 2\pi f/v$  ( $v$  = phase velocity), a variation in frequency  $\pm \delta f$  or in equivalent electrical length  $\pm \delta \beta$ , above or below the value for maximum, produces some other amplitude  $|V|_p^2$ .

$$|V|_p^2 = K \frac{\sinh^2 A + \cos^2(\beta w + \Phi_r \pm w \delta \beta)}{\sinh^2 A + \sin^2(\beta l + \Phi_0 + \Phi_r \pm l \delta \beta)}. \quad (27b)$$

Equation (17) relates the two amplitudes, as before, and in combination with (27a) and (27b) yields the desired expression.

$$\sinh^2 A(p^2 - 1 - p^2 \sin^2 \delta \beta w - \sin^2 \delta \beta l) = \sin^2 \delta \beta l. \quad (28)$$

At this point it is expedient to assume small damping, thus replacing the functions by their arguments. Also, the width of the resonance curve is assumed to be measured at the half-power point,  $p^2 = 2$ . The resultant formula is similar to (22).

$$A = \delta \beta l / \sqrt{1 - 2 \sin^2 \delta \beta w - \sin^2 \delta \beta l}. \quad (29a)$$

Since small arguments were assumed, the radical may be set equal to unity. However, an important and fundamental point is involved in (29a). Evidently the equation as it stands makes the damping depend on the length of line  $l$ , even if the line were lossless and only the termination dissipative. The difficulty is that  $l$  is in the argument of a periodic function in the fundamental equation (26). The proper result is given by choosing  $l$  equal to the period,  $\lambda/2$ . Here  $\lambda$  is the wavelength at the center frequency  $f$  and the equivalent lengths of  $\Phi_0$  and  $\Phi_r$  are lumped with  $\beta l$  to make the argument  $\pi$  in (26) and (27b). Physically, this is equivalent to requiring the system to operate in the fundamental mode. This is a standard assumption in lumped-constant circuits; in the case of a transmission line more than one-half wavelength is regularly involved, and hence a special interpretation is required. A more familiar form for (29a) is now obtained in terms of the frequency, which is proportional to  $\beta$ .

$$A = \pi \delta f / f = (\pi/2) \frac{f'' - f'}{f} \quad (29b)$$

where  $f''$  and  $f'$  are the frequencies at the half-power points.

The equation is in agreement with the standard definition of the  $Q$  of a transmission line, provided only one half-wavelength is considered with lossless terminations. The  $Q$  of a line circuit with dissipative terminations evidently differs from the standard value  $\beta/2\alpha$ ; it is best defined in terms of the resonance curve only.

#### LIMITATIONS OF THE FORMULAS FOR PHASE SHIFT AND DAMPING

In order to indicate more clearly the range of loads that can be handled by the formulas which have been derived, the conditions assumed are restated in modified form. Thus, the general relation (12) is more easily interpreted if it is stated in terms of the standing-wave ratio in power,  $S^2$ , and the quality factor  $Q = \beta/2\alpha$  for the measuring line. With the aid of (15a) the inequality (12) becomes

$$\frac{S^2}{Q^2(S^2 - 1)^2} \ll 1. \quad (30a)$$

A sufficient condition is obtained after inverting and dividing by  $S^2$ .

$$(S^2 + 1/S^2 - 2) \gg 1/Q^2. \quad (30b)$$

Evidently a good line with a  $Q$  of the order of a thousand or higher permits accurate phase measurements with very low standing-wave ratios.

Additional restrictions are implied in the methods involving the width of resonance and distribution curves. If measurements are made at the half-power points, (20a) and (22) may be combined to give the condition

$$Q \gg 1. \quad (31a)$$

Likewise, if  $2\delta l = \lambda/2$ , which is certainly as wide as any curve can be, (20b) yields

$$Q^2 \gg \pi^2/2. \quad (31b)$$

Therefore, in practice the  $Q$  of the line does not limit the values of damping that can be handled by measuring the curve width. On the other hand, the largest value of  $A$  that can be handled by (21) is

$$A = \sinh^{-1} \sqrt{\frac{1}{p^2 - 1}}. \quad (31c)$$

In some instances a value of  $p^2$  less than the usual 2 may be desirable. The extended range is purchased at the price of decreased accuracy, however, since the measured width of the curve is now multiplied by a larger number, i.e.,  $1/\sqrt{p^2 - 1}$ .

An interesting consequence of these conditions from the experimental point of view is that resonance and distribution curves should be symmetrical. In effect,  $A \pm \alpha \delta x \approx A$  if they are fulfilled ( $x = l$  or  $w$ ). A common difficulty with the resonance procedure is generator loading. As the line is tuned through resonance the generator output changes to produce an asymmetrical resonance curve. This corresponds to a variable  $K$  in (10) and invalidates all the results obtained. Lack of symmetry of the resonance or distribution curves is therefore a useful indicator of improper circuit adjustment.

Either the peak of the resonance curve at constant frequency or the minimum of the distribution curve may be treated by (21) and (22). Experimentally, the two methods differ materially. In the resonance method the detector is kept fixed at the point along the line giving maximum deflection. The line length is then changed to produce a resonance curve. If this procedure is carried out with the unknown load and then repeated with a known standard termination, the difference between the two values of  $A''$  yields the true termination damping for the unknown. All detector loading and reflected damping from the generator is eliminated. On the other hand, when the detector is moved to obtain the shape of the distribution curve, its loading is no longer a constant and cannot easily be eliminated. However, the change in loading is small in the interval between half-power points.

The peak of the resonance curve is at a fairly high voltage amplitude. Therefore, the detector in the resonance method is favorably located. However, the generator must be loosely coupled, as mentioned before. In contrast, measurements on the width of the distribution curve are near the minimum voltage amplitude. This demands a more sensitive detector, but by way of compensation there are no longer any restrictions on the coupling between line and generator. In general, it appears that the width of the resonance-curve peak or the distribution-curve minimum is a convenient index of termination damping, provided it is not too large.

The standing-wave-ratio method with (15b) seems best suited to large damping, i.e., low standing-wave ratio. In case the standing waves are high, the operating range of the detector becomes very great and probe

loading effects appear.<sup>5</sup> The calibration for the detector must then also be extended unduly. However, the standing-wave ratio  $S$  is often used as a variable in charts and its usefulness here is not limited by experimental considerations. If desired, results in damping  $A$ , in nepers, can be converted easily to  $S$ . Equation (15a) is rewritten here to illustrate this.

$$S^2 = \frac{\sinh^2 A + 1}{\sinh^2 A} \quad (32)$$

Results from the other methods using (21) are expressed directly in  $\sinh^2 A$ . Accordingly, (32) affords a convenient step for expressing all results in  $S$  if desired, even though all the data are not best obtained by the standing-wave-ratio method.

Finally, the method of frequency variation remains to be mentioned; its application is restricted, since low damping and operation in the fundamental mode are required.

#### CALIBRATION

All damping measurements, and particularly the standing-wave-ratio method, require a calibrated detector. With crystal detectors, it is often advisable to repeat the calibration at intervals. The standard method of obtaining the desired calibration curve makes use of the upper portion of the distribution curve along a line with high standing waves. The curve is assumed to be a sinusoid; this permits plotting detector output against the true (sinusoidal) voltage along the line.

In order to test the validity of this procedure the shape of the curve near the maximum must be examined.

<sup>5</sup> W. Altar, F. B. Marshall, and L. P. Hunter, "Probe error in standing wave detectors," *PROC. I.R.E.*, vol. 34, pp. 33-44; January, 1946.

Equation (25) may be applied to the distribution curve, as explained previously, by substituting  $A'$  for  $A''$  and  $w$  for  $l$ . The damping  $A'$  may be replaced by the standing-wave ratio with the aid of (32). A convenient form of the resulting relation is

$$\beta \delta w = \sin^{-1} \left[ \frac{1 - 1/p^2}{1 - 1/S^2} \right]^{1/2} \quad (33)$$

The half-width  $\beta \delta w$  of the distribution curve is given by this equation at a power level  $(1/p^2) \cdot |V|_{\max}^2$ . A sinusoidal distribution is strictly true only when  $S^2 \rightarrow \infty$ . The deviation from the limiting case is obtained by substituting the actual value of the standing-wave ratio  $S^2$  in (33). An approximate result is available by inspection: the curve is very nearly a sinusoid provided  $S^2 \gg p^2$ . In case a calibration is to be carried out at a fairly low standing-wave ratio, (33) is useful in determining the range and accuracy obtainable.

#### CONCLUSION

The preceding treatment of equations for terminal impedance is intended to bring out the basic similarity in all of the formulas which depend on amplitude measurement. The various relations are couched in terms of directly measurable quantities, and hence have a simple physical meaning in terms of the parameters  $\alpha$  and  $\beta$  of the line. Examples of the application of the methods described may be found elsewhere.<sup>6,7</sup>

The writer is indebted to Ronold King for suggesting the subject and to J. V. Granger for assistance in preparing the manuscript.

<sup>6</sup> D. D. King, "Microwave antenna impedance measurement with application to antennas," *Jour. Appl. Phys.*, vol. 16, pp. 435-444; August, 1945.

<sup>7</sup> D. D. King "The measured impedance of cylindrical dipoles," *Jour. Appl. Phys.*, vol. 17, pp. 844-852; October, 1946.

## Microwave Power Measurement\*

THEODORE MORENO†, ASSOCIATE, I.R.E., AND OSCAR C. LUNDSTROM‡, ASSOCIATE, I.R.E.

**Summary**—Possible methods of microwave power measurement are reviewed. The design requirements for bolometric wattmeters are outlined, and examples are given of bolometer elements that have been developed to meet these requirements. A recently developed bolometer element that may be used over an exceedingly

wide band of frequencies is included. The results of experiments to investigate sources of error and to determine the accuracy of these wattmeters are summarized. These experiments indicate that, although serious errors are possible, proper usage will hold errors to within a few per cent.

#### I. INTRODUCTION

AT ORDINARY radio frequencies, voltmeters and current meters may be used to determine power levels when impedances are known or measur-

ble. At higher frequencies, these meters are increasingly difficult to construct. When wave guides are used, voltage and current are ambiguous quantities that cannot be uniquely defined. It is therefore necessary at some point to turn to different methods of measurement.

#### II. SUMMARY OF TECHNIQUES

(A) *Low-frequency techniques applicable at higher frequencies.*

Rectifiers, which are useful at lower frequencies, still find application at microwave frequencies in the form of

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† Formerly, Engineering Laboratories, Sperry Gyroscope Company, Inc., Garden City, L. I., N. J., now, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass.;

‡ Engineering Laboratories, Sperry Gyroscope Company, Inc., Garden City, N. Y.

crystal rectifiers. However, these suffer from instability and temperature sensitivity, and require calibration against some absolute standard. Thermocouples are still of value at the highest frequencies, but also require calibration.

*(B) Techniques not extensively used at low frequencies.*

The techniques most suited for microwave power measurement generally involve the conversion of microwave power into heat. A further subdivision may be made into calorimetric and bolometric methods.

In the calorimetric method, the power to be measured is absorbed in a liquid or dissipated in a resistor which is cooled by a liquid. The power is then measured by determining the temperature rise and rate of flow of the liquid when equilibrium conditions have been established. Accuracy can be high, and calorimeters are frequently used as standards. However, the required equipment is bulky, time is required for equilibrium to be established, and the minimum power which can be measured accurately is a few watts. The calorimetric method is therefore suitable primarily for special laboratory measurements.

The bolometric method is generally more rapid, requires less bulky apparatus, and will handle lower powers. The required equipment is illustrated in Fig. 1. The heart of the wattmeter is a sensitive resistance element or bolometer whose resistance changes with temperature, and with power dissipated. This bolometer is usually a small bead of resistive material, such as the "thermistor," or a short length of fine wire. This element is placed in one arm of a direct-current bridge and heated by the direct current until the bridge is brought to balance. Dissipation of microwave power in the element will then unbalance the bridge.

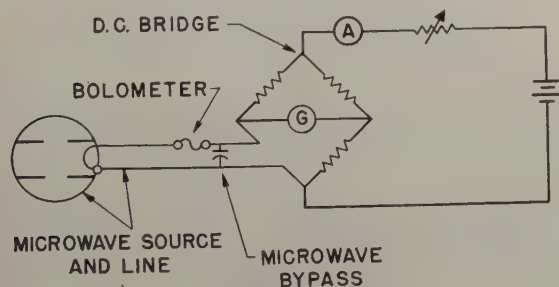


Fig. 1—Schematic circuit diagram of a bolometer wattmeter.

The degree of bridge unbalance may be used to measure the microwave power, with calibration accomplished by noting the unbalance caused by the addition of a known amount of low-frequency power to the sensitive element. Or the bridge may be rebalanced after addition of the microwave power, and the power measured is then said to equal the amount of direct-current power that it was necessary to subtract from the sensitive element in the rebalancing operation.

This type of measuring instrument is most useful at

microwave frequencies. But before discussing the design techniques that are applied to the components, it is well to look at the requirements which the instrument must meet.

### III. REQUIREMENTS TO BE MET BY A BOLOMETRIC WATTMETER

Certain fundamental requirements must be met by a bolometric wattmeter which is to be used for absolute power measurement. First among these is that all of the power to be measured be dissipated in the active element. Possible sources of loss which must be avoided include radiation, lossy dielectrics, poor conductors, and poor contacts between conducting surfaces. It is also necessary that the bolometer absorb substantially all of the power in the incident wave, and it must therefore be well-matched to the input transmission line.

An additional fundamental requirement is that direct-current and microwave power produce equivalent changes in resistance. This is necessary in order that the direct-current power which is subtracted from the bolometer in rebalancing the bridge be equal to the microwave power dissipated in the element.

Other bolometer characteristics are not so necessary for accuracy, but add to the ease and convenience of operation and minimize the possibility of error from improper use.

The sensitivity of the element is important, especially where small powers are to be measured. This sensitivity depends not only upon the ohms-per-watt sensitivity of the resistance, but also upon the current and resistance at which the element is operated. A high current and low resistance are desirable from this standpoint.

When larger powers are to be measured, the power-handling capacity of the element becomes more important than the sensitivity. High powers can always be cut down to the range of any element with the use of attenuators, but the design of suitable broad-band, high-power attenuators has proved in many ways to be more of a problem than the design of the wattmeter itself. So there is a definite need for bolometers which can handle large powers, and a corresponding loss of sensitivity can be tolerated.

The ability of the bolometer to withstand overloads without burning out must be considered. For some applications a large safety factor is essential, but there are many applications in which a 100 per cent safety factor is adequate.

The sensitivity of the bolometer element to changes in ambient temperature must also be considered. Ambient temperature sensitivity can be compensated for in the bridge design, but the bridge design is simpler if compensation is not necessary. To minimize the ambient temperature sensitivity, it is desirable to operate the element at a temperature far above ambient. This would indicate an incandescent element in a vacuum or in an inert gas.

A fast thermal time constant will increase the ease and rapidity with which measurements can be made. But if a bolometer with a very small time constant is used with certain bridges, errors in measurement may result when the microwave energy is amplitude-modulated. Some results of an investigation into this source of error are presented in Section V. In addition, a short thermal time constant increases the possibility of burn-out when pulsed power is being measured.

The construction and operating resistance of a bolometer depend to a great extent upon the holder with which it is used. A holder may be designed to adapt an existing element to a new type of transmission line or a new band of frequencies, but best results are usually obtained when an element and its holder are designed simultaneously, in order that they complement each other to the greatest possible extent.

It is advantageous, though, to have a bolometer element which may be adapted to a variety of holders, as this simplifies production and replacement problems. Distinct further advantages are gained if the elements are nearly enough identical to be interchanged or replaced without greatly altering the direct-current characteristics and without requiring retuning of the holder to maintain the desired impedance properties. It is also desirable that the desired impedance properties be maintained over the widest possible frequency band with the minimum number of adjustments.

#### IV. EXAMPLES OF MICROWAVE BOLOMETER ELEMENTS

Among the first bolometer elements to be used at microwave frequencies were the 5- and 10-milliamperere Littelfuses. The sensitive element is the platinum fuse wire, about 0.00006 inch in diameter. The resistance sensitivity is about 4 ohms per milliwatt, but the overload safety factor is not large. The impedance variation between elements is great enough so that at 9000 megacycles the holder must be retuned for each element. Better results are obtainable from a similar element which has been designed for the specific purpose of power measurement. An element of this type, often called a barretter, is shown in Fig. 2. These units have very uniform impedance properties, and may be interchanged in a 9000-megacycle holder without appreciably changing its microwave impedance. Fixed-tuned holders for this unit have been designed in wave guide for bands greater than  $\pm 10$  per cent with a standing-wave ratio under 1.26.

These bolometers, which employ very fine platinum wires suspended in air, are distinguished by their very short thermal time constant, in the order of 350 microseconds. They may therefore be used as detectors for audio-modulated signals and will measure rapid variations in power level. At the same time, as discussed in Section V, errors are possible when amplitude-modulated signals are being measured. Also, when measuring pulsed power as is used in radar sets, a typical unit will have

its power-handling capacity reduced from 30 milliwatts to an average power several times less. The elements are most satisfactorily used for measuring continuous power below 10 milliwatts.

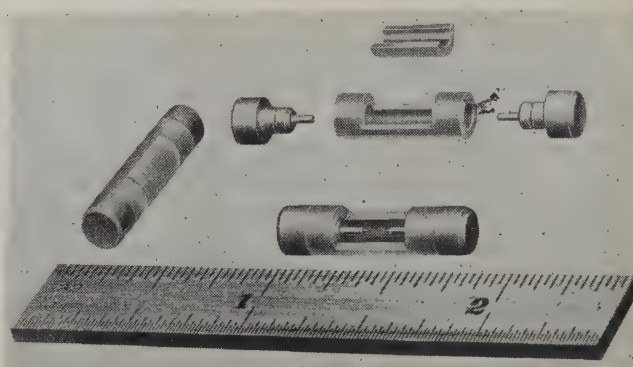


Fig. 2—Barretter element employing fine platinum wire for use in bolometric power measurement.

The "thermistor" has been extensively used as a bolometer element during the past few years. As used for power measurement, the thermistor takes the form of a small bead of resistive, temperature-sensitive oxides, suspended between two fine lead-in wires, and sometimes encased in a glass envelope as shown in Fig. 3. The resistor formed by the bead has a high negative temperature coefficient of resistance, which under typical operating conditions may be 15 ohms per milliwatt. The thermistor is therefore more sensitive than the platinum wire units, and is generally used for powers under 10 milliwatts.

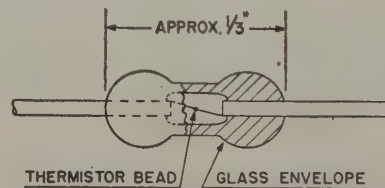


Fig. 3—Bead thermistor used in power measurement.

Thermistors will withstand much greater overloads than will platinum-wire barretters. Furthermore, their thermal time constant and heat capacity are relatively large. They are therefore ideally applicable to the measurement of pulsed power. The ambient temperature sensitivity is high, and the bridge must provide adequate compensation. The impedance variation is large enough to require retuning a 9000-megacycle holder when elements are changed, although a fixed-tuned holder may be used at 3000 megacycles.

For powers from 10 milliwatts to 15 watts, load lamps have been widely used as bolometer elements. A typical load lamp is shown in Fig. 4. It consists of a length of fine tungsten wire suspended in an evacuated or hydrogen-filled bulb. The lamp is intended for insertion in the center conductor of a coaxial line. This bolometer will give accurate results if properly used but is subject to possible gross error resulting from nonuniform current distribution along the filament when located near a

current node (see Section V). Furthermore, because of the resonant length of line beyond the filament, it is very difficult to match the lamp to a transmission line over any but a very narrow band of frequencies.

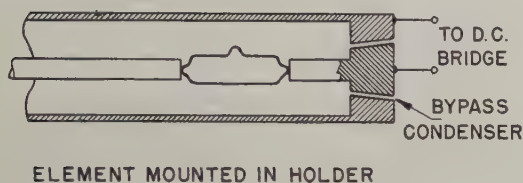
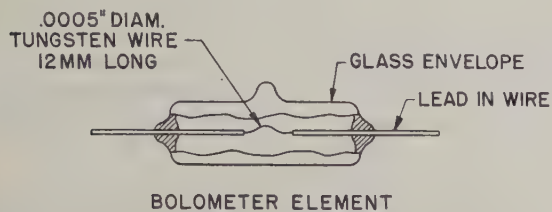


Fig. 4—High-power load lamp used for power measurement.

An improved type of load lamp and its associated holder are shown in Fig. 5. The sensitive element is again a short resistive wire, but is located at the end of the coaxial line. Mounting the wire adjacent to the short guarantees an essentially uniform current distribution along the filament, minimizing the possibility of error from this source. There is no resonant length of line beyond the filament, and matching problems are tremendously simplified.

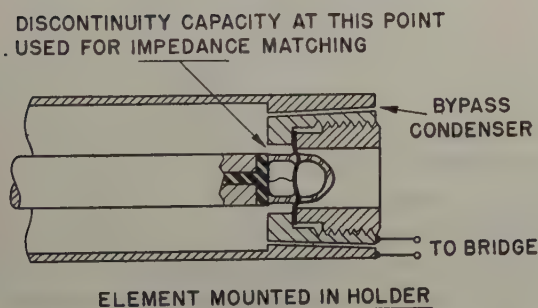
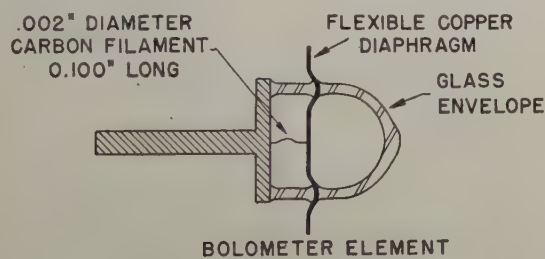


Fig. 5—Improved load lamp with broad-band impedance-matching properties.

If the filament is operated at a resistance near the characteristic impedance of the input line, the lamp in its holder will present a matched load to the line at all frequencies below that at which the inductance of the filament becomes appreciable. If this inductance is resonated with the discontinuity capacity at the filament input, the frequency range will be further ex-

tended. The high-frequency limit  $f_0$  which may be reached in this manner is given (within the approximation  $2\pi l/\lambda \cong \tan 2\pi l/\lambda$ ) by

$$f_0 = \frac{\sigma \left(1 - \frac{1}{\sigma^2}\right)^{1/2} \cdot Rc}{120\pi \ln b/a}$$

where  $\sigma$  is the maximum acceptable standing-wave ratio,  $R$  is the resistance per centimeter length of the filament at operating temperature,  $c$  is the velocity of light ( $c = 3 \times 10^{10}$  centimeters), and  $b/a$  is the diametric ratio of the line in which the filament forms the center conductor.

In one practical form of the bolometer, a carbon filament 2 mils in diameter has been chosen for its high resistivity, and bolometers have been matched to a 46-ohm coaxial line with a standing-wave ratio under 1.1 for all frequencies below 1100 megacycles, or with a standing-wave ratio under 1.4 for frequencies below 2900 megacycles. These values were predicted theoretically, and experimentally checked. They do not represent the maximum frequencies at which the lamps may be used. The range may be further extended with the use of more complex matching or equalizing networks, or holders may be designed which are not good at low frequencies but give satisfactory results over frequency bands higher than the above limits.

## V. ERRORS ENCOUNTERED IN BOLOMETER WATTMETERS, AND ACCURACY OF POWER MEASUREMENT

A question that can always be asked about microwave wattmeters is whether they are actually measuring power, or just something proportional to power. At present, there is no generally accepted standard of power measurement against which comparisons may be made, although calorimeters are sometimes used for this purpose. But calorimeters are subject to errors if improperly designed or employed, and considerable care is required to reduce these errors to less than 2 or 3 per cent.

It is usually necessary to compare the results of 2 or three instruments and, from the results, make a judicious estimate of the errors involved in each measurement. This technique may also be used to measure the relative error as a function of some variable when only one of the instruments is subject to error from the variable conditions.

A number of experiments of this type have been performed. In some unpublished work, E. Feenberg and R. Kahal have shown theoretically that serious errors can result from nonuniform current distribution when the power-resistance relation of the bolometer is not linear and when the power being measured is a considerable part of the total power dissipated in the bolometer. For short filaments, the error will be large if the filament is at or near a current node. The magnitude of the error increases sharply with increasing power level for a given bolometer.

The qualitative conclusions have been verified experimentally, although close quantitative checks have not been obtained. The results of a typical experiment are shown in Fig. 6. A holder and vacuum bolometer of the type shown in Fig. 4 were used at a frequency of 3000 megacycles. This bolometer has a power-resistance relation of the form  $R = A + BP^{1/3}$ , where  $R$  is resistance,  $P$  is power, and  $A$  and  $B$  are constants. The current distribution was varied by moving the position of the short behind the filament. The input power was continuously monitored by a calibrated probe and slotted section, and was adjusted to hold the power indicated

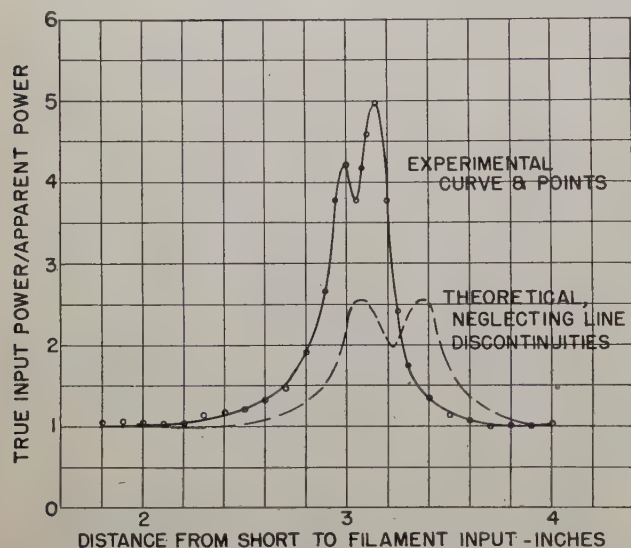


Fig. 6—Ratio of true input power to apparent input power with nonuniform current distribution along the sensitive filament.

by the bolometer wattmeter constant at 80 per cent of the total power required for balance. The ratio of true input power (as measured by the calibrated probe) to apparent power (as measured by the bolometer and bridge) is plotted in Fig. 6 as a function of the distance from the short to the filament input. Large errors were observed, larger than predicted by theory. This is believed the result of other sinks of power in the associated line and holder, whose effects were accentuated by the very high standing-wave ratio (up to 65) on the input line when the filament was located near a current node. This error is only one of maladjustment, but with the lamp and holder of Fig. 5 the error is not possible.

Errors may also arise with short-time-constant bolometers in certain bridge circuits if the input signal is amplitude-modulated at a frequency sufficiently low for the bolometer resistance to vary over the course of a cycle. This error was investigated by comparing the measurements of a thermistor to a platinum-wire barretter operating in an equal-arm, balanced bridge with matched source and galvanometer.

The power being measured was 100 per cent amplitude-modulated with a square wave, first at a frequency of 2 kilocycles, where the barretter resistance did not vary greatly over a modulation cycle, and second at a frequency of 20 cycles, where the barretter resistance

followed closely the modulation envelope. The thermistor resistance did not vary appreciably with the modulation.

The power indicated by the thermistor in these experiments is plotted in Fig. 7 as a function of the power indicated by the barretter. At the higher modulation frequency the two results were in agreement, but at the lower frequency the barretter error increased with power level, reaching a maximum of 40 per cent. A theoretical curve is also shown.

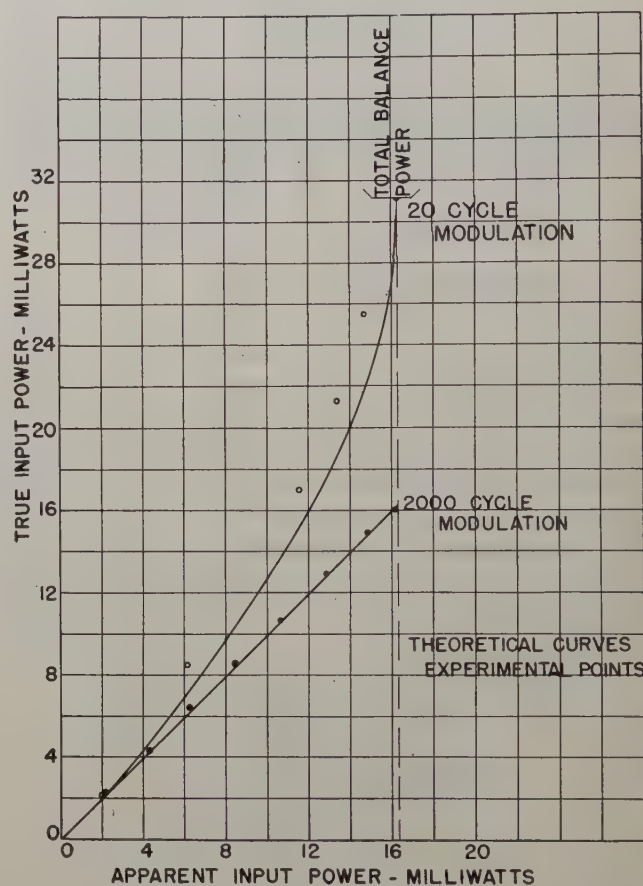


Fig. 7—Ratio of true input power to apparent input power for amplitude-modulated wave and equal-arm, matched bridge employing platinum-wire barretter.

Bolometers that are properly used, however, will give results in good agreement. In one experiment, carried out at 9400 megacycles, two thermistor mounts of Radiation Laboratory design, two thermistor mounts of English design, and a barretter mount of Sperry design were compared at a power level of 1 milliwatt. All results fell at random within 2 per cent of an arbitrary mean value, which was within experimental error.

Similar experiments at 3000 and 9400 megacycles which compared hot-wire bolometers and barretters with calorimeters have yielded agreement within 5 per cent, again within experimental error. It therefore seems reasonable to state that, at frequencies below 10,000 megacycles, a properly designed and properly employed bolometric wattmeter will measure true power within a few per cent.

# Design Values for Loop-Antenna Input Circuits\*

JAY E. BROWDER†, MEMBER, I.R.E., AND VICTOR J. YOUNG†

**Summary**—Design formulas and charts for the choice of inductances,  $Q$ 's, and coupling coefficients of loop-antenna coupling transformers are given on the basis of signal-to-noise characteristics. A method of determining these values when a cable or other primary capacitance is present is also given.

## INTRODUCTION

THIS ANALYSIS is based upon the fact that the power match of an antenna to a receiver is not necessarily the proper way to obtain best performance. In the transmitting case it is always desired that a maximum amount of power be transferred into the antenna, but with a receiver additional gain can always be supplied to make up for signal power. The signal-to-noise ratio that is obtained from the antenna cannot be improved in the receiver. The proper criterion for arranging receiver input circuits is that of obtaining a maximum signal-to-noise ratio. In the case of a loop receiving antenna, the power-matching criterion is never the proper one for determining the constants of the input circuit.

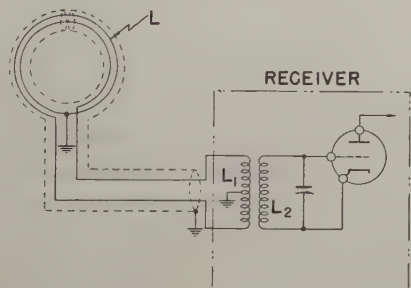


Fig. 1—A typical loop antenna in schematic form.

In Fig. 1 is shown a schematic drawing of a typical loop receiving-antenna input circuit. The shielded loop with its air gap is connected to a coupling transformer in the receiver either directly or through a transmission line. This coupling transformer, in turn, supplies the first vacuum tube and is tuned by a capacitor which must track with other tuning capacitors. It is the purpose of this paper to present the criteria that should be used in selecting the values of the various components for such an input circuit.

In specifying signal-to-noise ratios in terms of the circuit parameters, it will be necessary to assume that the resistive impedance of an antenna generates noise, just as does any other resistance, and that the normal strength of the noise generated in this way is obtained

by considering an equivalent resistor at a normal ambient temperature. It is well known that the noise power available from such a resistor is given by

$$N = kT\Delta f$$

where  $k$  is Boltzmann's constant,  $T$  is temperature in degrees Kelvin, and  $\Delta f$  is the bandwidth. In practice this may not always be true because the radiation resistance is concerned in this respect with external noise conditions. Such external noise fields may be thought of as changing the temperature of space. Thus external noise at a level other than the one used changes the quantitative results of the computation but does not affect their qualitative correctness.

## RELATIVE PERFORMANCE OF LOOP RECEIVING ANTENNAS

In order to compare the performance of various loop receiving antennas, it is necessary to assign a quantitative figure of merit to each antenna. As is common in most communication problems, the figure that has proven most versatile is a signal-to-noise ratio; it is the ratio of the signal strength to the thermal noise as measured at the open-circuited antenna terminals, under the assumption of a standard field strength and a standard bandwidth of the measuring equipment. It is also assumed in practice that the signal field strength in the neighborhood of the receiving antenna is larger than any noise fields which also exist in this region of space. This last assumption is, of course, not necessary as far as theoretical merit is concerned, but in practice if the noise field is larger than the signal field, nothing can be done to appreciably improve reception.

It is necessary to maintain both the standardization of bandwidth and a standard signal field strength in making absolute measurements of antenna performance. However, it is usually more convenient to deal entirely with ratios of performance between two or more antennas, whence it is necessary to specify the standard conditions to be maintained during any given set of tests. This means that the present discussion is not limited to any particular bandwidth or field strength.

The maximum open-circuit signal terminal voltage of any antenna is given by  $Eh_{eff}$ , and the thermal noise voltage of any resistor is given at room temperature by  $1.26 \times 10^{-10} \sqrt{R} \sqrt{\Delta f}$ . The condition for unity signal-to-noise ratio under the assumption of small noise field may then be written

$$E = \frac{1.26 \times 10^{-10} \sqrt{R} \sqrt{\Delta f}}{h_{eff}} \quad (1)$$

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† Sperry Gyroscope Company, Inc., Garden City, New York.

where

$E$  is the received signal field strength measured in volts per meter,

$R$  is the loop resistance, including radiation resistance

$\Delta f$  is the radio-frequency bandwidth

$h_{eff}$  is the effective height of the loop antenna, measured in meters.

But the effective height of a loop antenna may be given by

$$h_{eff} = \frac{2\pi A n}{\lambda} \quad (2)$$

where

$A$  is the area of the loop in square meters

$n$  is the number of turns

$\lambda$  is the wavelength in meters.

Making this substitution yields

$$E = \frac{1.26 \times 10^{-10} \sqrt{R} \sqrt{\Delta f} \lambda}{2\pi A n} \quad (3)$$

In this expression  $E$  is the minimum signal field strength which can be received with at least unity signal-to-thermal-noise ratio.

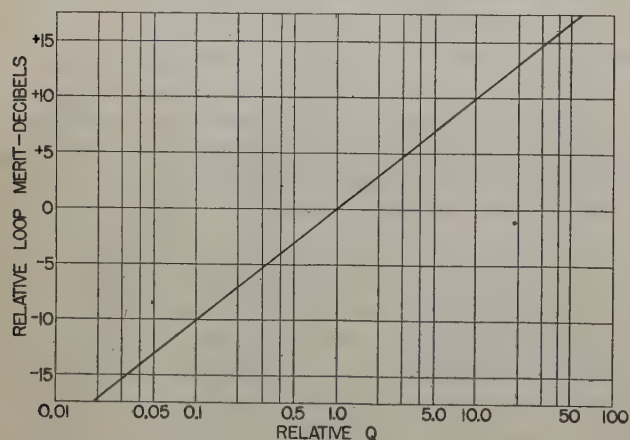


Fig. 2—Effect of loop  $Q$  on antenna merit for a loop having a constant area and form factor.

To compare one loop with another, it is only necessary to compare the corresponding minimum values of  $E$ . Doing this in ratio form, and using subscripts  $a$  and  $b$  to distinguish between the constants of the two installations,

$$\frac{E_b}{E_a} = \frac{A_a n_a \sqrt{R_b}}{A_b n_b \sqrt{R_a}} \quad (4)$$

This expression leads to the form

$$\text{Superiority of loop } b \text{ over loop } a = 20 \log \frac{A_b n_b \sqrt{R_a}}{A_a n_a \sqrt{R_b}} \text{ decibels.} \quad (5)$$

This formula gives a decibel comparison of the signal-to-noise merits of any two loops. When the expression

yields a positive number, loop  $b$  is that many decibels better than loop  $a$ . When the number computed is negative, it is loop  $a$  which is the best.

One interesting fact is immediately apparent from this expression. When loops of a given area and form factor (winding length divided by diameter) contain several turns (at least more than one), their inductance is proportional to the square of the number of turns. Thus, if under this condition the square roots of  $L_a$  and  $L_b$  are substituted for  $n_a$  and  $n_b$ , the formula can be modified to include only a constant and square roots of ratios of  $L_a$  to  $R_a$  and  $L_b$  to  $R_b$ . These ratios may, of course, be written as  $Q$ 's, and it can then be concluded that the merit of two loop antennas is identical as long as their  $Q$ 's, form factors, and areas are the same; or more specifically, that adding or subtracting turns to such a multiple-turn antenna will not change its merit.

In Fig. 2 is shown the relative merit of a direct-connected loop antenna which has multiple turns and constant area and form factor. As is shown, the merit of the loop is then dependent only on  $Q$ .

#### INPUT-CIRCUIT NOISE FIGURE

It will be assumed in what follows that the receivers operate at low radio frequencies. Under this condition the input conductance of the first vacuum tube is small compared to the circuit conductance and hence can be taken as a circuit characteristic. As a result, the noise of the first vacuum tube, which is very important, may be treated as a correction to the noise factor of the rest of the input circuit. The method of making such a correction will be discussed below, but first the operation of the input circuit without the vacuum tube connected must be considered.

To describe further the operating characteristics of a loop-antenna input circuit, it is necessary to develop what is called the inherent thermal noise of a receiving antenna. Consider a loop or any other type of receiving antenna which is exposed to a radio-frequency field of strength  $E$  in volts per meter. Such an arrangement will give rise to an open-circuit terminal voltage.

$$e = E h_{eff} \text{ volts} \quad (6)$$

where  $h_{eff}$  is the effective height of the antenna. This voltage  $e$  will be due to the received signal. In addition, there will also be a noise voltage present at the terminals. The series-resistance term in the antenna impedance as measured at the open-circuited terminals is the factor which determines this noise voltage. This is true irrespective of what fraction of this resistance is attributable to radiation and what fraction depends upon the ohmic resistance of the conductors. If this resistive component of the antenna impedance is  $R$ , then at room temperature the open-circuit terminal noise voltage is given by

$$e_n = 1.26 \times 10^{-10} \sqrt{R} \sqrt{\Delta f} \text{ volts.} \quad (7)$$

In case the radiation resistance of the antenna presents greater than the normal noise voltage, the excess noise can be considered to be caused by an external noise field, and will not be considered in this analysis.

Clearly, if one could use perfect input circuits, a signal of strength equal to that of the noise could just be received. This occurs when  $e = e_n$ , or when

$$E = \frac{1.26 \times 10^{-10} \sqrt{R} \sqrt{\Delta f}}{h_{eff}} \quad (1)$$

With practical input circuits,  $E$  has to be larger than this in order that the signal-to-noise level of the system be equal to unity. Expressed in decibels, the amount that  $E$  must be raised to yield the desired level with a practical input circuit is called the noise figure of the input circuit. Strictly, noise figure is always a negative quantity because it represents a reduction in system performance, as compared to a noiseless standard. In this paper it always appears as a negative number of decibels.

Although only loop receiving antennas are discussed herein, the concept of input-circuit noise figure can equally well be applied to any antenna connection. This quantity puts the calculation and measurement of the operation of an antenna coupling circuit on an absolute basis. Noise figure is most conveniently expressed in decibels. If it is zero decibels, then reception can occur at the minimum field strength as determined by  $h_{eff}$  and  $R$  of the antenna itself, and no further improvement in the coupling circuit can possibly be made. If, however, the noise figure of a given arrangement is minus 10 decibels, then considerable improvement can be made by redesign or improvement of the input circuit.

#### TRANSFORMER-COUPLED LOOP INPUT CIRCUITS

The actual circuit of a transformer-coupled loop input circuit is shown in Fig. 3(a).  $L$  and  $R$  represent the inductance and resistance of the loop. The other  $L$ 's and  $R$ 's specify the constants of the transformer as shown. For the purpose of analysis at the radio frequency, the circuit shown in Fig. 3(a) is replaced by the equivalent form shown in Fig. 3(b). Fig. 3(c) is also, in a sense, the same as that of Fig. 3(a) or 3(b). This follows from an application of Thevenin's theorem to the circuit of Fig. 3(b). Measurement of open-circuit inductance and resistance as seen at terminals  $AB$  will yield the values  $L_2'$  and  $R_2'$  and the complete circuit may be represented as shown in Fig. 3(c).

The circuits of Fig. 3 are used to determine the signal-to-noise ratio of the loop plus the input circuit. When a decibel comparison of this signal-to-noise ratio is made to the signal-to-noise ratio of the loop alone, the noise figure of the input circuit results.

When the loop is exposed to a radio-frequency field and properly oriented, a maximum signal voltage of strength  $e$  is induced in the antenna. In Fig. 3, this

voltage  $e$  may be thought of as being inserted in series with the loop at point  $P$ . As long as terminals  $AB$  are open,  $e$  will distribute itself only around the one available closed circuit and (assuming the  $R$ 's are small com-

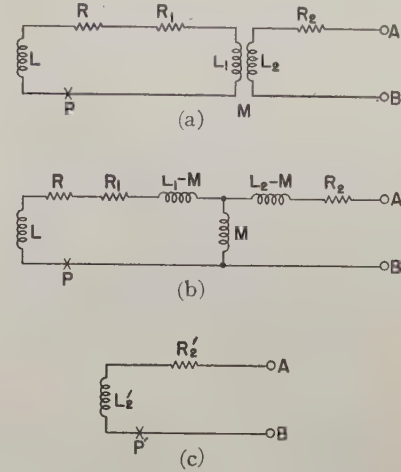


Fig. 3—Actual and equivalent loop-antenna transformer-coupling circuits.

pared to the  $X$ 's) the fraction of  $e$  which will appear across the inductance  $M$  of Fig. 3(b) will be given by

$$e_s = \frac{Me}{L + L_1} \quad (8)$$

Furthermore, as long as no load is placed on terminals  $AB$ , this is the voltage which will be observed at those terminals. Replacing  $M$  with  $k\sqrt{L_1 L_2}$ , where  $k$  is the coupling coefficient of the transformer, the voltage at  $AB$  may also be written as

$$e_s = \frac{k\sqrt{L_1 L_2}}{L + L_1} e \quad (9)$$

The impedance at terminals  $AB$ , called  $Z_{AB}$ , may be computed from Fig. 3(b). It is

$$Z_{AB} = R_2 + j[\omega(L_2 - M)] + \frac{(j\omega M)[R + R_1 + j\omega(L + L_1 - M)]}{R + R_1 + j\omega(L + L_1)} \quad (10)$$

This expression may be simplified because the  $R$ 's are normally small compared to the  $(\omega L)$ 's. It is specifically assumed that  $(R + R_1)^2$  is very small in comparison to  $[\omega(L + L_1)]^2$ . This is equivalent to saying that the  $Q^2$ 's must be large. In fact, for  $Q$ 's as small as 10, the value of  $Q^2$  is 100, and so the error of the approximation is equal to only 1 per cent. With this approximation and with the substitution of  $k\sqrt{L_1 L_2}$  for  $M$ ,  $Z_{AB}$  may be rewritten as

$$Z_{AB} = \left[ R_2 + \frac{(R + R_1)k^2 L_1 L_2}{(L + L_1)^2} \right] + j\omega \left[ L_2 - \frac{k^2 L_1 L_2}{L + L_1} \right] \quad (11)$$

We may calculate the thermal noise level at terminals  $AB$  from the resistive component of this impedance in the usual way. Denoting the noise voltage as  $e_n$ , then the signal-to-noise ratio at terminals  $AB$  is  $e_s/e_n$ . Written out completely, the expression is

Signal-to-noise ratio at terminals  $AB$

$$= \frac{\frac{k\sqrt{L_1 L_2}}{L + L_1} e}{1.26 \times 10^{-10} \sqrt{R_2 + \frac{(R + R_1) k^2 L_1 L_2}{(L + L_1)^2}} \sqrt{\Delta f}} \quad (12)$$

From this expression and from the signal-to-noise ratio of the loop terminals as obtained before, the noise figure of the coupling circuit is obtained by

Noise figure of coupling circuit

$$= 20 \log \frac{\text{Signal-to-noise at } AB}{\text{Signal-to-noise in loop}} \text{ decibels.} \quad (13)$$

Making the appropriate substitution yields

Noise figure of transformer coupling

$$= 20 \log \frac{k}{\sqrt{k^2 + k^2 \frac{R_1}{R} + \frac{R_2(L + L_1)^2}{RL_1 L_2}}} \quad (14)$$

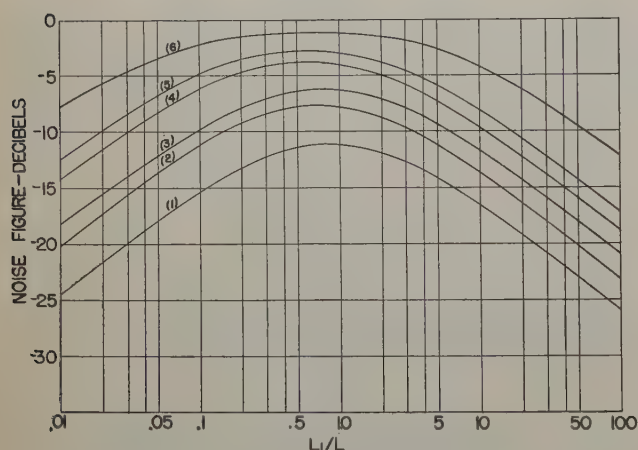


Fig. 4—Noise-figure variation with  $L_1/L$ .

- |                  |             |           |
|------------------|-------------|-----------|
| (1) $Q_2 = Q_1$  | $Q_1 = Q$   | $k = 0.6$ |
| (2) $Q_2 = Q_1$  | $Q_1 = Q$   | $k = 1.0$ |
| (3) $Q_2 = 2Q_1$ | $Q_1 = 2Q$  | $k = 0.6$ |
| (4) $Q_2 = 2Q_1$ | $Q_1 = 2Q$  | $k = 1.0$ |
| (5) $Q_2 = 2Q_1$ | $Q_1 = 3Q$  | $k = 1.0$ |
| (6) $Q_2 = 2Q_1$ | $Q_1 = 10Q$ | $k = 1.0$ |

Since it is more convenient to work with  $Q$ 's than with  $R$ 's, the appropriate  $\omega L/Q$  is next substituted for each  $R$ . The expression then becomes

Noise figure of transformer coupling

$$= 20 \log k - 10 \log \left[ k^2 + \frac{2Q}{Q_2} + \frac{k^2 Q}{Q_1} \frac{L_1}{L} + \frac{Q}{Q_2} \left( \frac{L_1}{L} + \frac{L}{L_1} \right) \right] \quad (15)$$

This last equation gives the variation and values of the transformer-coupling noise figure with any values or changes of the parameters of the input circuit. In Fig. 4,

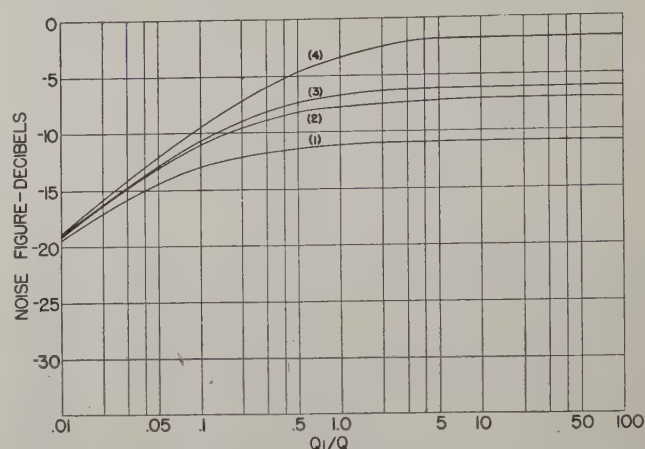


Fig. 5—Noise-figure variation with  $Q_1$ .

- |                    |             |           |
|--------------------|-------------|-----------|
| (1) $L_1/L = 0.75$ | $Q_2 = Q$   | $k = 0.6$ |
| (2) $L_1/L = 0.75$ | $Q_2 = Q$   | $k = 1.0$ |
| (3) $L_1/L = 0.75$ | $Q_2 = 2Q$  | $k = 1.0$ |
| (4) $L_1/L = 0.75$ | $Q_2 = 10Q$ | $k = 1.0$ |

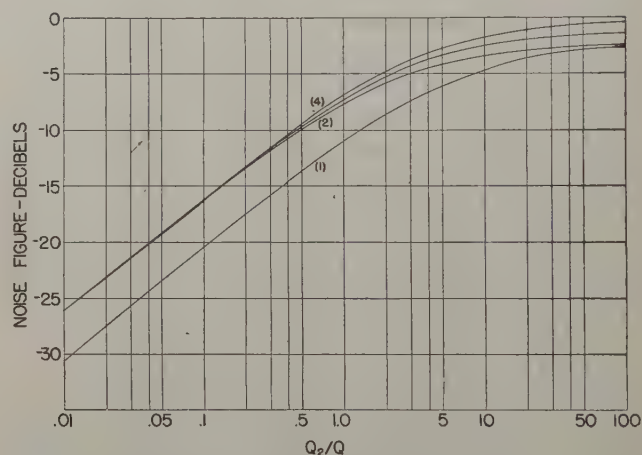


Fig. 6—Noise-figure variation with  $Q_2$ .

- |                    |             |           |
|--------------------|-------------|-----------|
| (1) $L_1/L = 0.75$ | $Q_1 = Q$   | $k = 0.6$ |
| (2) $L_1/L = 0.75$ | $Q_1 = Q$   | $k = 1.0$ |
| (3) $L_1/L = 0.75$ | $Q_1 = 2Q$  | $k = 1.0$ |
| (4) $L_1/L = 0.75$ | $Q_1 = 10Q$ | $k = 1.0$ |

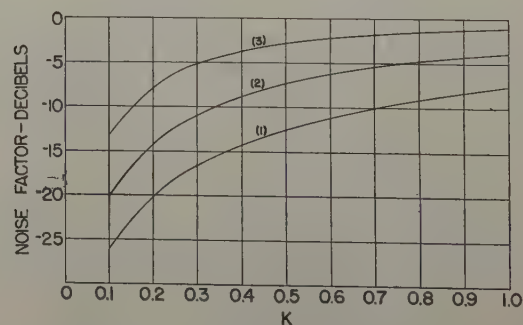


Fig. 7—Noise-figure variation with  $k$ .

- |                    |              |             |
|--------------------|--------------|-------------|
| (1) $L_1/L = 0.75$ | $Q_2 = Q_1$  | $Q_1 = Q$   |
| (2) $L_1/L = 0.75$ | $Q_2 = 2Q_1$ | $Q_1 = 2Q$  |
| (3) $L_1/L = 0.75$ | $Q_2 = 2Q_1$ | $Q_1 = 10Q$ |

this equation is plotted so as to show the variation of noise figure with  $L_1/L$  for various values of the  $Q$  ratios. Since all of these curves show optimum operation for  $L_1/L$  equal to about 0.75, this value of  $L_1/L$  is chosen, and in Figs. 5, 6, and 7 it is used in showing variation with the various  $Q$  ratios.

#### EQUIVALENT NOISE OF INPUT TUBE

In the actual loop-coupling arrangement, two elements are connected across the terminals  $AB$  of Fig. 3. One is a capacitor which is tuned to resonate the loop, and the second is the grid of the first vacuum tube. The capacitor does not affect the noise figure which has been computed, and the tube adds a noise correction which may be evaluated.

To show that the capacitor does not affect the noise figure of the loop, refer to Fig. 3(c) and imagine that a source  $e$  is connected at point  $P'$  and a resonating capacitor connected across terminals  $AB$ . The resonant current may be computed so that the drop across the reactance, which is the terminal voltage at  $AB$ , becomes

$$e_s = \frac{\omega L_2'}{R_2'} e. \quad (16)$$

To obtain the signal-to-noise ratio under this condition, this quantity must be divided by  $1.26 \times 10^{-10} \sqrt{R_{res}} \sqrt{\Delta f}$ , where  $R_{res}$  is the resistance component of the resonant impedance as measured at  $AB$ . It can easily be shown that  $R_{res} = (L_2' \omega)^2 / R_2'$ . Thus the signal-to-noise ratio can be written as

Signal-to-noise ratio

$$= 20 \log \frac{1}{\sqrt{R_2'}} \frac{e}{1.26 \times 10^{-10} \sqrt{\Delta f}} \text{ decibel.} \quad (17)$$

Likewise, with no capacitor connected at terminals  $AB$ ,

$$e_s = e$$

and dividing by the noise voltage,  $1.26 \times 10^{-10} \sqrt{R_2'} \sqrt{\Delta f}$ , gives the same value of the signal-to-noise ratio. These results also show that resonating a transformer coupling does not affect the noise figure of that coupling.

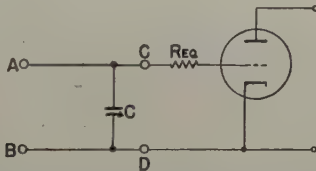


Fig. 8—Equivalent noise resistance of the input tube.

To evaluate the noise correction due to the presence of the tube, refer to Fig. 8. In this figure is shown the antenna connection to the first vacuum tube, and also the capacitor used for resonating the loop. The loop, or the loop and the coupling transformer, are considered

to be to the left of the terminals  $AB$ . The tube noise is represented by the equivalent resistant  $R_{eq}$  connected in series with grid. Aside from the noise it generates, this resistor has no circuit significance as long as the grid is assumed to draw little or no current.

The effect of  $R_{eq}$ , expressed in the same terms as were used for the input circuit proper, yields

Noise figure correction due to tube

$$= -10 \log \left( 1 + \frac{R_{eq}}{R_{res}} \right). \quad (18)$$

Values of this quantity may be obtained from Fig. 9. This measure of noise increase due to the first vacuum tube must be added to the noise figure of the input circuit to obtain the complete noise figure for the over-all performance of the system.

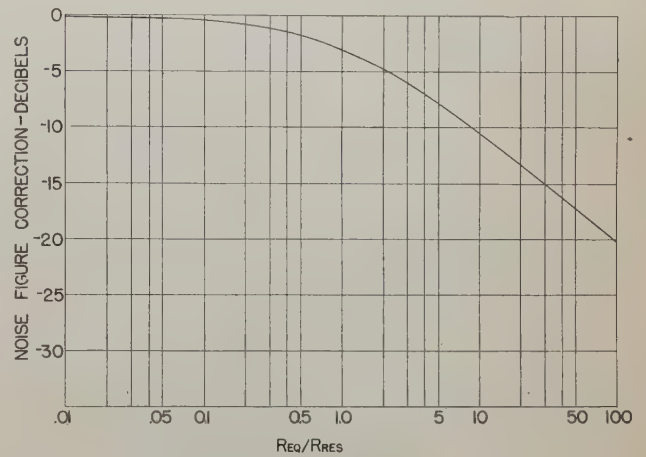


Fig. 9—Noise-figure correction variation with  $R_{eq}/R_{res}$ .

#### EFFECT OF LOOP CABLE

When the length of the cable connecting the loop to the transformer is at all significant, its effect on the performance of the system must be taken into account. If  $L'$  is defined as the inductance of the loop and connecting cable as seen at the primary terminals of the transformer, and  $R'$  is the resistance as measured at this same point, then in the formula which has been given for noise figure,  $L'$  can be substituted for  $L$ , and  $R'$  can be substituted for  $R$ .

Two cases are of interest: (1) when the connecting cable has appreciable electrical length; and (2) when the frequency and the length of the cable which is used is such that the cable length is very short in comparison to a quarter-wavelength.

For case (1), the most important parameter of the connecting cable is its characteristic impedance  $Z_0$ . In this case the cable is comparatively long and is being used as a transmission line. The values of  $L'$  and  $R'$  can be calculated from the equation

$$R' + j\omega L' = Z_0 \frac{Z_0 \sinh l + (R + j\omega L) \cosh l}{Z_0 \cosh l + (R + j\omega L) \sinh l} \quad (19)$$

where  $l$  is the electrical length of the transmission line measured as an angle.

Case (2), in which the cable is relatively short in comparison to a quarter wave, covers most of the cases generally encountered. Now the cable can be considered to represent lumped-constant elements in the circuit. Along with the capacitance of the cable, which is usually the most important factor, may well be included the distributed capacitance occurring at the loop and transformer ends of the cable.

The values of  $L'$  and  $R'$  for this lumped-constant case are obtained from the expression

$$R' + j\omega L' = \frac{(R + j\omega L) \left( -j \frac{1}{\omega C} \right)}{R + j\omega L - j \frac{1}{\omega C}} \quad (20)$$

where  $C$  is the capacitance just referred to. If  $C$  is small enough so that

$$\left( \frac{1}{\omega C} - \omega L \right)^2$$

is very large compared to  $R^2$ , this expression may be simplified to yield

$$R' = \frac{R}{(1 - \omega L \omega C)^2} \quad (21)$$

and

$$L' = \frac{\frac{L}{C} \left( \frac{1}{\omega C} - \omega L \right) \frac{R^2}{\omega C}}{\left( \frac{1}{\omega C} - \omega L \right)^2} \quad (22)$$

Frequently  $R$  and  $C$  are small enough so that

$$\frac{R^2}{\omega C \left( \frac{1}{\omega C} - \omega L \right)^2} \ll \frac{L}{C \left( \frac{1}{\omega C} - \omega L \right)},$$

and in this case the expression for  $L'$  may be further simplified to

$$L' = \frac{L}{1 - \omega L \omega C} \quad (23)$$

These formulas are of particular importance in calculating the apparent inductance as seen at the primary terminals of the coupling transformer. Graphical data concerning their solution are given in Fig. 10.

In those cases where the cable inductance is of importance in comparison to the loop inductance, it is usually adequate to add these two inductances and assume that the capacitances are in shunt with the total inductance. This is an approximation, and care must be

taken not to use this approximation where extreme accuracy is expected.

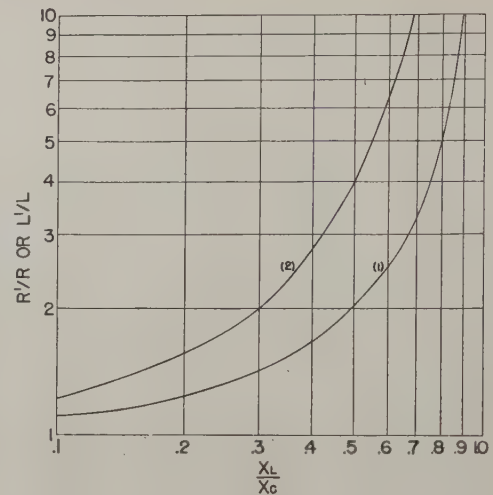


Fig. 10—Change of loop parameters with shunt capacitance.  
(1) Change of reactance with cable capacitance.  
(2) Change of resistance with cable capacitance.

#### SYSTEM PERFORMANCE CHANGE WITH LOOP $Q$

If a loop antenna is coupled to a receiver, and if the  $Q$  of the loop is varied while all other circuit parameters are held constant, two factors will change. As is shown in Fig. 2, both the merit of the loop itself and the noise figure of the coupling circuit will be changed.

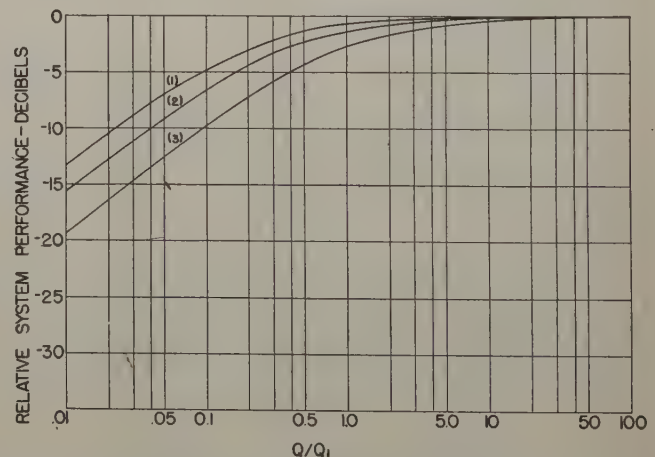


Fig. 11—Relative change of system performance with change of  $Q$ .  
(1)  $L_1/L = 0.75$   $Q_2 = Q_1$   $k = 1.0$   
(2)  $L_1/L = 0.75$   $Q_2 = 2Q_1$   $k = 1.0$   
(3)  $L_1/L = 0.75$   $Q_2 = 10Q_1$   $k = 1.0$

From the computations which have been made, these two decibel measurements of operation can be added so as to yield a measure of over-all system performance as a function of  $Q$ . This has been done, and under the restrictive conditions already mentioned, Fig. 11 shows the result in graphical form.

The curve of Fig. 11 increases steadily with increased  $Q$ . The practical limitation on  $Q$  depends upon

bandwidth. Just how this dependence occurs is discussed below.

#### APPARENT $Q$ OF THE TRANSFORMER SECONDARY

The apparent  $Q$ , the ratio of the reactance to the resistance, of the loop and coupling arrangement as shown at terminals  $AB$  of Fig. 3 is the quantity that determines the antenna bandwidth. We shall call this quantity  $Q_2'$ .  $Q_2'$  also determines the manufacturing tolerances which can be allowed for the loop inductance, transformer inductances, and tuning-capacitor capacitance.

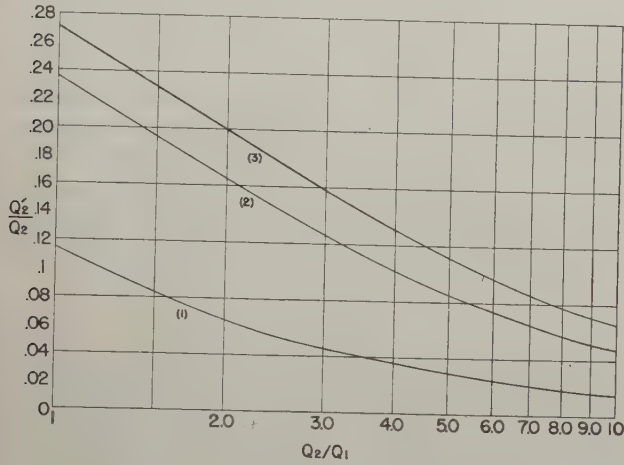


Fig. 12—Apparent  $Q$  of the secondary of the coupling transformer.

- |     |                |             |           |
|-----|----------------|-------------|-----------|
| (1) | $L_1/L = 0.75$ | $Q_1 = 10Q$ | $k = 1.0$ |
| (2) | $L_1/L = 0.75$ | $Q_1 = 2Q$  | $k = 1.0$ |
| (3) | $L_1/L = 0.75$ | $Q_1 = Q$   | $k = 1.0$ |

To calculate  $Q_2'$  we must evaluate  $\omega L_2'$  and  $R_2'$ . From Fig. 3(b),  $Z_{AB}$  has already been found, and since only inductances are involved,  $R_2'$  and  $\omega L_2'$  can be immediately taken from the real and imaginary parts of that expression. Thus

$$Q_2' = \frac{\omega \left( L_2 - \frac{M^2}{L + L_1} \right)}{R_2 + \frac{(R + R_1)M^2}{(L + L_1)^2}} \quad (24)$$

Substituting  $Q$ 's for  $R$ 's, this may be rewritten as

$$\frac{Q_2'}{Q_2} = \frac{1 - k^2 \left( \frac{L_1}{L + L_1} \right)}{1 + k^2 \frac{Q_2}{Q} \frac{L}{(L + L_1)} \frac{L_1}{(L + L_1)} + k^2 \frac{Q_2}{Q_1} \left( \frac{L_1}{L + L_1} \right)^2} \quad (25)$$

Here  $Q_2'$  is actually given as a ratio to  $Q_2$ . This has been done only for convenience in the graphical representation of Fig. 12.

#### APPARENT INDUCTANCE OF THE TRANSFORMER SECONDARY

In order to design the loop-coupling transformer so that it will track with the other coils in the receiver, it is necessary to know the inductance of the secondary with the loop connected to the primary of the transformer. An expression for  $L_2'$  was obtained as the imaginary part of (11). For convenience of graphical representation, this expression may be rewritten as

$$\frac{L_2'}{L_2} = 1 - k^2 \frac{L_1}{L + L_1} \quad (26)$$

Fig. 13 shows graphical values of this expression.

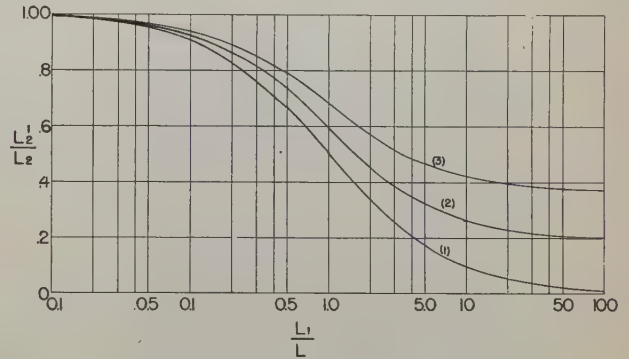


Fig. 13—Apparent inductance of the secondary of the coupling transformer.

- |     |           |
|-----|-----------|
| (1) | $k = 1$   |
| (2) | $k = 0.9$ |
| (3) | $k = 0.8$ |

#### CONCLUSION

The concept of the noise figure of an input circuit at comparatively low radio frequencies is a powerful aid in the analysis and improvement of input circuits. Formulas and charts which have been given cover the loop-coupling-transformer input circuit. Further design development of these transformers will proceed more rapidly if effort is expended on those parameters which theoretically will give the greatest benefits with the least change of the circuit elements. Unquestionably, there is much to be gained over those designs which have been so generally used in the past. The formulas and charts which have been given can be used directly for that purpose.

# Radio Control of Model Flying Boats\*

V. WELGE†, ASSOCIATE, I.R.E.

**Summary**—A new method of carrying on flying-boat hull design involves radio control of free-flight models. The scale factors of the models are such that severe weight and space limitations are imposed on the radio-control equipment, considering the complexity of control. Seven independent channels capable of simultaneous operation are needed. Five accomplish proportional positioning and two perform switching operations. A single radio-frequency carrier is used, amplitude-modulated by seven control tones.

The factors affecting the positioning accuracy are discussed. A compensating circuit is described which corrects for variations in receiver output with charges of battery voltage and distance from the control station. Over-all positioning accuracy is plus or minus 4 per cent.

Operating experience includes nearly 1000 test runs utilizing scale models of three different types of flying boats.

## I. INTRODUCTION

FLYING-BOAT hull research has followed a course roughly paralleling that of aerodynamic development. It has been expedient to employ scale models to allow research to be carried out in towing basins of convenient size and to minimize the cost of construction of the various hull configurations to be tested. Much useful information has been gathered in this manner in spite of the fact that the mounting of the model on the towing device imposes certain restrictions on the motion of the model. However, during the war emergency the existing towing basins (all at a distance from San Diego) became overloaded, and additional facilities became necessary. Radio control of free-flight models was proposed as an alternative to the construction of a towing basin at San Diego, and was selected for reasons of economy, availability of material, and freedom from restrictions on motion.

Considerable test data were available on  $\frac{1}{8}$ -scale towing-basin models, and it was desired to employ a radio-controlled model of the same scale to provide a direct comparison of results on the initial tests. Since, in a dynamically similar scale model, the weight scales down as the third power of the scale factor, this imposed severe weight restrictions on the radio equipment in the airplane. In the case of the first model, a two-engined flying boat, the weight allowance was 15 pounds. Inasmuch as no seven-channel proportional system existed in this weight bracket, it was necessary to develop one.

## II. SPECIFICATIONS

The following specifications were set up to serve as a guide in the development:

1. Seven independent controls are to be provided, capable of simultaneous operation. Five of these con-

trols are to be continuously variable throughout their range and are to accomplish a positioning which is proportional to the setting of the control levers at the ground station. These are to be used for rudder, ailerons, elevator, and two engine throttles. The other two controls switch off the ignition and lower the flaps, respectively.

2. The time lags between movement of the control wheel or rudder pedals at the ground station and movement of the control surfaces on the airplane must be held to a minimum to provide effective control of flight, particularly in landings.

3. The weight of the radio installation in the airplane, including batteries, is to be held within 15 pounds.

4. The battery capacity must be sufficient for 30 minutes of operation.

5. The radio equipment must provide for a maximum operating range of 2000 feet.

## III. METHOD USED

Preliminary studies indicated that a single radio-frequency carrier, modulated with seven amplitude-controlled tones, held the best promise from the standpoint of over-all simplification of equipment and weight reduction in the airplane installation.

The arrangement of the airplane equipment is shown in Fig. 1. Note that, in contrast to the nonproportional "bang-bang" type of control used in target airplanes,

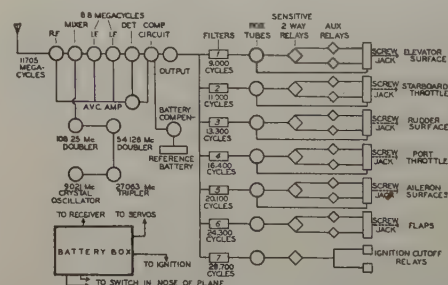


Fig. 1—Functional diagram of airplane installation.

which uses two tones for each control surface, this method uses only one. For example, bidirectional control is accomplished in the case of the elevator surface by having it position to the full-down attitude when the tone is at zero amplitude. Maximum amplitude of the tone positions the elevator to the full-up attitude. Any intermediate attitude can be achieved by the transmission of the corresponding amplitude of the elevator channel tone.

At the ground station a replica of the controls found in a cockpit are provided. Adjustment of the amplitude of a tone is accomplished very simply by mechanically coupling a control lever to a potentiometer wired across

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† Consolidated Vultee Aircraft Corp., San Diego, California.

the output of the tone generator. The arrangement of the ground-station equipment is indicated in Fig. 2.

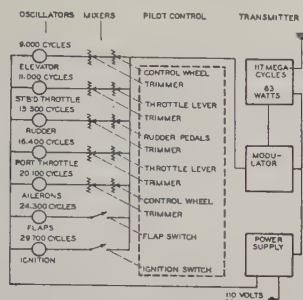


Fig. 2—Functional diagram of ground-station equipment.

#### IV. AIRPLANE EQUIPMENT

The receiver is a superheterodyne with one radio-frequency stage preceding the first detector. An oscillator unit consisting of a crystal oscillator followed by one tripler and two doubler stages keeps the receiver in tune with the transmitter. The two intermediate-frequency amplifiers are tuned to 8.8 megacycles. The normal variations in transmitter frequency and receiver tuning are accommodated by a 300-kilocycle pass band in these amplifiers.

Interstage shielding was a luxury that could not be afforded with the weight and space available. Shielding of the oscillator unit was essential, however. This unit was mounted in the light duraluminum can visible in the photograph of the receiver, Fig. 3. Plexiglas panels standing on edge form the chassis, which offers good space utilization and reasonable access to component parts.

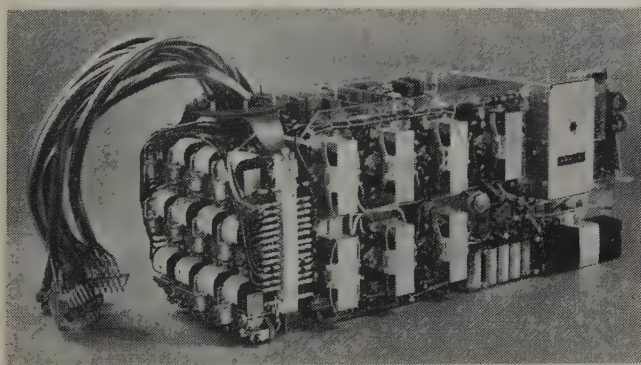


Fig. 3—Receiver removed from case.

Most of the space in the receiver is taken up by the filters and positioning circuits. Simple parallel-resonant circuits are used as filters. The filter coils are wound on powdered iron cores and are tuned to resonance by mica capacitors. Secondary windings couple the output signals to the positioning circuits. Each of these consists of a tube, a sensitive polarized relay, two auxiliary relays, a servo, and a potentiometer. The potentiometer is an integral part of the servo, which in turn is mounted in the model at some point convenient to the control it

actuates. The remaining components of the positioning circuits are mounted in the receiver. The sensitive relays are not capable of operating the servomotors directly, because of the small contact forces developed, and because of the slow make and break which occurs as the positioning circuit goes into and out of balance. Thus auxiliary relays are required to handle the motor currents.

The receiver is housed in a waterproof dural case measuring 6 by 6 by 16 inches. Sponge-rubber gaskets provide a seal which withstands submersion in three feet of water. Slide clamps on the cover provide a quick means of removing the chassis for service work. The leads out of the case are brought through a tar seal. The receiver is shock-mounted on rails in a model. This arrangement is of value for tests which involve shifting of the airplane's center of gravity. A typical installation in a model is shown in Fig. 4.

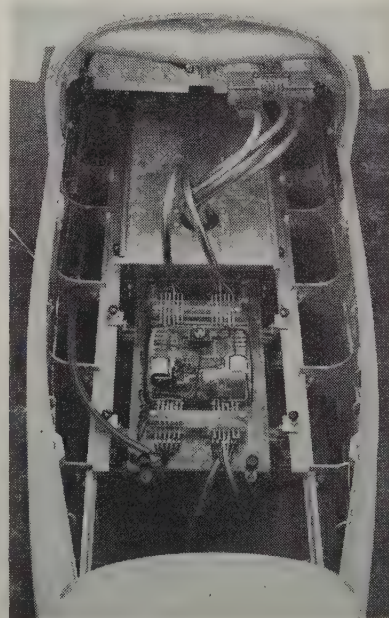


Fig. 4—Typical installation of radio equipment in a model.

The battery box, which is not waterproofed, may be seen clamped on the rails in front of the receiver. Dry batteries are used to supply B and C voltages, which are 90 and  $22\frac{1}{2}$  volts respectively. Storage batteries are used to heat the filaments, supply ignition current, and operate the servos.

The servomotors, Fig. 5, develop a working thrust of 5 pounds. With this load, the jack moves through its 2 inches of motion in 2 seconds. The efficiency is on the order of 10 per cent. In addition to the positioning potentiometer, servo accessories include limit switches and a spring-loaded brake, which is released electromagnetically when motor current flows.

The weight of the airplane installation comes to 21 pounds. Although this exceeds the specification, it falls

within the capabilities of the airplanes, which have gross weights ranging from 75 to 175 pounds.

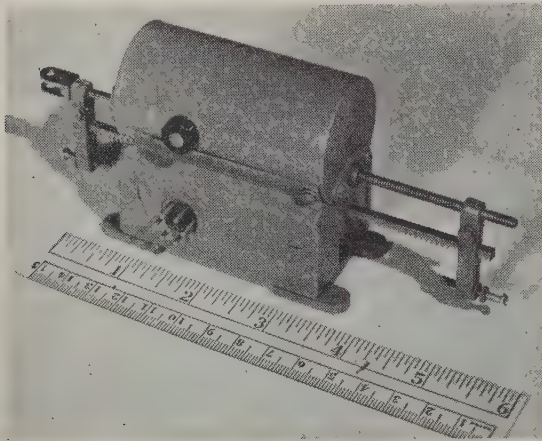


Fig. 5—Electric servomotor used to actuate control surfaces and throttles.

### V. GROUND-STATION EQUIPMENT

A cabinet, rudder pedals, and pilot's seat are mounted on a platform, as shown in Fig. 6. The control and pilot's seat are standard parts, tending to make the pilot feel as much at home as possible. An instrument panel is provided which includes an elapsed-time clock; ignition



Fig. 6—Ground station.

switch, flap switch, and trimmers. The latter permit compensation for variations in the system which cause control surfaces to be "off center" when the levers are in mid-position.

The seven-channel oscillators are of the transitron type to obtain a high degree of stability and good wave shape. Low-impedance outputs to connect across the potentiometers coupled to the control levers are obtained by winding secondaries on the coils in the tuned circuits. The signals leaving the potentiometers are

combined through resistance networks to limit interaction of one channel upon another to prescribed limits.

Two stages of amplification are needed to raise the combined signal voltages to the level necessary to drive the modulator tubes, which are push-pull 845 tubes in class AB<sub>1</sub>. They are capable of 50 watts output at the plate voltage used.

The modulation transformer is an air-core type with an interlaced pie-wound construction. Its transmission varies less than plus or minus 0.4 decibels over the frequency range of 6000 to 30,000 cycles per second.

Seven frequencies between 116.0 and 118.5 megacycles were assigned to this project. It was considered necessary to be able to change from one to another in the event interference developed. Because of the crystal procurement problem, a resonant concentric transmission line was used. A vernier adjustment on the line allows selection of any of the operating frequencies. The stability is satisfactory for this class of service. Drift during a warm-up cycle approximates 0.02 per cent. Changes in ambient temperature at San Diego are not great, and line adjustments are seldom required to maintain the required plus or minus 0.05 per cent frequency tolerance.

Two buffer stages provide isolation for the oscillator and sufficient power amplification to excite properly the final amplifier. Resistance-loaded band-pass circuits are employed in these buffer amplifiers to reduce the number of tuning controls. The circuits pass the frequencies from 116.0 to 118.5 megacycles without adjustment.

The final amplifier is an 829 tube rated at 63 watts output. This is coupled into a vertical half-wave doublet antenna fed and supported by a concentric line.

### VI. FACTORS AFFECTING ACCURACY OF SYSTEM

The over-all error of plus or minus 4 per cent is a function of the sensitivity of the positioning circuits and

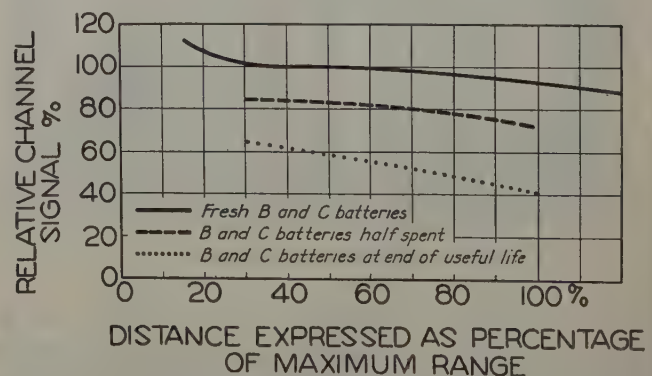


Fig. 7—Effect of distance from ground station upon receiver output for various battery conditions.

the ability of the radio link to transmit correctly the control signals. The major problem in the latter case is illustrated in Fig. 7. The output variation tolerated by the automatic volume control is on the order of 6 decibels. While this is perfectly satisfactory in the case of a

home radio, in this application it would result in a rudder shift from full left to trim position. To reduce this undesired variation, a compensating circuit, shown in Fig. 8, is used. A variable-mu tube is used in the first "audio" stage. A portion of the automatic-volume-control voltage furnishes the bias for this tube and compensates for the normal decrease in receiver output as the signal strength falls off. A comparator tube compares the B voltage to the fixed voltage across an un-

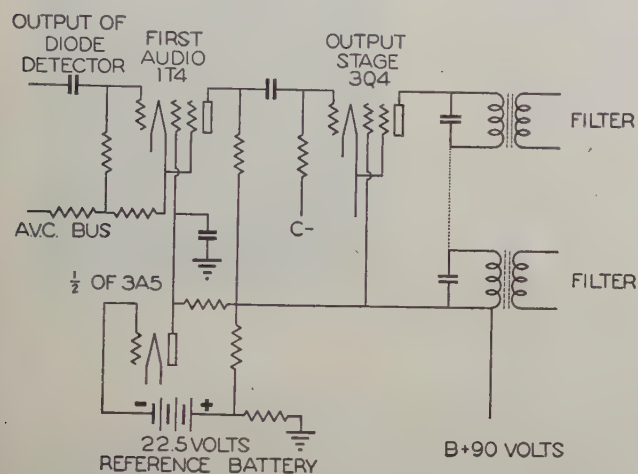


Fig. 8—Schematic diagram of compensating circuit.

loaded battery. This tube regulates the screen voltage on the first "audio" stage and compensates for the normal decrease in receiver output as the battery voltage falls off. The receiver performance is considerably improved, as may be seen by comparing Fig. 9 to Fig. 7.

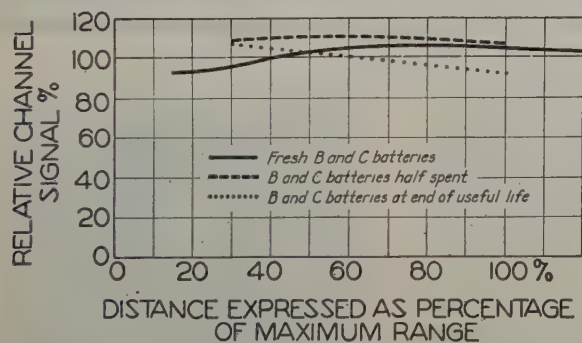


Fig. 9—Variations in receiver output remaining after installation of compensating circuit.

By using batteries which have a useful life that greatly exceeds the duration of a test run, and by arranging the geometry so that the distance from the ground station to the airplane remains essentially constant, the remaining variations do not exceed plus or minus 2.0 per cent.

Minor errors result from insufficient channel isolation in the receiver filter system and nonlinearity. The latter appears in the form of sum and difference beat notes, in the presence of harmonics, and in a change in channel amplitude when the other channels are varied. By conservative design and use of a moderate percentage of modulation, these errors are kept small as compared to the other errors in the system.

The filter selectivity is such that 6 per cent of a signal appears in the adjacent channel as interference. Inasmuch as this interfering voltage adds to the desired signal to give a resultant which is equal to the square root of the sum of the squares, the adjacent channel is vulnerable only when its signal is at a low amplitude. To limit this error, the positioning systems are arranged to be at the limits of their travels when the channel amplitudes are reduced to 10 per cent of full amplitude.

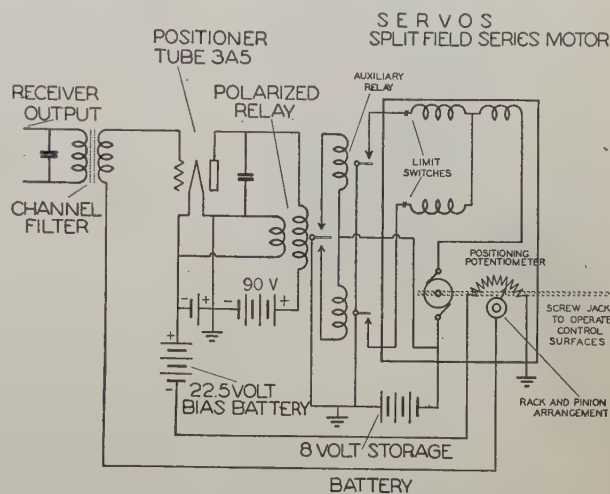


Fig. 10—Schematic diagram of positioning system.

The positioning-circuit performance is largely a function of the sensitivity of the polarized relay shown in Fig. 10. In addition to other modifications to the stock item, an auxiliary winding installed on the relay bucks

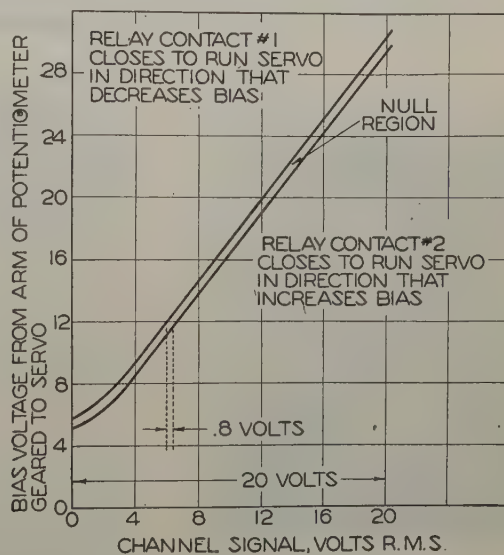


Fig. 11—Performance of positioning system.

the null current of the tube. This positioning tube operates on the lower bend of its plate-current/grid-voltage curve, and responds to a change in signal by a change in plate current. The relay closes, causing the servo to run in the appropriate direction until the bias matches the signal. As can be seen in Fig. 11, the sensitivity is one part in 25, or plus or minus 2 per cent.

## VII. OPERATION OF RADIO-CONTROL SYSTEM

The radio-control system has been in use for nineteen months at the time of writing. During this time nearly 1000 test runs of flying boats have been made, involving three different models fitted with one engine, two engines, and four engines, respectively. Some flights have been made, although emphasis in this particular program is placed upon performance while the flying boat is waterborne. Some crashes have occurred. These were due largely to operating a flying boat under conditions known to be too unstable to accumulate complete data. Herein lies one advantage of this method of testing. Operating a flying boat manned by personnel in such a manner would be dangerous. Fortunately, the construction of the model is such that major repairs can usually be made in a few days. With the exception of the first experimental receiver, which was not waterproof, the receivers have weathered the crashes with virtually no damage. The battery boxes are usually ruined, but these are cheap and easy to replace.

In such extensive operations reliability of the equipment is at a premium. In order to keep lost time down to a minimum, a comprehensive maintenance schedule is adhered to. Fifty trips constitute a normal service period for a receiver. It is of interest to note the number of operations performed during this period. Observation

of the pilot indicates that, on the average, each control lever is reset ten times a minute. During fifty trips of six minutes' average duration this represents 3000 adjustments per channel, or 15,000 operations in the five proportional channels.

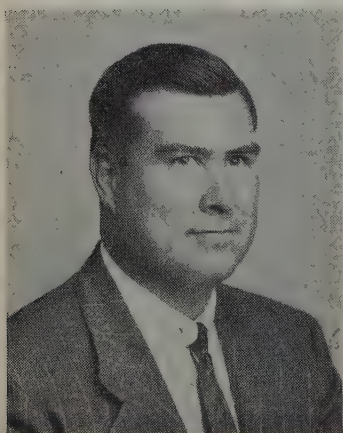
## VIII. CONCLUSION

The equipment described above was constructed under the pressure of wartime urgency and was in satisfactory operation in August, 1944, thirteen months after the start of the project. Since that time some improvements have been made in the system; for example, waterproofing the receiver and enclosing the electric servomotors. Some work has been done on a simple receiver of an expendable nature for certain prospective tests. Recently, however, the main effort on this radio-control project has been on the development of an autopilot. This development is nearly complete, and it is expected that precision control of the airplane will be enhanced by its adoption.

## IX. ACKNOWLEDGMENT

The help and co-operation of the personnel, and of C. J. Breitwieser, chief of the Consolidated Vultee Radio and Electrical Laboratories, is gratefully acknowledged.

# Contributors to Waves and Electrons Section



JAY E. BROWDER

Jay E. Browder (A'38-M'44) was born at Wichita, Kansas, on November 24, 1911. He attended Kansas City, Kansas, Junior College from 1930 to 1932, and the University of Kansas from 1933 to 1934.

Mr. Browder was employed by Transcontinental and Western Airlines, Inc. from 1934 to 1939, stationed first at Camden, New Jersey, and later at Kansas City, Missouri. During this later period he was associated with the communication laboratory of T.W.A., working on airborne communication and navigation equipment.

In 1940 he joined the Sperry Gyroscope Company and has been associated with that company since that time working on airborne navigation, communication, and radar equipments.



D. D. King (M'46) was born on August 7, 1919, at Rochester, New York. He received the A.B. degree in engineering sciences from Harvard College in 1942, and the Ph.D. degree in physics from Harvard University in 1946. He was a teaching fellow in physics and communication engineering in 1943, serving as a staff member of the pre-radar Officer's Training School at Cruft Laboratory, Harvard University. During 1945 he was a research associate at Cruft

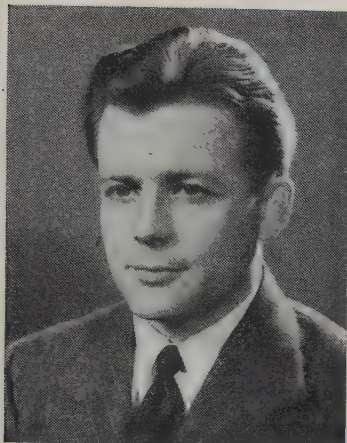


D. D. KING

Laboratory. At present he is engaged in work on antennas and ultra-high-frequency circuits as a research fellow at Harvard University.

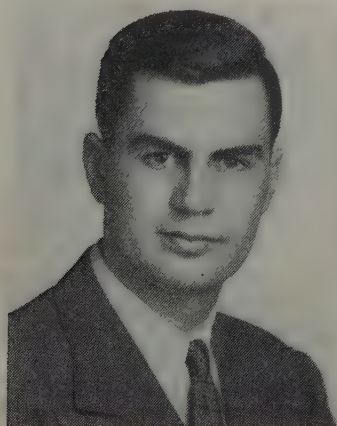
Dr. King is a member of Sigma Xi and the American Physical Society.

# Contributors to Waves and Electrons Section



OSCAR C. LUNDSTROM

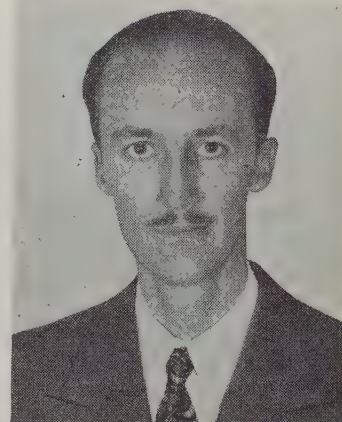
Oscar C. Lundstrom (S'42) was born at Morristown, South Dakota, on September 12, 1917. He was graduated from the University of California in 1941 with the B.S. degree in electrical engineering. From 1942 to 1944 he was a graduate student and acting instructor at Stanford University, from which he received the A.M. and E. E. degrees in electrical engineering. Since 1944 Mr. Lundstrom has been employed by the Sperry Gyroscope Company as an assistant project engineer. He is a member of Tau Beta Pi, Eta Kappa Nu, and Sigma Xi.



THEODORE MORENO

Theodore Moreno (S'41-A'44) was born at Palo Alto, California, on September 2, 1920. He received the B.A. degree from Stanford University in 1941, and the M.A. degree in 1942. From 1942 to 1946 he was employed in the research laboratories of the Sperry Gyroscope Company, first as assistant project engineer, and later as project engineer. Since 1946, he has been a research associate at the Massachusetts Institute of Technology.

Mr. Moreno is a member of Phi Beta Kappa, Tau Beta Pi, and Sigma Xi.



V. WELGE

V. Welge (S'33-A'43) was born in Johannesburg, South Africa, in 1912. He received the B.S. degree in electrical engineering from the University of California in 1933. This was supplemented in 1941 by graduate work in electromagnetic waves at the University of California. From 1933 to 1935 he was engaged in designing public-address equipment and broadcast-studio equipment for Rember Radio Co., San Francisco, California. In 1935 he joined the Associated Broadcasters Incorporated, in San Francisco, as transmitter engineer at KSFO. Mr. Welge's duties included the development of radio-relay equipment, and assistance in the design and installation of the directional antennas for KWID, a 100,000-watt short-wave broadcast station installed in 1942.

In 1943 Mr. Welge joined the Radio and Electric Laboratory of Consolidated Vultee Aircraft Corp., San Diego, California, where he has been engaged as a research engineer on the development of radio control and autopilot systems. He holds a pilot's license and he is a member of Phi Beta Kappa, Tau Beta Pi and Sigma Xi.



Victor J. Young was born at Albion, Michigan, on April 11, 1913. He received the A.B. degree from Albion College in 1935, and the Ph.D. in physics from the State University of Iowa in 1940.

Dr. Young was an instructor at New York University from 1940 to 1942, and has

been employed as an engineer by the Sperry Gyroscope Company since 1942. At the State University of Iowa and at New York University, he did research in nuclear physics and worked with the electronic tools necessary in that field. At the Sperry Gyroscope Company Laboratory, he has taken an active part in the development of radar and microwave communication systems.

He is a member of Sigma Xi and of the American Physical Society. His book "Understanding Microwaves," was published in May, 1946.



VICTOR J. YOUNG

# Abstracts and References

Prepared by the National Physical Laboratory, Teddington, England, Published by Arrangement with the Department of Scientific and Industrial Research, England, and *Wireless Engineer*, London, England

NOTE: The Institute of Radio Engineers does not have available copies of the publications mentioned in these pages, nor does it have reprints of the articles abstracted. Correspondence regarding these articles and requests for their procurement should be addressed to the individual publications and not to the I.R.E.

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Correction: Abstract 38 of February, 1947, should read *Jour. British I.R.E.* instead of *Jour. I.E.E.*

## ACOUSTICS AND AUDIO FREQUENCIES

534+621.395.625.3 985

Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "A Method for Changing the Frequency of a Complex Wave," by E. L. Kent; "The Reduction of Background Noise in the Reproduction of Music from Records," by H. H. Scott; and "Recent Developments in Magnetic Recording," by R. B. Vaile, Jr. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.

534.321.9:620.179 986

Supersonic Flaw Detector—J.H.J. (*Electronics*, vol. 19, pp. 198, 202; December, 1946.) 1-megacycle quartz-crystal generator to detect flaws in sheet aluminum alloy.

534.321.9.001.8 987

Supersonic Vibrations and Their Applications—E. G. Richardson. (*Jour. Roy. Soc. Arts*, vol. 95, pp. 90-105; January 3, 1947.)

534.62 988

Realization of Dead Rooms [Chambres Soudes] for Acoustic Tests—A. Moles. (*Radio en France*, no. 4, pp. 14-20; 1946.)

534.756+621.39 989

Theory of Communication—Gabor. (See 1057.) Part 1, analysis of information; Part 2, analysis of hearing; Part 3, frequency compression and expansion.

534.851:621.395.813 990

Periodic Variations of Pitch in Sound Reproduction by Phonographs—U. R. Furst. (PROC. I.R.E. AND WAVES AND ELECTRONS,

The Annual Index to these Abstracts and References, covering those published from January, 1946, through December, 1946, may be obtained for 2s. 8d., postage included, from the *Wireless Engineer*, Dorset House, Stamford St., London S. E., England.

vol. 34, pp. 887-895; November, 1946.) The causes of "wow" are discussed and an account given of the design and construction of an instrument for its measurement.

534.851:621.395.813 991

Dynamic Suppression of Phonograph Record Noise—H. H. Scott. (*Electronics*, vol. 19, pp. 92-95; December, 1946.) A method of suppression of high- and low-frequency noise by automatic control of the bandwidth of an audio amplifier. Details of performance are given; a description of circuits and equipment will be published later.

534.862.4 992

Perfect v. Pleasing Reproduction—Moir. (See 1185.)

621.395.623.73 993

Wide-Range Loudspeaker Developments—H. F. Olson and J. Preston. (*Jour. Soc. Mot. Pic. Eng.*, vol. 47, pp. 327-352; October, 1946.) The duo-cone loudspeaker, consisting of two coaxial, congruent, separately driven cones appears to possess many advantages over other types. A detailed investigation was carried out to determine the best values of the constants of such a loudspeaker. Methods and results are described.

621.395.623.8 994

A Problem in Outdoor Sound—A. B. Ellis and J. P. Gilmore. (*Electronics*, vol. 19, pp. 126-129; December, 1946.) High-quality amplifying equipment to cover a large open-air stage and audience.

621.395.625.6 995

Factors Governing the Frequency Response of a Variable-Area Film Recording Channel—M. Rettinger and K. Singer. (*Jour. Soc. Mot. Pic. Eng.*, vol. 47, pp. 299-326; October, 1946.)

621.396.615.1 996

RC Low-Frequency Generators (See 1038.)

621.396.615.11 997

Low-Frequency Generators—Aschen. (See 1039.)

621.396.645.029.3 998

Amplifier with Very-High Musical Fidelity: Part 9—Chrétien. (See 1047.)

621.396.645.029.4 999

Resistance Amplifiers: Part 1—Low Frequencies—L. Chrétien. (*Toute la Radio*, vol. 13, pp. 4-7; December, 1946.) A general treatment, with special reference to the amplification of rectangular signals and the correction of phase distortion.

534.861+621.396.712.3 1000

Radio Sound Effects [Book Review]—J. Creamer and W. B. Hoffman. Ziff-Davis Publishing Co., New York, N. Y., 1945, 61

pp., \$1.50. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, pp. 865-866; November, 1946.) Gives a general background for broadcasting studio work. For another review see 1444 of 1946.

## AERIALS AND TRANSMISSION LINES

621.315.2:621.365.5 1001

Low Reactance Flexible Cable for Induction Heating—M. Zucker. (*Trans. A.I.E.E. (Elec. Eng.)*, December, 1946; vol. 65, pp. 848-852; December, 1946.)

621.315.2.029.5:621.317.333.4 1002

New Methods for Locating Cable Faults, Particularly on High-Frequency Cables—F. F. Roberts. (*Jour. I.E.E. (London)*, part III, vol. 93, pp. 385-395; November, 1946.) Discussion, pp. 395-404.) A theoretical survey is given of the possibilities of frequency modulation and pulse methods of locating cable faults. It is concluded that the pulse method is preferable, and in particular the direct-current pulse rather than the carrier pulse system. An instrument of the direct-current-pulse type is described which gives a location accuracy of faults on coaxial cables within 1 per cent at ranges up to 10 miles.

621.315.2.029.5:621.395 1003

High-Frequency Telephone Cables—A. C. Holmes. (*Jour. I.E.E. (London)*, part I, vol. 93, p. 568; December, 1946.) Construction, attenuation, and current transmission are discussed and the advantages of coaxial cables over wave guides indicated. At the higher frequencies coaxial cables may be limited by the problems of equalization and temperature control, with an upper limit of about 10 megacycles and a maximum of 1000 to 1500 channels. Abstract of Chairman's address to the North Midlands Students' Section of the Institution of Electrical Engineers.

621.392+621.396.67 1004

Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "The Theory and Design of Several Types of Wave Selectors," by N. I. Korman; "Aircraft Antenna Pattern Measuring System," by O. Schmitt; "Problems in Wide-Band Antenna Design," by A. G. Kandoian; and "Slot Radiators," by A. Alford. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.)

621.392 1005

Wave Propagation in Curved Guides—M. Jouguet. (*Compt. Rend. Acad. Sci. (Paris)*, vol. 222, pp. 537-538; March 4, 1946.) Expressions are derived for the field components on the assumption that the curvature does not exceed a certain value. Phase velocity and at-

tenuation are shown to be independent of the curvature to a first approximation.

- 621.392 1006  
**Reflection of an Electromagnetic Wave by a Disk Located in a Waveguide**—T. Kahan. (*Compt. Rend. Acad. Sci. (Paris)*, vol. 222, pp. 998–1000; April 24, 1946.) It is shown that (1) if a wave guide is terminated by an infinite section of the same cross section, no reflection occurs and therefore no standing waves; (2) if a guide is terminated by a semi-transparent disk, whose impedance is equal to the characteristic impedance of the guide, a system of stationary waves is produced; (3) if the guide is prolonged beyond the disk by an infinite similar guide, stationary waves appear; and (4) if the guide is terminated by a semi-transparent disk with a reflection coefficient of  $1/3$  and whose impedance is equal to the characteristic impedance of the guide, followed by a cavity whose length is an odd number of quarter waves, no reflection occurs. See also 1795 of 1946.

- 621.392:621.396.677 1007  
**A Wide-Band Directional Coupler for Wave Guide**—H. C. Early. (*Proc. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 883–886; November, 1946.) This uses a small loop responding to both electric and magnetic fields. With a special section of ridge wave guide, a two-to-one frequency range can be covered with good directional characteristics.

- 621.392.22 1008  
**Propagation of an Electromagnetic Signal along a Heterogeneous Line**—F. Raymond. (*Compt. Rend. Acad. Sci. (Paris)*, vol. 222, pp. 1000–1002; April 24, 1946.) A method has previously been given (2317 of 1945) for solving the equations of propagation by developments in series, introducing the idea of two waves propagated in opposite directions along the bifilar line considered. Application to the case of a discontinuity in the line results in formulas for the reflection and transmission coefficients. Equations are derived, for signals which are any functions whatever of time  $t$  and distance  $x$  expressing the law of signal variation as a function of  $t$  and  $x$ . When the line has no loss, simple results are obtained which exhibit the reflection coefficient at the discontinuity.

- 621.396.67 1009  
**Antennas for Circularly Polarized Waves**—(*Electronics*, vol. 19, p. 214; December, 1946.) Construction and polar diagrams for a transmitting aerial are given; the receiving aerial is briefly described. Orientation of the receiving aerial is less critical for circular than for horizontal or vertical polarization.

- 621.396.67 1010  
**The Wide-Band Dipole**—F. Duerden. (*Elec. Eng.*, vol. 18, pp. 382–384; December, 1946.) The approximate bandwidth of a dipole is derived from a knowledge of the radiation resistance and the characteristic impedance, the latter being calculated from the dimensions. This gives a considerably greater value than can normally be expected since the impedance of the dipole changes with frequency; this causes serious mismatches in the feeding transmission line. Curves are therefore plotted of bandwidth against dipole dimensions for various values of permissible standing-wave ratios.

- 621.396.67.011.2 1011  
**Simplifications in the Consideration of Mutual Effects between Half-Wave Dipoles in Collinear and Parallel Orientations**—K. J. Affanasiev. (*Proc. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, p. 863; November, 1946.) Fig. 7 of this paper, omitted from 23 of February.

- 621.396.674 1012  
**Radiation from Large Circular Loops**—E. B. Moullin. (*Jour. I.E.E. (London)*, part I, vol. 93, p. 609; December 1946.) Summary of 24 of February.

- 621.396.677 1013  
**Metal-Lens Antennas**—W. E. Kock. (*Proc. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 828–836; November, 1946.) A discussion of their properties, methods of construction, and applications. The fundamental principles are given, and the equations of the plate profiles are derived for both plain and stepped lenses. Index of refraction, bandwidth, tolerances, and matching to the feed line must be considered in the design. Methods of construction for centimeter or meter wavelengths and of feeding by directive dipole or wave guide and horn are described. Parabolic reflectors and metal lenses are compared and possible applications of the latter are discussed. Field patterns are given for an experimental 40-wavelength-aperture lens, used with a conical feed horn having an aperture 2 wavelengths in diameter, and also illustrations of several types of lens.

- 621.396.677 1014  
**A Practical Calculator for Directional Antenna Systems**—H. A. Ray, Jr. (*Proc. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 898–902; November, 1946.) This easily-constructed device can be used to calculate the ground and sky-wave patterns of two- and three-element arrays. The gain can be found by graphical integration.

- 621.396.677 1015  
**A Generalised Radiation Formula for Horizontal Rhombic Aerials: Part 3**—H. Caferata. (*Marconi Rev.*, vol. 9, pp. 102–108; July–September, 1946.) A formula is developed which is believed to be more inclusive than those previously published and expresses the radiation in any given direction in a spherical co-ordinate system at large distances from the source. The source considered is that of a multiple array of horizontal rhombic elements arranged  $n$  in cascade, with  $m$  cascades in parallel, and all contained in the same horizontal plane. The formula takes into account arbitrary phase relations between elements and between cascades and also includes the effect of attenuation along the conductors comprising the radiating system. Parts 1 and 2 of this article (1456 and 3188 of 1946) dealt with the development of the general formula for imperfectly conducting, and perfectly conducting earth. The list of symbols used in this development was presented in part 2 immediately adjacent to the general formulas in order to make reference as easy as possible. Part 3, now presented, deals with the derivation of formulas for particular cases representing the application of the general formula to radiation in the principal vertical and horizontal planes through the origin and principal axis of the system.

## CIRCUITS AND CIRCUIT ELEMENTS

- 621.526 1016  
**Application of Circuit Theory to the Design of Servomechanisms**—A. C. Hall. (*Jour. Frank. Inst.*, vol. 242, pp. 279–307; October, 1946.) Laplace transform theory is applied to determine the response characteristics of servomechanisms, and a method is given for the determination of the sensitivity. Comparison is made with the feedback amplifier. Steady-state performance and dynamical characteristics of servomechanisms are discussed, and design methods given for lag-compensated networks and for circuits giving minimum steady-state errors.

- 621.315.2.011.3 1017  
**The Inductance of Wires and Tubes**—A. H. M. Arnold. (*Jour. I.E.E. (London)*,

part II, vol. 93, pp. 532–540; December, 1946.) Maxwell's equations are used to develop formulas for the inductance of a single-phase system of wires or tubes arranged side by side or concentrically. Various functions required for numerical calculation are tabulated.

- 621.316.86:546.281.26 1018  
**Silicon Carbide Non-Ohmic Resistors**—Ashworth, Needham, and Sillars. (See 1115.)

- 621.316.86.023 1019  
**High Value Deposited Resistances at High Frequency**—A. Klemt. (*Arch. Tech. Messen.*, no. 112, pp. T117–118; October, 1940.) Cylindrical or helical deposits of carbon on a ceramic base provide resistances of low temperature coefficient and small phase angle. The skin effect is negligible since the thickness of the layer is usually less than 0.1 millimeters which is the penetration depth in carbon at 1000 megacycles. The influence of internal capacitances and their loss on the apparent high-frequency resistance is analyzed and the possibility of compensation is discussed.

- 621.319.4:533.5 1020  
**Vacuum Condensers**—H. A. H. Griffiths. (*Wireless World*, vol. 53, pp. 23–24; January, 1947.) Capacitors with large diameter glass to metal seals and of rigid cylindrical construction are described. They are much smaller in size than any other type of capacitor of equivalent rating and have low power factor and temperature coefficient of capacitance.

- 621.392.4 1021  
**Wideband Phase Shift Networks**—R. B. Dome. (*Electronics*, vol. 19, pp. 112–115; December, 1946.) Wide-band characteristics are achieved by the use of two networks, each variable with frequency but having a constant phase difference. Design calculations are indicated for a pair of resistance-capacitance networks which are preferable to inductance capacitance. Such networks are used for single side-band telephony, for increasing transmitter efficiency, for carrier-frequency adjustment, and for frequency shift keying.

- 621.392.5:621.396.96 1022  
**Precision Resistance Networks for [Fire-Control] Computer Circuits**—E. C. Hagemann. (*Bell Lab. Rec.*, vol. 24, pp. 445–449; December, 1946.) Design and development are considered in terms of the accuracy required for use under widely varying wartime conditions.

- 621.392.52 1023  
**Rational Calculation of Ladder-Type Filters**—P. Coulombe. (*Bull. Soc. Franç. Elec.*, vol. 6, pp. 103–110; March, 1946.) The problem of the attenuation due to any cell in a filter can be standardized by writing it in the form  $\Sigma \log [(\mu+x)/(\mu-x)]$ , where  $x$  is a function of the frequency and  $\mu$  a parameter characterizing the filter. Methods are developed, based on Tchebycheff's theory, for determining the values of  $\mu$  giving the most economical filter for assigned attenuation in one or two bands.

- 621.392.52 1024  
**Filters and Filter Problems**—F. Locher. (*Tech. Mitt. Schweiz. Telegr.-Teleph. Verw.*, vol. 24, pp. 194–203; October 1, 1946.) A review of present day practice in the audio-frequency and high-frequency ranges (to about 300 kilocycles). Damping and loss effects are discussed and examples given of crystal filters, some using quartz and others ammonium phosphate, and of electromagnetic narrow-band audio-frequency filters.

- 621.394/.397].645 1025  
**The Cathode Follower Driven by a Rectangular Voltage Wave**—M. S. McIlroy. (*Proc. I.R.E. AND WAVES AND ELECTRONS*, vol. 34,

pp. 848–851; November, 1946.) When a voltage wave is transmitted from a high-impedance source to a low-impedance load through resistance-capacitance coupling and a cathode follower, the position of the grid-return tap on the cathode resistor determines whether the output voltage is linearly related to the input or is affected by cut-off or over-driving. Four operating conditions are discussed, determined by the presence or absence of grid current and of plate current.

**621.396 1026**  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.*, vol. 65, pp. 569–574; December, 1946.) Abstracts are given of the following papers read at the conference: "The Mechanical Transients Analyzer," by G. D. McCann; "An Oscillographic Method of Presenting Impedances on the Reflection Coefficient Plane," by A. L. Samuel; "A Permeability-Tuned 100-Mc Amplifier of Specialized Coil Design," by Z. Benin; "Very-High-Frequency Tuner Design," by G. Wallin and C. W. Dymond; "Frequency Modulation of High-Frequency Power Oscillators," by W. R. Rambo; "Design of Wide-Range Coaxial Cavity Oscillators using Reflex Klystron Tubes," by J. W. Kearney; and "Reflex Oscillators for Radar Systems," by J. O. McNalley and W. G. Shepherd. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46–47, 111; December, 1946.

**621.396.6.029.6 1027**  
**The Interstage Auto-Transformer at Television Frequencies**—P. Feldmann. (*Electronic Eng.*, vol. 19, pp. 21–22; January, 1947.) Formulas are derived for the evaluation of the stage gain of an auto-transformer coupled system and of the bandwidth of the response curve.

**621.396.611:518.61 1028**  
**Calculation of the Electromagnetic Field, Frequency and Circuit Parameters of High-Frequency Resonator Cavities**—H. Motz. (*Jour. I.E.E.* (London), part I, vol. 93, pp. 610–611; December, 1946.) Summary of 52 of February.

**621.396.611.015.33 1029**  
**The Transient Response of a Tuned Circuit**—D. G. Tucker. (*Electronic Eng.*, vol. 18, pp. 379–381; December, 1946.) The circuit considered consists of a combination of inductance, capacitance, and resistance in parallel to which a constant current source is applied. The general differential equation 
$$e/R + (1/L) \int e \, dt + C \, de/dt + B = i$$
 is solved for (a) a direct-current applied signal, (b) an alternating-current applied signal. Oscillograms are given to illustrate the waveform of the solutions.

**621.396.611.1 1030**  
**The Relay Oscillator and Related Devices**—R. L. Ives. (*Jour. Frank. Inst.*, vol. 242, pp. 243–277; October, 1946.) Relay oscillators can be made to cover the frequency range 100 cycles per second to 10 cycles per hour. A simple theoretical discussion is given of such oscillators and various related circuits. Many practical examples of their application to switching, sequence of operation devices, etc., are outlined.

**621.396.611.1:621.385.38 1031**  
**Multiple Thyatron Circuits**—I. Sager. (*Electronics*, vol. 19, pp. 158, 178; December, 1946.) It is shown that the power capabilities of thyatrons can be improved by series, series-shunt, and shunt operation. A combination circuit is described which can produce powers up to 100 megawatts at 30 kilovolts.

**621.396.611.1.015.33 1032**  
**Effect of a Differentiating Circuit on a**

**Sloping-Wave Front**—L. S. Schwartz. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, p. 862; November, 1946.) Mathematical analysis showing the importance of specifying as steep a wave front as possible for a trigger pulse used to actuate a timing circuit after passing through differentiators.

**621.396.611.4 1033**  
**Methods of Driving a High Q Cavity with Many Self-Excited Oscillators**—J. R. Woodyard, E. A. Martinelli, W. Toulis, and W. K. H. Panofsky. (*Phys. Rev.*, vol. 70, p. 447; September 1–15, 1946.) Discussion of the problem of driving a long cavity, to be used as a linear accelerator, in the lowest longitudinal mode by a large number of oscillators. Summary of American Physical Society paper.

**621.396.611.4 1034**  
**Apertures in Cavities**—J. H. O. Harries. (*Electronics*, vol. 19, pp. 132–135; December, 1946.) The loading, internal-field distortion, energy transfer, and leakage loss vary with the size and position of slots or apertures in resonators. The variation of these factors with slot dimensions was measured for cylindrical and rectangular resonators at wavelength of 80 centimeters.

**621.396.611.4:621.317.32 1035**  
**Measurement of Electric Field Strength in a Cavity Resonant at 200 Mc/s**—Panofsky. (See 1131.)

**621.396.611.4.012.8 1036**  
**Application of the Dynamical Theory of Currents to Cavity Resonators**—A. Banos, Jr. (*Phys. Rev.*, vol. 70, p. 448, September 1–15, 1946.) The equivalent circuit of a cavity is derived from the Lagrangian equations with perturbations to include the effects of wall losses and of the external circuit. Summary of American Physical Society paper.

**621.396.615 1037**  
**Locking Phenomena in Oscillators**—Z. Jelonek. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, 863; November, 1946.) Letter describing results given in a Polish paper published in 1938. The method of analysis is compared with that of Adler (2522 of 1946).

**621.396.615.1 1038**  
**RC Low-Frequency Generators**—(*Toute la Radio*, vol. 13, pp. 8–12; December, 1946.) Gives two circuits, one with continuously variable frequency and the other with a certain number or fixed frequencies.

**621.396.615.11 1039**  
**Low-Frequency Generators**—R. Aschen. (*Radio en France*, no. 2, pp. 3–12; 1946.) Describes, with full circuit details, the Ferisat beat-frequency oscillator and discusses, with numerous diagrams, modern resistance-capacitance oscillators.

**621.396.615.17:621.317.755 1040**  
**Linear Sweep Circuits**—R. P. Owen. (*Electronics*, vol. 19, pp. 136, 138; December, 1946.) Eight methods for the improvement of the linearity of saw-tooth generators are analyzed, with typical basic circuits for a 60-cycle sweep as examples.

**621.396.621 1041**  
**Selectivity by Counter-Retroaction**—X. de Maistre. (*Radio en France*, no. 3, pp. 29–32; 1946.) A variable-selectivity circuit, controlled manually or responding automatically to signal intensity variation.

**621.396.64:621.318 1042**  
**Magnetic Amplifiers**—L. Schoerer. (*Radio en France*, no. 4, pp. 24–30; 1946.) A static alternating-current system, based on magnetic saturation effects.

**621.396.645 1043**  
**Direct-Coupled R.F. Amplifier**—E. Travis. (*Electronics*, vol. 19, pp. 154, 158; December, 1946.) For amplification of signal-generator output in bridge methods of aerial resistance measurement. A circuit diagram is given and advantages are indicated.

**621.396.645 1044**  
**Load Conditions in Class-A Triode Amplifiers**—H. G. Foster. (*Electronic Eng.*, vol. 19, pp. 11–16; January, 1947.) Analysis of the ideal triode shows that there is no optimum value of the load resistance, but that the power increases with increase of load resistance and in the limit is equal to half the maximum safe anode dissipation of the tube, the anode efficiency approaching 50 per cent under Class-A conditions. Methods are given for selecting the load resistance and supply voltage for particular tubes. With equal supply voltages, the effective anode-to-anode load for two triodes in push-pull is about 50 per cent greater than the corresponding load resistance for a single triode and the power output is about  $2\frac{1}{2}$  times that of the single triode.

**621.396.645 1045**  
**Wide-Band Amplifiers**—(*Wireless World*, vol. 53, pp. 25–26; January, 1947.) Design data for 3- and 4-stage amplifiers with stagger tuning.

**621.396.645:621.396.822 1046**  
**Background Noise in Amplifiers**—Zelbststein. (See 1200.)

**621.396.645.029.3 1047**  
**Amplifier with Very High Musical Fidelity: Part 9**—L. Chrétien. (*T.S.F. Pour Tous*, vol. 22, pp. 176–179; September, 1946.) One of a series of eleven articles in *T.S.F. Pour Tous*, vol. 22, 1946. This part describes the complete apparatus and gives a detailed diagram; it also recapitulates the previous articles. Parts 10 and 11 deal with assembly and testing.

**621.396.645.35 1048**  
**D. C. Amplifiers**—B. E. Noltingk and R. A. Lampitt. (*Electronic Eng.*, vol. 18, p. 389; December, 1946.) Correspondence on 679 of March.

**621.396.645.35 1049**  
**Electrometer Input Circuits**—H. A. Thomas. (*Electronics*, vol. 19, pp. 130–131; December, 1946.) It is shown theoretically that the use of negative feedback will reduce the time constant of the direct-current amplifier. A typical circuit is described and details of its performance given.

**621.396.622:621.396.62 1050**  
**The Design of Band-Spread Tuned Circuits for Broadcast Receivers**—Hughes. (See 1195.)

**621.396.662.2:621.396.615.17 1051**  
**Non-Linear Coils for [Radar] Pulse Generators**—H. A. Stone, Jr. (*Bell Lab. Rec.*, vol. 24, pp. 450–453; December, 1946.) The design and operation of these coils are fully explained. Magnetron voltage of about 25 kilovolts can be obtained from a 6-kilovolt rectifier. The peak power of the pulse can reach 1 megawatt.

**621.396.69 1052**  
**Components**—M. Chauvierre. (*Radio en France*, no. 1, pp. 9–18; 1946.) Review of an exhibition of components including transformers, tuning units, fixed and variable capacitors, coils and coil formers, resistors, multiplugs and sockets, switches, etc.

**621.396.694.001.8:621.385.5 1053**  
**New Uses for Pentagrids**—A. H. Taylor. (*Electronics*, vol. 19, pp. 142, 154; December, 1946.) A pentagrid can be used as a phase inverter and a high-stability single-coil oscillator, to supply alternate positive and negative syn-

chronization to a cathode-ray oscilloscope, or to act as a variable frequency oscillator for a transmitter.

- 621.3.01 1054  
**Circuit Analysis by Laboratory Methods [Book Review]**—C. E. Skroder and M. S. Helm. Prentice-Hall, New York, N. Y. 1946, 282 pp., \$5.35. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, p. 865; November, 1946.) Gives theory and general laboratory material for the study of direct-current and alternating-current circuits and "is apparently intended for junior electrical engineering students."

- 621.3.011.3:518.2 1055  
**Inductance Calculations. [Book Review]**—F. W. Grover. D. Van Nostrand Co., Inc., New York, N. Y. 1946, 286 pp. \$5.75. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, p. 865; November, 1946.) A collection of formulas and tables for the calculation of mutual and self-inductance at low frequencies. Magnetic forces between coils are also treated and data provided for determining inductance changes as the frequency is increased.

- 621.396.621 1056  
**Most-Often-Needed 1946 Radio Diagrams [Book Review]**—M. N. Beitman. Supreme Publications, Chicago, Ill., 1946, 192 pp., \$2.00. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, p. 865; November, 1946.) An assembly of the available wiring diagrams and service information on radio-receiver models produced by approximately 40 manufacturers during the early part of 1946.

## GENERAL PHYSICS

- 534.756+621.39 1057  
**Theory of Communication**—D. Gabor. (*Jour. I.E.E.* (London), part III, vol. 93, pp. 429-457; November, 1946.) Part 1 describes a new method of analysis of signals in which there is symmetry between time and frequency, manifested in the Fourier transform relationships between the time function of the signal and its frequency spectrum. The accuracy with which the time  $t$  and frequency  $f$  can be simultaneously specified is limited by an uncertainty relation  $\Delta t \Delta f \geq \frac{1}{2}$ , which is closely linked to Heisenberg's principle of indeterminacy, and is derived by consideration of the 'effective duration' and 'effective frequency width' of the signal. This inequality defines a minimum area in the two dimensional 'information diagram' in which signals may be represented with time and frequency as co-ordinates. It is shown that the elementary signal which occupies this minimum area is the modulation product of a harmonic oscillation of any frequency with a pulse of the form of a probability function. Any signal can be expanded into elementary signals so that their representative areas cover the whole time-frequency area, and each elemental area, or logon, represents one quantum of information. The use of the information diagram is illustrated by two examples, a simple frequency-modulated signal, and time division multiplex telephony. The signal which can be transmitted in the shortest time through a channel, within a specified frequency band, is shown to have a sinusoidal frequency spectrum, and its effective duration and frequency width are found.

In part 2, an analysis of hearing sensations is made, the experimental results of various observers being considered in the light of the methods of part 1. It is deduced that in the frequency range 60 to 1000 cycles the human ear can discriminate nearly every second datum of information, but over the whole auditory range the efficiency is much lower since the discrimination falls off sharply at higher frequencies. Similar results obtained with signals of widely differing durations indicate that the threshold information sensitivity of the ear is

independent of the duration from 20 to 250 milliseconds, which means that the time constant of the ear appears to be adjustable over this range. To explain this it is necessary to postulate a new mechanism in the process of hearing, the nature of which is uncertain, but it is suggested that a new effect in nerve conduction may be responsible.

In part 3 the possibility is considered of transmitting audio signals in comparatively narrow wave bands by means of frequency compression in transmission and re-expansion on reception. A 'kinematical' method of doing this is described and analyzed theoretically, in which a record is scanned by a succession of moving slits in front of a 'window' of continuously graded transparency, behind which is a photocell. In order to simplify the analysis, the transparency variation is taken as following a probability law, though it is shown later that under optimum conditions a triangular or trapezoidal variation would produce less noise. For a sinusoidal 'input' the reproduced signal collected by the photocell is shown to have a line spectrum consisting of all combinations of the original frequency with the split repetition frequency. The action of the converter is shown graphically and the optimum operating conditions deduced. The full cycle of compression and re-expansion is considered and the result shown diagrammatically. It is found that the reproduction improves with increase in frequency, while the distortion occurring does so almost entirely in the expansion process.

An experimental test of the above mechanical system confirms the theoretical deductions. It is found that speech can be compressed into a frequency band as narrow as 500 cycles and still remain intelligible. Alternative kinematical methods of frequency condensation for transmission are described, of which the most convenient uses magnetic tape recorders.

In contrast, electrical methods of condensation are discussed, in which a nonsinusoidal carrier is used. The case in which a sinusoidal signal is modulated with repeated probability pulses is analyzed: compression only and not expansion is possible by this method. The original signal may however be restored in the receivers by a second modulation of the same type, and it is found that distortion similar to that in the kinematical case is again produced.

- 535.14 1058  
**Photophysics**—M. Boll. (*Télévis. franç.* no. 17, Supplement *Électronique*, pp. 1-2, 5; September, 1946.) A brief résumé of modern theory.

- 537+538 1059  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.* vol. 65, pp. 569-574; December, 1946.) Abstracts are given for the following papers read at the conference: "Bunching Conditions for Electron Beams with Space Charge," by L. Brillouin; and "Generalized Boundary Condition in Electromagnetic Problems," by S. A. Schelkunoff. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47; 111; December, 1946.

- 537.523.4 1060  
**Physical Mechanism of the Spark**—S. Teszner. (*Bull. Soc. Franç. Élec.*, vol. 6, pp. 61-80; February, 1946.) Consideration of the initial stage phenomena of spark discharges shows that the theories of Townsend, Rogowski, and Loeb and Meek are not completely satisfactory, especially when the ignition is ultrarapid. For this case an explanation is given, based on an extremely rapid passage of electrons, combined with the development of the first avalanche. A theory is also presented for the effects observed in the established regime of high-frequency sparks.

- 537.525:538.551.25 1061  
**Excitation of Plasma Oscillations**—D. Bohm. (*Phys. Rev.*, vol. 70, p. 448; September 1-15, 1946.) Two cases are considered: (a) a beam of fast electrons injected from the cathode into the plasma, and (b) plasma containing high speed electrons moving at random. The energy of the resulting oscillations is deduced. Summary of American Physical Society paper.

- 537.291:621.396.615.14 1062  
**Induction Currents Produced by Moving Electrons**—Colino. (*See* 1269).

- 537.311.2 1063  
**What is Ohm's Law?**—G. W. Stubbings. (*Elect. Rev.* (London), vol. 140, pp. 225-226; January 31, 1947.) A survey of a representative sample of scientific books reveals two views of the nature of Ohm's law: (a) that it is virtually the assignment of a name to the  $E/I$  ratio, and (b) that for certain materials the ratio  $E/I$  is constant and independent of  $I$ , provided all other conditions of the electric circuit remain constant.

- 537.533.7:538.245 1064  
**Forces on Ferromagnets Through Which Electrons are Moving**—D. L. Webster. (*Phys. Rev.*, vol. 70, pp. 446-447; September 1-15, 1946.) Summary of American Physical Society paper.

- 538.3 1065  
**Discovery and Title of the Elementary Law of Electromagnetism**—L. Bouthillon. (*Bull. Soc. Franç. Élec.*, vol. 5, pp. 352-363; November, 1945.) A comprehensive historical review shows that the formula  $dH = idl (\sin \omega)/r^2$  is due primarily to Biot, Laplace contributing to a much less extent. It is recommended that it should be known as the law of Biot and Laplace.

- 538.32:621.385.832 1066  
**An Analysis of Electromagnetic Forces**—W. A. Tripp. (*Elec. Eng.*, vol. 65, pp. 596-598; December, 1946.) Reply to criticism by A. Gronner (3253 of 1946) of an article by Tripp (587 of 1946). It is shown that the return to pre-Maxwell theory is necessary and that Gronner's relativity-cum-orthodoxy argument leads to an absurdity.

- 539.1 1067  
**The Nucleus of Atoms (Protons, Neutrons, Mesons)**—L. Leprince-Ringuet. (*Bull. Soc. Franç. Élec.*, vol. 6, pp. 43-55; February, 1946.)

- 539.22 1068  
**Relaxation Phenomena in Anisotropic Liquids in a Rotating Electrical Field**—V. N. Tsvetkoff. (*Bull. Acad. Sci. (U.R.S.S.)*, sér. phys., vol. 5, no. 1, pp. 57-67; 1941. In Russian, with English summary.) An investigation of the properties of anisotropic liquids composed of long chain molecules in a rotating electric field. It is found that the relaxation time in such liquids depends upon two factors, the time of polarization of a molecular group and the time of its revolution in the field. If the first of these factors is of the order of  $10^{-10}$  seconds (as is usual in polar liquids), then the second factor is a few tenths of a second for magnetic fields of several thousand gauss, or for electric fields of several centimeter-gram-second units.

- 621.31 1069  
**On the Theory of Electric Contacts between Metallic Bodies**—J. Frenkel. (*Zh. Eksp. Teor. Fiz.*, vol. 16, no. 4, pp. 316-325; 1946. In Russian.) An electric contact between two metals is treated as a gap which the electrons cross by the mechanism of thermionic emission. The effect of image forces is to reduce the potential difference between the two metals by an amount inversely proportional to the width of the gap. This explains why the electrical conductivity of fine metallic powders and

thin layers increases with temperature according to a law similar to that for semiconductors. For full English translation see *Jour. Phys.*, (U.S.S.R.) vol. 9, no. 6, pp. 489-495; 1945.)

621.396.615.142 1070  
Elementary Treatment of Longitudinal Debunching in a Velocity Modulation System—Feenberg. (See 1272.)

531.19 1071  
Statistical Thermodynamics [Book Review]—E. Schrödinger. Cambridge University Press, Macmillan, New York, N. Y., 1946, 86 pp., \$1.50. (*Amer. Jour. Sci.*, vol. 245, p. 59; January, 1947.) An inquiry confined to macroscopic systems. The points of view of Boltzmann, Gibbs, and Fowler are united "into one homogeneous aspect by combining the Gibbsian ensemble with the Darwin-Fowler mathematical tools."

539 1072  
Nucleonics [Book Review]—Progress Press, Washington, D. C. 38 pp., \$1.00 (*Nature* (London), vol. 158, p. 731; November 23, 1946.)

#### GEOPHYSICAL AND EXTRATERRESTRIAL PHENOMENA

523.16:621.396.822.029.62 1073  
Variation of Cosmic Radiation with Frequency—L. A. Moxon. (*Nature* (London), vol. 158, pp. 758-759; November 23, 1946.) Results are given of measurements at 40, 90, and 200 megacycles in Britain and in other latitudes, using directional aerials. A diagram shows the increase of aerial noise temperature from the equatorial plane of the galaxy as a function of galactic longitude from 0 to 360 degrees. At 350 degrees longitude noise level is found to vary approximately as  $f^{-3}$ , while the minimum noise level varies approximately as  $f^{-2}$ , where  $f$  is the frequency.

523.72:621.396.822.029.62/.63 1074  
Origin of Radio-Waves from the Sun and the Stars—M. N. Saha. (*Nature* (London), vol. 158, pp. 717-718; November 16, 1946.) The emission of radio waves may be the result of nuclear transitions caused by the presence of a strong magnetic field. For the sodium atom two types of transition are possible; one gives rise to meter waves for fields of a few hundred gauss, while the other gives rise to centimeter waves for fields of the order of  $10^4$  gauss. Similar results hold for hydrogen and various hydrides occurring in solar and stellar atmospheres. The magnetic fields in sunspots are of the right order of magnitude to produce such transitions, if the assumption of a small additional cross field is made. This theory appears to explain the observed effects on meter wavelengths at times of sunspot activity, and suggests that there may also be emission of the centimeter range. The emission of centimeter waves from the Milky Way may indicate the development of spots in some of its stars, which would probably be too small for spectroscopic observation. See also 404 and 402 of March and back references.

539.16.08:537.591 1075  
Cloud Chamber for Airborne Cosmic-Ray Observations—W. E. Hazen. (*Phys. Rev.*, vol. 70, p. 445; September 1-15, 1946.) Summary of American Physical Society paper.

550.389 1076  
North Magnetic Pole Believed Moved 200 Miles—(*Sci. News Letter*, Wash. vol. 50, p. 280; November 2, 1946.) Recent observations indicate that the north pole is now at least 200 miles north and slightly east of its former position in Boothia peninsula.

551.510.535:523.746 1077  
The Current Sunspot Trend—H. T. Stetson. (*Sci. Mon.*, vol. 63, pp. 399-402; November, 1946.) Description of the correlation between

E-layer electron density and mean sunspot number. The next sunspot maximum should occur during the early part of 1948.

551.515.827:621.396.812 1078  
Reflection of Radio Waves from Tropospheric Layers—Smyth and Trolese. (See 1180.)

551.556.3 1079  
Wind Energy: Its Value and the Search for [Installation] Sites—P. Ailleret. (*Rev. Gén. Élec.*, vol. 55, pp. 103-108; March, 1946.)

551.578.1:621.396.812 1080  
Attenuation of 1.24-Centimeter Radiation through Rain—Anderson, Freres, Day, and Stokes. (See 1181.)

551.594.11 1081  
The Definition and Theory of the Potential Acquired by a Conductor in Atmospheric Electricity—R. Lecolazet. (*Compt. Rend. Acad. Sci.* (Paris), vol. 222, pp. 331-332; February 4, 1946.) A precise definition of the potential acquired by a conductor in the earth's electric field is used to develop an approximate theory of arrangements for measurement of the field.

551.594.13:621.317.723 1082  
Measurement of the Electric Conductivity of the Air by a Tetrode Electrometer—Lacaze. (See 1140.)

551.5(02) 1083  
Meteorology with Marine Applications [Book Review]—W. L. Donn. McGraw-Hill Book Co., New York, N. Y. 1946, 465 pp., \$4.50. (*Amer. Jour. Sci.*, vol. 244, pp. 813-815; November, 1946.)

551.5(023) 1084  
Handbook of Meteorology [Book Review]—E. A. Berry, Jr., E. Bollay, and N. R. Beers (Editors). McGraw-Hill Book Co., New York, N. Y. 1946, 1068 pp., \$7.50. (*Amer. Jour. Sci.*, vol. 244, pp. 813-815; November, 1946.)

#### LOCATION AND AIDS TO NAVIGATION

621.396.9.001.8 1085  
Guided Missiles in World War II—Selvidge. (See 1171.)

621.396.93 1086  
Frequency, Power, and Modulation for a Long-Range Radio Navigation System—P. R. Adams and R. I. Colin. (*Elec. Commun.*, vol. 23, pp. 144-158; June, 1946.) Ground stations must have a minimum range of 1500 miles. All aircraft within range must receive useful signals irrespective of climatic or propagation conditions. These assumptions restrict the frequency to below 300 kilocycles or between 2 and 30 megacycles. The transmission characteristics, signal-to-noise ratios, and aerial efficiency for the two bands are discussed and it is concluded that for maximum reliability and most economical working, the frequency should be about 70 kilocycles and the aerial power 10 to 100 kilowatts depending on static level and assuming a receiver bandwidth of 10 to 20 cycles. There is a comprehensive bibliography.

621.396.931/.933:22.029.62 1087  
Ultra-High-Frequency Radiosonde Direction Finding—L. C. L. Yuan. (*Proc. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 852-857; November, 1946.) Description of a single direction-finder for measuring azimuth to within  $\frac{1}{2}$  degree and elevation to within  $\frac{1}{2}$  degree at 183 megacycles for a fixed transmitter; these errors are somewhat increased for a moving balloon transmitter. Measurements using Adcock and single dipole aerials are described. Various types of reflector systems for shielding a dipole aerial from ground-reflected waves were tested; the corner reflector type of shield

with a simple half-wave dipole was found to be the most effective. See also 732 of April.

621.396.932 1088  
Electronics on World's Largest Liner—(See 1222.)

621.396.933 1089  
Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "Improvements in 75-Megacycle Aircraft Marker Systems," by B. Montgomery, "Automatic Radio Flight Control," by F. L. Moseley and C. B. Watts, "Naviglobe—Long Range Air Navigation System," by P. R. Adams and R. I. Colin; and "Teloran—Air Navigation and Traffic Control by means of Television and Radar," by D. H. Ewing and R. W. K. Smith. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.)

621.396.933 1090  
New Radio Beacon Methods—E. Aisberg. (*Toute la Radio*, vol. 13, pp. 274-277; November, 1946.) A short historical survey, with discussion of the basic principles and operation of the loran system.

621.396.933 1091  
The Block System for Airway Control—(*Elec. Ind.*, vol. 5, pp. 54-56; December, 1946.) A method based upon railway block-signaling practice, using pulse technique for signaling codes and ensuring automatic safety on air routes.

621.396.933 1092  
[Long and Short Range] Aerial Navigation [Including Instrument and Automatic Landing] and Traffic Control with Navaglobe, Navar, Navaglide, and Navascreen—H. Busignies, P. R. Adams, and R. I. Colin. (*Elec. Commun.*, vol. 23, pp. 113-143; June, 1946.) Co-ordinated proposals for the step-by-step provision of these facilities over a period of years, using the minimum amount of aircraft equipment.

About 75 navaglobe transmitters on about 100 kilocycles provide equipped aircraft with fixes anywhere in the world. Each aircraft compares the amplitudes of three figure-of-eight diagrams to obtain a bearing, and two such bearings give a fix; an independent check is given by normal loop direction finding.

Navar is basically a 3000-megacycle ground surveillance radar at each airport and provides (a) normal plan position indicator, (b) plan position indicator on all aircraft carrying responders, or only on those at a selected height, (c) as (b), but on aircraft intending to land. Video signals from (b) are rebroadcast for display with a ground wind vector on the aircraft plan position indicator. Aircraft height can be measured, identification announced by means of coded responders, and prearranged orders automatically displayed to the pilot of any selected aircraft. The rotating beam of the ground radar and a timing signal provide the pilot with azimuth, while the aircraft pulse transmitter and a ground responder give him range from the airport or from the end of the runway when using navaglide.

Navaglide also uses the 3000-megacycle band to provide four signals for blind or automatic landing. Time sharing permits these and similar signals from nearby navaglides to use the same frequency so that only one receiver per aircraft is required. The above indications are automatically displayed to the pilot and where appropriate can be used for auto-pilot control.

Navascreen is a semiautomatic system for displaying on a large scale information about all nearby aircraft for the controllers use. An accelerated time device permits prediction of collisions and forecasts traffic density at the airport.

621.396.933

**The Problems of Blind Landing**—H. C. Pritchard. (*Jour. R. Aero. Soc.*, vol. 50, pp. 935-958; December, 1946. Discussion pp. 958-973.) A review of recently developed methods and a general discussion of the various problems involved. The methods include the American Civil Aeronautics Authority (see 2237 and 2655 of 1945), and SCS 51 systems, the GCA (ground-controlled approach) system and the BABS (beam approach beacon system). The GCA system comprises a mobile ground radar equipment which determines aircraft position and gives instructions to the pilot. It also includes a search system giving the plan position of all aircraft within about 15 miles and below, about 4000 feet. In the BABS system, aircraft transmissions are received by the ground station and reradiated on a slightly different frequency after a short time delay, with arrangements to give on a cathode-ray tube in the aircraft, short and long pulses whose relative amplitudes are determined by the angular position of the aircraft relative to the runway. A continuous range indication is also given and this, together with a barometric altimeter, is relied on for vertical guidance. The provision of adequate information for vertical guidance appears to be the most difficult landing problem and some form of radio altimeter the most promising solution.

621.396.933.1.029.62

**An Ultra-High-Frequency Radio Range with Sector Identification and Simultaneous Voice**—A. Alford, A. G. Kandoian, F. J. Lundberg, and C. B. Watts, Jr. (*Elec. Commun.*, vol. 23, pp. 179-189; June, 1946.) Reprint of 932 of 1946.

621.396.96

**Minimum Detectable Radar Signal and Its Dependence upon Parameters of Radar Systems**—A. V. Haeff. (*PROC. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 857-861; November, 1946.) Discussion of the influence on the sensitivity of a pulse radar system of pulse repetition rate  $r$ , pulse length  $t$ , intermediate-frequency bandwidth  $B$ , and video bandwidth  $b$ . Experimental determination of the absolute value of the minimum pulse signal ( $V_{min}$ ) detectable visually through noise, is described with the apparatus used, and the results are expressed by the formula:

$$V_{min} = \frac{1}{2} E_n B^{1/2} (1 + 1/tB) (1670/r)^{1/6}$$

where  $E_n$  is the noise voltage per unit of intermediate-frequency bandwidth.

621.396.96

**Radar Demonstration by Hasler A. G. at Berne**—W. Schiess. (*Tech. Mitt. Schweiz. Telegr.-Teleph. Verw.*, vol. 24, pp. 211-218; October 1, 1946.) Demonstration to representatives of the technical press of British 3-centimeter naval radar, and a simple explanation of radar and its operation, with some details of plan-position-indicator equipment.

621.396.96:621.392.5

**Precision Resistance Networks for [Fire-Control] Computer Circuits**—Hagemann. (See 1022.)

621.396.96(43)

**Great Britain's Part in the Creation of Radar**—R. W. Hallows. (*Toute la Radio*, vol. 13, pp. 201-203; September, 1946.) "Many mistaken ideas are current on the origins of radar. . . . In order to re-establish the truth, we have thought it useful to complete the study by M. Ponte (see 1099 below). . . . by another which brings to light the important contributions of British technicians."

621.396.96(44)

**French Contributions to Radar Technique**—M. Ponte. (*Toute la Radio*, vol. 13, pp. 204-

1093

207; September, 1946.) Reprint of 3290 of 1946. See also 1098 above.

621.396.96.001.8

**Radar-Guided Bomb**—(*Electronics*, vol. 19, pp. 186, 194; December, 1946.) Automatic radar control for glider-type Bat bomb.

## MATERIALS AND SUBSIDIARY TECHNIQUES

531.788.12

**Improvements in the MacLeod Gauge**—P. Tarbès. (*Le Vide*, (Paris), vol. 1, pp. 9-11; January, 1946.)

533.5+621.52

**Vacuum Pumps**—P. Pensa. (*Le Vide*, (Paris), vol. 1, pp. 4-8 and 48-53; January and March, 1946.) A short historical survey, with a description of Holweck's molecular pump, various rotary oil pumps, and the principles and construction of diffusion and condensation pumps.

533.5+621.521]:614.8

**Safety Arrangement for Vacuum Plant**—M. Schérer. (*Compt. Rend. Acad. Sci.* (Paris), vol. 222, pp. 997-998; April 24, 1946.) A simple device is described which depends on the action of a magnetic field on a gaseous discharge and which can be used to operate a stopcock.

535.37

**Luminescence Processes in Zinc Sulphide Phosphors**—G. F. J. Garlick and A. F. Gibson. (*Nature* (London), vol. 158, pp. 704-705; November 16, 1946.)

535.37:546.46.784

**The Fluorescence of Pure Magnesium Tungstate**—C. G. A. Hill. (*Trans. Faraday Soc.*, vol. 42, pp. 685-689; November, 1946.) Under optimum conditions of preparation, maximum fluorescence efficiency is given by the composition  $MgWO_4$ , the fluorescence being associated with a particular monoclinic crystal structure.

535.376

**The Light Output of Zinc Sulphide on Irradiation with Alpha Rays**—H. A. Klasens. (*Trans. Faraday Soc.*, vol. 42, pp. 666-668; November, 1946.) A critical review of the literature leads to the conclusion that the value of 80 per cent given by Riehl is much too high, most measurements giving values of the order of 10 to 15 per cent.

546.287

**Silicones, New Electrical Insulating Materials**—F. Appell. (*Rev. Gén. Elec.*, vol. 55, pp. 99-103; March, 1946.) A short account of the methods of preparation of silicone liquids, greases, rubbers, and resins; also of the physical properties of the various products and their applications in the electrical industry.

546.431.826:621.3.011.5

**Oscillograph Study of Dielectric Properties of Barium Titanate**—A. de Bretteville, Jr. (*Jour. Amer. Ceram. Soc.*, vol. 29, pp. 303-307; November, 1946.) See also 3625 of January.)

620.193.21:669.018.2.21

**Weathering—Appreciation and a Study**—G. D. Chapman. (*Light Metals*, vol. 9, pp. 593-608; November, 1946.) The significance of atmospheric corrosion of cast-light alloys is reviewed and illustrated by some experimental work. The possible use of X-rays to determine the type and degree of attack is demonstrated.

620.197.6

**Preventing Corrosion in Steel Chassis**—(*Wireless World*, vol. 53, p. 22; January, 1947.) Extensive corrosion tests under tropical conditions have confirmed the superiority of electrodeposited tin-zinc alloys over either metal alone. Alloys with between 50 and 80 per cent tin show the highest corrosion resistance.

621.31

**Contacts and Contact Materials**—(*Elec. Times*, vol. 110, p. 841; December, 26 1946.) See also 781 of April. Notice of book published by Johnson Matthey and Co. and Mallory Metallurgical Products. Properties and tables of use for various alloys and pure materials are listed.

621.315.59:546.655.78

**Cerium Tungstate as a Semi-Conductor**—J. B. Nelson and J. H. McKee. (*Nature* (London), vol. 158, pp. 753-754; November 23, 1946.)

621.315.612.6.017.143.029.4+666.11

**The Application of Low Frequency Spectra to Glass Technology**—N. J. K. (*Glass Ind.*, vol. 27, pp. 609, 624; December, 1946.) Results obtained by P. Girard and P. Abadie published recently in *Bull. Inst. Verre* indicate the importance of power-loss measurements at wavelengths from 1 to  $10^6$  meters. With rubber, a direct correlation was found between the shape of the loss curve and the mechanical properties. The results for glass suggest the possibility of similar correlation.

621.315.612.6.017.143.029.64+666.11

**Dielectric Properties of Glasses at Ultra-High Frequencies and Their Relation to Composition**—L. Navias and R. L. Green. (*Glass Ind.*, vol. 27, pp. 615, 618; December, 1946.) Summary of paper already noted in 452 of March.

621.316.86:546.281.26

**Silicon Carbide Non-Ohmic Resistors**—F. Ashworth, W. Needham and R. W. Sillars. (*Jour. I.E.E.* (London), part I, vol. 93, pp. 595-958; December, 1946.) Discussion on 141 of February.

621.316.86.023

**High Value Deposited Resistances at High Frequency**—Klemt. (See 1019.)

621.318:539.16.08

**Magnetic Field with Small Axial Variations**—C. E. Nielsen. (*Phys. Rev.*, vol. 70, p. 450; September, 1-15, 1946.) Combinations of coils, iron bars, and rings to produce the types of field required for cloud chamber experiments. Summary of American Physical Society paper.

621.318.23:539.16.08

**The Strengths of Some Model Magnets**—S. B. Jones. (*Phys. Rev.*, vol. 70, p. 450; September 1-15, 1946.) Designs of permanent magnets made of Alnico V for use with airborne cloud chambers in cosmic ray studies. Summary of American Physical Society paper.

621.318.323.2.042.15.029.64

**Iron Cores at Very-High Frequencies**—J. Gouveritch. (*Radio en France*, no. 4, pp. 4-8; 1946.) Describes iron-dust materials and the best core shapes to realize  $Q$  values up to 280 at frequencies up to 50 megacycles.

669.26:621.385.032.2

**Zirconium in Electron Tubes**—Foote Mineral Co. (*Rev. Sci. Inst.*, vol. 17, p. 517; November, 1946.) Zirconium absorbs oxygen, nitrogen, and other gases at temperatures above 300 degrees centigrade, and may therefore be used as a continuous "getter" for certain types of tube. It can also act as a flux in welding tungsten, molybdenum, and tantalum and has a low coefficient of secondary emission. Abstract of article by G. A. Espersen in *Foote Prints*, vol. 18.

679.5

**Unique Plastic makes Excellent Insulator**—(*Sci. News Letter*, (Washington), vol. 50, p. 312; November 16, 1946.) "Teflon" is unharmed by temperatures up to 300 degrees centigrade and can be bent without cracking at +80 de-

grees centigrade. Its losses are low even at 3000 megacycles and it withstands every known solvent. Summary of an address to the Society of the Plastic Industry by E. B. Yelton.

679.5:621.315.616 1122

Plastics as Insulation—(*Elec. Times.*, vol. 110, pp. 837-838; December 26, 1946.) Summary and discussion of a paper read before the Institution of Electrical Engineers Installations Section, on "The Growing Importance of Plastics in the Electrical Industry," by G. Haefely.

679.5 1123

Experimental Plastics and Synthetic Resins [Book Review]—G. F. D'Alelio, J. Wiley and Sons, New York, N. Y. Chapman Hall, London, 1946, 185 pp., \$3. (*Nature* (London), vol. 158, p. 689; November 16, 1946.)

## MATHEMATICS

517.54 1124

Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) An abstract is given of a paper read at the conference entitled "Conformal Transformations in Orthogonal Reference Systems," by C. S. Roys. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.

518.5:621.38 1125

A.C.E. The Automatic Computing Machine—Department of Scientific and Industrial Research. (See 1157.)

518.61 1126

Nomographs—(*Rev. Sci. Instr.*, vol. 17, p. 527; November, 1946.) A 'projected scale chart' used in conjunction with simple equations permits the construction of nomographs involving up to five variables. Developed by W. H. Burrows.

530.162:621.396.822 1127

On the First Passage Time Problem for a One-Dimensional Markoffian Gaussian Random Function—A. J. F. Siegert. (*Phys. Rev.*, vol. 70, p. 449; September 1-15, 1946.) The probability distribution of the first passage of a random variable through any given value is calculated. Summary of American Physical Society paper.

512.99:621.3 1128

Elementary Vectors for Electrical Engineers [Book Review]—G. W. Stubbings, Pitman and Sons, London, second edition, 6s.6d. (*Engineering* (London), vol. 162, p. 507; November 29, 1946.)

## MEASUREMENTS AND TEST GEAR

621.3.011.3:389.6 1129

The Possibility of a Comparison of the Inductance Standards of the Various National Laboratories and the Part such a Comparison might play in Determinations of the Electromagnetic Unit of Resistance—M. Romanowski and R. Hérou. (*Bull. Soc. Franç. Élec.*, vol. 6, pp. 355-362; June-July, 1946.)

621.317.3.029.63/.64 1130

Radio Measurements in the Decimetre and Centimetre Wavebands—R. J. Clayton, J. E. Houlden, H. R. L. Lamont, and W. E. Willshaw. (*Jour. I.E.E.* (London), part III, vol. 93, pp. 457-459; November, 1946.) Discussion on 1914 of 1946.

621.317.32:621.396.611.4 1131

Measurement of Electric Field Strength in a Cavity Resonant at 200 Mc/s—W. K. H. Panofsky. (*Phys. Rev.*, vol. 70, p. 447; September 1-15, 1946.) Fields up to 25 kilovolts per centimeter are determined by measurement either of magnetic field by a loop method or of electrostatic force on the diaphragm of a con-

denser microphone. Summary of American Physical Society paper.

621.317.32.027.3 1132

High Voltage Measurement—(*Elec. Times.*, vol. 110, pp. 723-724; November 28, 1946; *Electrician*, vol. 137, pp. 1517-1518; November 29, 1946.) Summaries of two papers by F. M. Bruce noted in 472 of March.

621.317.333.4:621.315.2.029.5 1133

New Methods for Locating Cable Faults, Particularly on High-Frequency Cables—Roberts. (See 1002.)

621.317.335:621.396.694 1134

Measuring Inter-Electrode Capacitances—C. H. Young. (*Bell Lab. Rec.*, vol. 24, pp. 433-438; December, 1946.) A new type of bridge circuit with a double 3-terminal star network which can measure capacitance to within  $10^{-6}$  picofarad and conductance to within  $5+10^{-6}$  micromho.

621.317.34:621.396.44 1135

Receiver for Measurements on Carrier-Frequency Systems—H. G. Thilo. (*Arch. Tech. Messen.*, no. 112, p. T108; October, 1940.) Receiver covering the frequency range 120 to 280 kilocycles using a bridge-rectifier mixer circuit with a 2-stage intermediate-frequency amplifier at 350 kilocycles (bandwidth about  $\pm 10$  kilocycles). The circuit is arranged to measure the peak values of the modulated signal by suitable choice of time constants. Internal calibration is obtained by using the local oscillator as an intermediate-frequency generator after adjusting its amplitude to a standard value with the aid of the output voltmeter.

621.317.4.013.24:621.384.6 1136

A Method for Measuring Small Changes in Alternating Magnetic Fields—Powell. (See 1160.)

621.317.44+ [621.38:531.765 1137

Electronic Meters—(*Elec. Times.*, vol. 110, p. 812; December, 1946.) Summary of two papers read before the Institution of Electrical Engineers' Measurements Section. The first entitled "A Millisecond Chronoscope," by R. S. J. Spilsbury and A. Felton, describes an instrument on the capacitor charging principle, with a working range from 2 milliseconds to 1 second. The second, entitled "A Sensitive Recording Magnetometer," by A. Butterworth, describes a temperature compensated instrument using the change of alternating-current resistance of a mumetal wire when subjected to an axial magnetic field.

621.317.7 1138

Measurement Apparatus at the Paris Fair—G. Giniaux. (*TSE Pour Tous*, vol. 22, pp. 182-186) September, 1946.)

621.317.7 1139

Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "The Notch Wattmeter for Low-Level Power Measurement of Microwave Pulses," by D. F. Bowman; and "Microwave Frequency Stability," by A. E. Harrison. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.

621.317.723:551.594.13 1140

Measurement of the Electric Conductivity of the Air by a Tetrode Electrometer—J. Lacaze. (*Compt. Rend. Acad. Sci.* (Paris), vol. 222, pp. 1242-1244; May 20, 1946.)

621.317.725 1141

Simple Valve Voltmeter—H. W. Baxter. (*Wireless World*, vol. 53, pp. 9-10; January, 1947.) The voltage to be measured is applied

to a diode, with a capacitor in the anode circuit, and the rectified output from the diode is balanced by a direct voltage from a potential divider, using the anode current of a second tube as a balance indicator. Peak voltages up to 90 volts can be measured using 9-volt batteries.

621.317.73 1142

Z-Meter—L. E. Packard. (*Elec. Ind.*, vol. 5, pp. 42-45; December, 1946.) This instrument measures the complex impedance of communication circuits, microphones, lines, transformers, and speakers at audio frequencies, giving both impedance and phase-angle independently of frequency. Stray capacitance and coupling are rendered negligible by means of a shielded and balanced input transformer. The range of impedance is from 0.5 to 100,000 ohms and of phase-angle from  $+90$  to  $-90$  degrees, over a frequency range of 30 to 20,000 cycles. The instrument will also measure  $R$  from 0.5 to 100,000 ohms,  $L$  from 5 microhenrys to 500 henrys,  $C$  from 0.0012 to 10,000 microfarads, the dissipation factor  $D$  of capacitors, and the  $Q$  of inductors.

621.317.76:389.6 1143

A Standard of Frequency and Its Applications—C. F. Booth and F. J. M. Laver. (*Jour. I.E.E.* (London), part III, vol. 93, pp. 427-428; November, 1946.) Discussion of 2973 of 1946.

621.317.76:389.6 1144

Frequency Standard—H. Gilloux. (*Radio en France*, no. 2, pp. 13-16; 1946.) Describes a secondary frequency standard, using a 500-kilocycle quartz crystal with multivibrator frequency-division stages, permitting direct-frequency measurement in 1-kilocycle steps to about 4 megacycles, in 20-kilocycle steps to 40 megacycles and in 100-kilocycle steps to 100 megacycles.

## OTHER APPLICATIONS OF RADIO AND ELECTRONICS

539.16.08 1145

An Improvement on the Copper Evaporation Geiger-Müller Counter—P. J. G. de Vos, K. Gürgen and S. J. du Toit. (*Rev. Sci. Instr.*, vol. 17, p. 516; November, 1946.)

620.179:534.321.9 1146

Supersonic Flaw Detector—J.H.J. (See 986.)

621.3:629.1.001.4 1147

Vibration Exciter for Structural Tests—P. J. Holmes. (*Electronics*, vol. 19, pp. 96-100; December, 1946.) Description of electromagnetic shakers for vibration tests at mechanical resonance on aircraft structures. Circuits of carrier-type audio-frequency amplifier, phase shifter and phase indicator are given.

621.317.761:621.165 1148

An Electronic Frequency Meter and Speed Regulator—E. Levin. (*Trans. A.I.E.E.* (*Elec. Eng.*, December, 1946), vol. 65, pp. 779-786; December, 1946.) An attachment to a steam turbine which will indicate speeds to an accuracy of the order of 0.1 per cent and which holds the speed steady (within  $\pm 1/2$  per cent) at any speed between 10,000 and 100,000 revolutions per minute.

621.318.572 1149

High Speed Pulse Recording Circuit—B. E. Watt. (*Rev. Sci. Instr.*, vol. 17, pp. 338-342; September, 1946.) The circuits described are capable of driving a Cenco low-impedance counter at rates greater than 130 counts per second. Hard tubes are used, and the circuit cannot jam.

621.318.572 1150

Design and Operation of an Improved Counting Rate Meter—A. Kip, A. Bousquet, R. Evans, and W. Tuttle. (*Rev. Sci. Instr.*, vol. 17, pp. 323-333; September, 1946.) De-

sign and operation of the various components, viz., amplifiers, pulse equalizer, integrating circuit, degenerative vacuum-tube voltmeter, and the stabilized high- and low-voltage power supplies. Practical methods with curves are developed for determining the mean counting rate and its probable error directly from the output records.

621.318.572 1151  
Electronic Switch—F. Haas. (*Toute la Radio*, vol. 13, pp. 22-23; December, 1946.)

621.318.572 1152  
Current Integrator—B. E. Watt. (*Rev. Sci. Instr.*, vol. 17, pp. 334-338; September, 1946.) An integrator for the range 50 microamperes to below  $4 \times 10^{-3}$  microamperes, using the author's pulse recording circuit (1149 above). The charge per count is constant to within  $\pm 2$  per cent over the entire range, and is unaffected by power supply variations.

621.335.029.52 1153  
An Application of High Frequencies—L. Guerrier. (*Tele. Franç.*, Supplement *Électronique*, p. 5; September, 1946.) Russian experiments on traction powered by induction at 50 kilocycles.

621.36+621.38.001.8 1154  
Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "Detectors for Buried Metallic Bodies," by L. F. Curtis; "Electron Optics of Deflection Fields," by R. G. E. Hutter; "Microwaves and Their Possible Use in High-Frequency Heating," by T. P. Kinn and J. Marcum; "Ignitron Converters for Induction Heating," by R. J. Ballard and J. L. Boyer; "Dielectric Preheating in the Plastics Industry," by D. E. Watts, G. F. Leland and T. N. Willcox; "The Problem of Constant Frequency in Industrial High-Frequency Generators," by E. Mittelmann; "Large Electronic D.C. Motor Drives," by M. M. Morack; "Electronic Speed Control of A.C. Motors," by W. H. Elliott; "The Electronic Method of Contouring Control [following a master template by a tracer]," by J. Morgan; "Modulation of Infrared Systems for Signaling Purposes," by W. S. Huxford; "Photo Detectors for Ultraviolet, Visible, and Infrared Light," by R. J. Cashman; "Military Applications of Infrared Viewers," by G. E. Brown; "The Use of Radioactive Materials in Clinical Diagnosis and Medical Therapy," by J. T. Wilson; "The Mass Spectrometer as an Industrial Tool," by A. O. Nier; "The Cathode-Ray Spectrograph," by R. Feldt and C. Berkley; "Some Fundamental Problems of Nuclear Power Plant Engineering," by E. T. Neubauer; "An Accelerator Column for Two to Six Million Volts," by R. R. Machlett; "The Betatron Accelerator applied to Nuclear Physics," by E. E. Charlton and G. C. Baldwin; and "The Pressuregraph," and by A. Crossley. For titles of other papers read, see other sections. For other abstracts see *Elec. Indus.*, vol. 5, pp. 46-47, 111; December, 1946.

621.362 1155  
Vacuum Thermocouples for Radiation Measurement—F. Kerkhof. (*Arch. Tech. Messen*, no. 112, pp. T115-116; October, 1940.) The sensitivity (deflection for a given irradiation/mean zero fluctuation) of a vacuum thermocouple is far superior to that of a bolometer. The influence of the gas pressure, the material of the element and its blackening on the sensitivity is discussed. Some applications are given.

621.365:666.1 1156  
H. F. Glass Working—E. M. Guyer. (*Elec. Ind.*, vol. 5, pp. 65-67; December, 1946.)

High-frequency heating alone, or in combination with flame, simplifies many processes.

621.38:518.5 1157  
A.C.E. The Automatic Computing Machine—Department of Scientific and Industrial Research. (*Electronic Eng.*, vol. 18, pp. 372-373; December, 1946.) Gives some details of the automatic computing engine planned at the National Physical Laboratory.

621.38:531.765]+621.317.44 1158  
Electronic Meters—(See 1137.)

621.38.001.8 1159  
Punch Press Protector—J. Isaacs. (*Electronics*, vol. 19, pp. 101-103; December, 1946.) An electronic device for protecting a punch press from damage resulting from failure to eject disks from the die.

621.384.6:621.317.4.013.24 1160  
A Method for Measuring Small Changes in Alternating Magnetic Fields—W. M. Powell. (*Phys. Rev.*, vol. 70, p. 444; September 1-15, 1946.) Method using two search coils in compensating circuit with an amplifier and cathode-ray oscilloscope which enables variations of 0.1 oersted to be detected, e.g., in the betatron or synchrotron. Summary of American Physical Society paper.

621.384.6 1161  
Description of a Frequency Modulated Cyclotron and a Discussion of the Deflector Problem—E. J. Lofgren and B. Peters. (*Phys. Rev.*, vol. 70, p. 444; September, 1-15, 1946.) Summary of American Physical Society paper.

621.384.6 1162  
Frequency Modulation for Berkeley 37" Cyclotron—K. R. MacKenzie and F. H. Schmidt. (*Phys. Rev.*, vol. 70, p. 445; September 1-15, 1946.) Summary of American Physical Society paper.

621.384.6 1163  
Efficiency of Frequency Modulated Cyclotron—L. Foldy and D. Bohm. (*Phys. Rev.*, vol. 70, p. 445; September 1-15, 1946.) Summary of American Physical Society paper.

621.384.6 1164  
Frequency Modulated Cyclotron Characteristics—B. T. Wright and J. R. Richardson. (*Phys. Rev.*, vol. 70, p. 445; September 1-15, 1946.) Summary of American Physical Society paper.

621.384.6 1165  
Synchrotron Radiofrequency System—A. C. Helmholtz, J. V. Franck, and J. M. Peterson. (*Phys. Rev.*, vol. 70, p. 448; September 1-15, 1946.) Summary of American Physical Society paper.

621.385.833 1166  
The Electrostatic Electron Microscope—P. Grivet and H. Bruck. (*Ann. Radioélect.*, vol. 1, pp. 293-310; April-July, 1946.) For another account see 3706 of January.

621.385.833 1167  
The C.S.F. Electrostatic Microscope—P. Grivet. (*Le Vide*, Paris, vol. 1, pp. 29-47; March, 1946.) A general discussion of electrostatic and electromagnetic electron microscopes and a detailed description of the C.S.F. instrument, with micrographs of crystals and microbes obtained with it. For another account see 3706 of January. (Bruck and Grivet.)

621.385.833:535.317.6 1168  
Reduction of Spherical Aberration in Strong Magnetic Lenses [in Electron Microscope]—L. Marton and K. Bol. (*Phys. Rev.*, vol. 70, p. 447; September 1-15, 1946.) Compound instead of single objective lenses are used. Summary of American Physical Society paper.

621.385.833:537.533.73 1169  
The Universal Electron Microscope as a High Resolution Diffraction Camera—R. G. Picard and J. H. Reisner. (*Rev. Sci. Instr.*, vol. 17, pp. 484-489; November, 1946.) Electron diffractions corresponding to spacings above 7 angstroms can be obtained by magnifying the camera image by means of an electron lens. A critical description is given of the modifications required to adapt an electron microscope for this purpose.

621.389:531.76 1170  
Aids to Stroboscopic Measurement—E. L. Thomas. (*Electronic Eng.*, vol. 18, pp. 369-371; December, 1946.) Devices to improve the accuracy and reliability of speed measurements.

621.396.9.001.8 1171  
Guided Missiles in World War II—H. Selvidge. (*Proc. Radio Club Amer.*, vol. 23, pp. 3-15; October, 1946.) An account of the principal features and methods of control of many types developed in the United States and in Germany.

621.398:629.13 1172  
Radio Control Systems for Guided Missiles—S. L. Ackerman and G. Rappaport. (*Electronics*, vol. 19, pp. 86-91; December, 1946.) A detailed description of methods and equipments. Circuit diagrams are given, and different control techniques discussed.

537.533.7(02) 1173  
Introduction to Electron Optics [Book Review]—V. E. Cosslett. Clarendon Press, Oxford; Oxford University Press, London, 1946, 272 pp., 25s. (*Nature* (London), vol. 158, pp. 685-686; November 16, 1946; *Jour. Sci. Instr.*, vol. 23, p. 302; December, 1946.)

621.385.833 1174  
Electron Optics and the Electron Microscope [Book Review]—V. K. Zworykin, G. A. Morton, E. G. Ramberg, J. Hillier, and A. W. Vance. John Wiley and Sons, New York, N. Y.; Chapman and Hall, London, 1945, 766 pp., \$10. (*Trans. Faraday Soc.*, vol. 42, pp. 702-704; November, 1946.) See also 1960 and 2358 of 1946. The present review is thorough, comprehensive, and critical. "... this book is valuable alike for its wealth of practical detail of the technique and for its mostly well-chosen presentation of the available theoretical considerations."

PROPAGATION OF WAVES  
523.72:621.396.822.029.62/.63 1175  
Origin of Radio-Waves from the Sun and the Stars—Saha. (See 1074.)

551.510.535:621.396.24 1176  
Short-Wave Long Distance Links by Means of the Ionosphere—de Gouvenain. (See 1211.)

551.510.535:621.396.24 1177  
Application of the Theories of Indirect Propagation to the Calculation of Links using Decametre Waves—Aubert. (See 1210.)

621.396.11 1178  
Second National Electronics Conference, Chicago, Autumn, 1946—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) An abstract is given of a paper read at the conference entitled: "Radio Propagation at Frequencies above 30 Megacycles," by K. Bullington. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.

621.396.812 1179  
Five Metre Propagation Characteristics—D. W. Heightman and E. J. Williams. (*R.S.G.B. Bull.*, vol. 22, pp. 98-102; January, 1947.) Discusses chiefly tropospheric propagation of 5-meter waves, with special reference to

the effects of refraction, temperature inversions, humidity gradients, and general weather conditions.

621.396.812:551.515.827 1180  
**Reflection of Radio Waves from Tropospheric Layers**—J. B. Smyth and L. G. Trolse. (*Phys. Rev.*, vol. 70, p. 449; September 1-15, 1946.) Reflection coefficients are calculated for an elevated layer produced by a warm, dry air mass over a cool, humid air mass. Experimental data over a 90-mile-link on 52, 100, and 547 megacycles are compared with theory. Summary of American Physical Society paper.

621.396.812:551.578.1 1181  
**Attenuation of 1.24-Centimeter Radiation through Rain**—L. J. Anderson, C. H. Freres, J. P. Day, and A. P. D. Stokes. (*Phys. Rev.*, vol. 70, p. 449; September 1-15, 1946.) Experimental determination over a path of 6400 feet. A value of 0.37 decibel per mile per millimeter per hour was obtained, which is somewhat greater than the theoretical value deduced assuming incoherent scattering. Summary of American Physical Society paper.

621.396.812.029.64 1182  
**Measurement of the Angle of Arrival of Microwaves**—W. M. Sharpless. (*PROC. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 837-845; November, 1946.) A description of the apparatus is given and a summary of the data collected during the summer of 1944 for two short optical paths using a wavelength of  $3\frac{1}{4}$  centimeters. The beams of two receiving aerials of width 0.36 degree are swept through small arcs, one vertically and the other horizontally, across the line of incoming waves. The signal level output is observed on a recorder giving an indication of the arrival direction.

The horizontal angle of arrival was found not to deviate by more than  $\pm 1$  degree, but vertical angles as much as  $1\frac{1}{2}$  degree above the true angle of elevation were measured. Observations on reflected rays from land and sea were also made and evidence of trapping for both the direct and reflected rays was obtained.

621.396.812.029.64 1183  
**Further Observations of the Angle of Arrival of Microwaves**—A. B. Crawford and W. M. Sharpless. (*PROC. I.R.E. AND WAVES AND ELECTRONS*, vol. 34, pp. 845-848; November, 1946.) A metal lens aerial of 0.12 degree beam width and wavelength  $1\frac{1}{4}$  centimeters was used. Results are compared with those for  $3\frac{1}{4}$  centimeters described in an earlier paper (1182 above) and correlated with meteorological conditions. Six curves of modified atmospheric refractive index against height, representative of conditions over the path, were obtained daily from balloon and tower observations. A detailed analysis is given of multiple path transmission which occurred on two nights when there was definite evidence of inversions in the refractive index versus height curve near the line of transmission.

621.397.81 1184  
**A Modification to Ray Theory Allowing for Ground Contours**—H. P. Williams. (*Electronic Eng.*, vol. 19, pp. 17-20; January, 1947.) A simple empirical law is given which, by allowing for multiple, instead of single, reflection from the ground, gives greatly improved correlation with experimental field strength curves for television transmissions on wavelengths of 7 meters from Alexandra Palace to points of reception near the ground and within 20 miles. The law should be of value at meter wavelengths and whenever the points of transmission and reception are intervisible. Its application to shorter wavelengths may be open to question.

## RECEPTION

534.862.4 1185  
**Perfect v. Pleasing Reproduction**—J. Moir. (*Electronic Eng.*, vol. 19, pp. 23-27; January, 1947.) The results of comprehensive tests, in which a reproducer of the highest quality was used to determine the frequency range which would produce the most pleasing reproduction, indicate a decided public preference for a restricted frequency range. This optimum range is approximately 70 to 6500 cycles, but is somewhat dependent on program material. A similar preference appears to exist when electrical reproduction is not involved. It is suggested that the present practice of transmitting with the flat or slightly rising characteristic and providing each listener with a tone control is the correct procedure.

621.396.44:621.317.34 1186  
**Receiver for Measurements on Carrier-Frequency Systems**—Thilo. (*See* 1135.)

621.396.619.13 1187  
**Twelve-Channel F.M. Converter**—J. E. Young and W. A. Harris. (*Electronics*, vol. 19, pp. 110-111; December, 1946.) Telephone-dial selection of any of twelve stations in the new frequency-modulated band is achieved remotely with a prewar frequency-modulated receiver by use of a three-tube converter in which mixer input and oscillator are tuned by preset trimmers connected to a 12-position rotary selector switch.

621.396.62+621.396.82 1188  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "Interference between Very-High-Frequency Radio Communication Circuits," by W. R. Young, Jr.; "Front-End Design of Frequency Modulation Receivers," by C. R. Miner; and "A Single-Stage Frequency Modulation Detector," by W. E. Bradley. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.

621.396.62 1189  
**Tendencies in the Design of the Communication Type of Receiver**—G. L. Grisdale and R. B. Armstrong. (*Jour. I.E.E.* (London), part I, vol. 93, p. 605; December, 1946.) Summary of 223 of February.

621.396.62.029.62 1190  
**The Professional Receiver**—R. Aschenbrenner. (*Radio en France*, no. 3, pp. 14-25; 1946.) Discusses in some detail the design of all the component stages of a communications receiver for the range 15 to 60 meters.

621.396.621 1191  
**Towards the Radio Receiver De Luxe**—L. Boë. (*Radio en France*, no. 1, pp. 26-33; 1946.) Reviews the conditions for quality reproduction and discusses a circuit fulfilling them.

621.396.621 1192  
**V55R Communication Receiver**—(*Wireless World*, vol. 53, p. 36; January, 1947.) A commercial modification of the Royal Air Force type R1155. A complementary unit contains a 5- and 10-meter converter.

621.396.621.029.64 1193  
**First Steps in V.H.F. Exploration. A Practical Super-Regenerative Receiver**—"Cathode Ray." (*Wireless World*, vol. 53, pp. 15-17; January, 1947.) Details of a simple set of high sensitivity and low selectivity, permitting easy tuning.

621.396.662:621.396.62 1194  
**Tuning Devices for Broadcast Radio Receivers**—R. C. G. Williams. (*Jour. I.E.E.*

(London), part III, vol. 93, pp. 405-423; November, 1946. Discussion, pp. 423-427.) An historical discussion of tuning device evolution is followed by a description of listening tests on the degree of mistuning required for observable deterioration of quality. As a result of these a target tolerance of 1 kilocycle for long and medium waves and 2 kilocycles for short waves is suggested. Problems of design imposed by these criteria are discussed together with detailed descriptions of the chief systems of preset tuning and bandspreading hitherto used. In an appendix experimental measurements of the effect of mistuning on audio frequency response and harmonic distortion are described, and a theoretical analysis of the problem is given.

621.396.662:621.396.62 1195  
**The Design of Band-Spread Tuned Circuits for Broadcast Receivers**—D. H. Hughes. (*Jour. I.E.E.* (London), part III, vol. 93, pp. 459-460; November, 1946.) Discussion of 1803 of 1946.

621.396.822 1196  
**Noise Factor: Part 2. Methods of Measurement. Sources of Test Signals**—L. A. Moxon. (*Wireless World*, vol. 53, pp. 11-14; January, 1947.) Either a continuous-wave signal generator or a noise source can be used, and the latter is preferable. The most satisfactory noise source is a temperature-limited diode with a pure tungsten filament. For measurements in the 3000 to 10,000 megacycle range, the filament is mounted axially in a small-bore copper tube about 10 centimeters long. The correction factors for various possible errors can then be calculated. For part 1, see 864 of April.

621.396.822:530.162 1197  
**On the First Passage Time Problem for a One-Dimensional Markoffian Gaussian Random Function**—Siebert. (*See* 1127.)

621.396.822:621.396.619.16 1198  
**Pulse Distortion: the Probability Distribution of Distortion Magnitudes Due to Inter-Channel Interference in Multi-Channel Pulse-Transmission Systems**—D. G. Tucker. (*Jour. I.E.E.* (London), part I, vol. 93, pp. 611-612; December, 1946.) Summary of 230 of February.

621.396.822:621.396.621 1199  
**Note on the Theory of Noise in Receivers with Square Law Detector**—M. Kack and A. F. J. Siebert. (*Phys. Rev.*, vol. 70, p. 449; September 1-15, 1946.) The probability density of the noise emerging from a receiver consisting of intermediate-frequency amplifier, detector, and video amplifier, is calculated using the characteristic function method. Summary of American Physical Society paper.

621.396.822:621.396.645 1200  
**Background Noise in Amplifiers**—U. Zeltstein. (*Toute la Radio*, vol. 13, pp. 2-3, 24; December, 1946.) Discusses thermal and shot effects.

621.396.822.029.62:523.16 1201  
**Variation of Cosmic Radiation with Frequency**—Moxon. (*See* 1073.)

621.396.828+621.396.665 1202  
**Noise and Output Limiters: Part 2—E. Toth. (Electronics, vol. 19, pp. 120-125; December, 1946.)** The operation of six types of radio- and audio-frequency limiters and a frequency modulated discriminator for amplitude modulation limit are described, and the characteristics of thermionic and crystal diodes when used as limiters are considered. Series-type noise-peak limiters are recommended for modulated continuous-wave reception and full wave audio-frequency shunt output types for continuous-wave reception. For part 1 see 867 of April.

## STATIONS AND COMMUNICATION SYSTEMS

- 621.315.668.2 1203  
**The Reconstruction of Ten 305-Foot Tubular Steel Radio-Masts in Reinforced Concrete**—J. P. Harding. (*Jour. Inst. Civil Eng.*, vol. 27, pp. 113-179; December, 1946.) An account of the encasing in reinforced concrete of the ten masts erected at Leafield in 1912 by the Marconi Company.
- 621.384.3:621.391.64 1204  
**Lamp Enables Two-Way Talk Over Invisible Searchlight Beam**—R.H.O. (*Jour. Frank. Inst.*, vol. 242, pp. 339-340; October, 1946.) A description of a 'talking lamp' using caesium vapour, which makes two way conversation possible by means of an invisible infrared beam. Secrecy is claimed, and speech is transmitted practically instantaneously with true telephone quality at conversational speed.
- 621.39+534.756 1205  
**Theory of Communication**—Gabor. (See 1057.) Part 1, analysis of information; part 2, analysis of hearing; and part 3, frequency compression and expansion.
- 621.394/.395[1939/1945] 1206  
**Telephony and Telegraphy**—W. G. Radley. (*Jour. I.E.E.* (London), part I, vol. 93, pp. 569-576; December, 1946.) A review of improvements and developments in British Post Office equipment and methods in the last six years.
- 621.395.44:621.315.052.63 1207  
**Field Tests on Power-Line Carrier-Current Equipment**—R. H. Miller and E. S. Prud'homme. (*Trans. A.I.E.E.* (Elec. Eng., December, 1946), vol. 65, pp. 824-827; December, 1946.) Tests on the Pacific Gas and Electric Company's new telephone system included a comparison of the performances of frequency- and amplitude-modulation equipment. The results are expressed graphically.
- 621.395.5:621.317.34 1208  
**Transmission Rating of Telephone Systems**—W. A. Codd. (*Trans. A.I.E.E.* (Elec. Eng., October 1946), vol. 65, pp. 694-698; October, 1946.)
- 621.396 1209  
**Colonial Telecommunication**—(*Electrician*, vol. 137, pp. 1519-1520; November 29, 1946.) Summary and discussion of two papers read before the Institution of Electrical Engineers: "The Development and Design of Colonial Telecommunication Systems and Plants," by C. Lawton, and "The General Planning and Organization of Colonial Telecommunication Systems," by V. H. Winsor. For another account see *Elec. Times*, vol. 110, pp. 729-750; November 28, 1946.
- 621.396.24:551.510.535 1210  
**Application of the Theories of Indirect Propagation to the Calculation of Links Using Decametre Waves**—R. Aubert. (*Bull. Soc. Franç. Élec.*, vol. 6, pp. 111-128; March, 1946.) A résumé is first given of the principal laws of propagation in an ionized medium. The general characteristics of the ionosphere and the seasonal variations of equivalent heights and critical frequencies are then surveyed; from a knowledge of these and of the variations of solar activity it is possible to predict similar data for several months in advance and to deduce maximum usable frequencies for given distances.
- The paper is mainly concerned with the calculation of the operating conditions for a short-wave link. This calculation involves the following factors: (a) The determination of the path of the wave—the angle of elevation of the ray must be as low as possible to give a minimum number of hops, but sufficient to avoid reflection at the E layer. Curves are given of

minimum angle of elevation and corresponding distance per hop for a series of E-layer critical frequencies. (b) The determination of the maximum usable frequency for the given path—this is derived from seasonal world charts, published in Washington, which can be interpreted in terms of local time. (c) The choice of an optimum working frequency, usually about 20 per cent below the maximum usable frequency to ensure greater security of communication. (d) The calculation of loss by reflection at the F layer—this is a function of the ratio of optimum working frequency to maximum usable frequency; curves are given and approximations explained. (e) The calculation of loss by absorption in the E layer—during the day the absorption is proportional to  $\lambda^2$  and is a function of the altitude of the sun; curves proposed at the Bucharest meeting of the C.C.I.R. are reproduced. At night absorption is proportional to  $\lambda^{0.2}$  for the first 3 or 4 hours and is negligible for the later hours.

A detailed example of the use of the above data is given for the link Paris-New York, with explanations of the approximations necessary in practice.

- 621.396.24:551.510.535 1211  
**Short-Wave Long Distance Links by Means of the Ionosphere**—A. de Govenain. (*Toute la Radio*, vol. 13, pp. 264-269; November, 1946.) Discusses the effects of the E and F layers and explains the use of ionospheric charts for the choice of optimum operational frequencies and hours of traffic. See also 1210.
- 621.396.324.029.3 1212  
**Multi-Channel Two-Tone Radio Telegraphy**—L. C. Roberts. (*Bell Lab. Rec.*, vol. 24, pp. 461-465; December, 1946.) This voice frequency telegraph system can handle a large amount of traffic over a single radio frequency with comparatively low power per channel. It gives independent start-stop teletypewriter circuits which can be extended by land lines to teletypewriters situated at different places.

For two-tone transmission, one channel is used for marking and an adjacent channel at a frequency 170 cycles higher is used for spacing. Selective fading is counteracted by simultaneous transmission of the same two-tone signals on two pairs of frequencies, corresponding members of which differ by about 1000 cycles. A channel shifter is used to enable 6 channels of the frequency-diversity system—24 tones—to be transmitted over a single radio channel.

- 621.396.41:621.396.619.16 1213  
**Pulse-Time-Modulated Multiplex Radio Relay System—Terminal Equipment**—D. D. Grieg and A. M. Levine. (*Elec. Commun.*, vol. 23, pp. 159-178; June, 1946.) The principles and advantages of the system are explained with numerous diagrams. A description is given of the physical and electrical characteristics of the terminal equipment of an operating 24-channel system which takes advantage of transmission technique developed during the war.
- 621.396.44 1214  
**Carrier-Frequency Broadcasting**—(*Electrician*, vol. 137, pp. 1503-1504; November 29, 1946.) An outline description of the new Rugby wired broadcasting system, including monitoring facilities, put into public service on November 22, 1946. Six programs are to be made available to subscribers over two polyvinyl-chloride covered wires by means of modulated carrier waves of frequencies up to 200 kilocycles spaced 20 kilocycles apart. It is estimated that the 6-watt output of the equipment is sufficient to provide for 3000 subscribers. For another account see *Elec. Times*, vol. 110, p. 725; November 28, 1946.
- 621.396.1:621.396.82 1215  
**Interference Considerations Affecting Channel-Frequency Assignments**—M. Reed

and S. H. Moss. (*Jour. I.E.E.* (London), part I, vol. 93, pp. 603-604; December, 1946.) Summary of 241 of February.

- 621.396.619.11 1216  
**Amplitude Modulated Waves**—H. Moss. (*Electronic Eng.*, vol. 18, pp. 375-378; December, 1946.) The theory of amplitude modulation is considered in general and sinusoidal modulation in particular. The effects of modulation index above unity, of suppressing the carrier and of attenuating one sideband are discussed and examples of oscillographic measurements of modulation depth and distortion given. For previous parts in this series see 2966 of 1946 and back references.

- 621.396.619.13 1217  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) Abstracts are given of the following papers read at the conference: "Frequency Modulation of High-Frequency Power Oscillators," by W. R. Rambo; "A Microwave Relay Communication System," by G. G. Gerlach; "Pulse Time Multiplex Broadcasting of the Ultrahigh Frequencies," by D. D. Grieg and A. G. Kandoian; and "Signal Systems for Improving Railroad Safety," by K. W. Jarvis. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.

- 621.396.645:621.396.7 1218  
**Remote Amplifier for Broadcast Service**—P. Wulfsberg. (*Elec. Ind.*, vol. 5, pp. 70-71, 102; December, 1946.) An audio-frequency four-channel and master self-contained unit for operation with either alternating current or batteries. The gain is 92 decibels, and frequency response  $\pm 1$  decibel from 30 to 12,000 cycles; the unit gives 50 milliwatt output with 1 per cent or less distortion.

- 621.396.65.029.62/.64:621.396.619.16 1219  
**A Multichannel Microwave Radio Relay System**—H. S. Black, J. W. Beyer, T. J. Grieser, and F. A. Polkinghorn. (*Trans. A.I.E.E.* (Elec. Eng. December, 1946), vol. 65, pp. 798-806; December, 1946.) The AN/TRC-6 is an 8-channel relay system operating at about 5000 megacycles. Sharp beaming and the complete absence of interference greatly reduce the required transmitter power. "Short pulses of microwave power carry the intelligence of the eight messages utilizing pulse position modulation to modulate the pulses and time division to multiplex the channels."

The eight high-grade telephone circuits can be used for various purposes. Two-way transmissions over distances of 1600 miles and one-way over 3200 miles have been achieved. See also 2315 of 1946 and back references.

- 621.396.712.3 1220  
**Studio Control Unit**—N. J. Peterson. (*Elec. Ind.*, vol. 5, pp. 68-69, 109; December, 1946.) All controls and amplifiers for one or two broadcasting studios, together with announcing booth, contained in a single desk cabinet.

- 621.396.93 1221  
**Frequency, Power, and Modulation for a Long-Range Radio Navigation System**—Adams and Colin. (See 1086.)

- 621.396.932 1222  
**Electronics on World's Largest Liner**—(*Electronics*, vol. 19, pp. 84-85; December, 1946.) Navigational aids aboard the liner Queen Elizabeth include radar, loran, depth-sounding, and radio installations.

- 621.396.712 1223  
**Broadcasting Stations of the World [Book Review]**—Wireless World, Iliffe and Sons, London, 1s. (*Elec. Rev.* (London), vol. 140,

p. 264; February 7, 1947.) Details of over a thousand stations, arranged for easy reference.

**621.526+621.396.016.2.029.64** 1224  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.*, vol. 65, pp. 569–574; December, 1946.) Abstracts are given of the following papers read at the conference: "High-Performance Demodulators for Servomechanisms," by K. E. Schreiner; and "Continuous-Wave Ultrahigh-Frequency Power at the 50-kW Level," by W. G. Dow and H. W. Welch. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46–47, 111; December, 1946.

**621.314.2.018.8** 1225  
**Primary Oscillation Damper, using a Shunted Rectifier, for Transformers feeding X-Ray Tubes**—L. Maurice. (*Bull. Soc. Franç. Elec.*, vol. 6, pp. 98–102; March, 1946.)

**621.314.6+621.319.4+621.383]:669.018** 1226  
**Light Alloys in Metal Rectifiers, Photocells, and Condensers**—Continuing the series in *Light Metals* mentioned in 544 of March and 3768 of January.

(xii) vol. 8, pp. 348–359; July, 1945. "The properties of interleaving papers are critical and physically and chemically are related to the electrode metals."

(xiii) vol. 8, pp. 409–414; August, 1945. Summarizes practical requirements for interleaving papers.

(xiv) vol. 8, pp. 459–462; September, 1945. A detailed survey of the electrical, chemical, physical, and mechanical characteristics of impregnating media for fixed paper capacitors.

(xv) vol. 8, pp. 479–491; October, 1945. Continuing (xiv), with particular attention to chemical properties in relationship to the metal with which impregnating media come in contact.

(xvi) vol. 8, pp. 559–576; November, 1945. A discussion on the relationship between impregnated media and the service characteristics and life of fixed-paper capacitors.

**621.316.53** 1227  
**Some Technical Considerations concerning Contactors**—F. Bertholet. (*Bull. Sci. Ass. Inst. Électrotechn. Montefiore*, vol. 59, pp. 271–277, 319–341, 353–381; June–August, 1946.) In 13 chapters, dealing with the magnetic circuit, shape, and pressure of contacts, phenomena at make and break, arc extinction, effect of current intensity and of the connected circuits, contactors in oil, contact life, and auxiliary contacts.

**621.316.935.078** 1228  
**Saturable Reactors for Automatic Control**—W. D. Cockrell. (*Elec. Ind.*, vol. 5, pp. 48–53; December, 1946.) Theory and application of power control reactors using magnetic saturation effects.

**621.385.4** 1229  
**A Neon Stroboscopic Lamp**—(*Electronic Eng.*, vol. 18, p. 374; December, 1946.) Details of construction and operation of the Ferranti Neosteon Type NSP1, designed for short discharge flashes up to 250 per second.

**621.391.64+621.384.3** 1230  
**Some Developments in Infrared Communications Components**—J. M. Fluke and N. E. Porter. (*Proc. I.R.E. and Waves and Electrons*, vol. 34, pp. 876–883; November, 1946.) For communication at wavelengths 0.8 to 1.2 microns, the most suitable source is a caesium vapour lamp; power supply for it is considered. Plastic filters are more efficient than glass but cannot operate at such a high temperature. The three main types of receiver-phosphors, electron image tubes, and photocells are briefly discussed.

**621.392.032.53:533.5** 1231  
**Resonant Windows for Vacuum Seals in Rectangular Wave Guides**—M. D. Fiske. (*Rev. Sci. Instr.*, vol. 17, pp. 478–483; November, 1946.) They consist of a thin dielectric plate (glass) hermetically sealed to a Fernico metal frame. They are ground for tuning, deoxidized and soldered across the guide. They may be represented by a parallel resonant circuit shunted across a transmission line. Windows having a  $Q$  less than unity and giving 97 per cent power transmission have been constructed. Detailed design information is given for 3-centimeter windows.

**621.394.652** 1232  
**Telegraph Manipulating Key Design**—H. J. H. Wassell. (*Marconi Rev.*, vol. 9, pp. 109–115; July–September, 1946.) Conditions to be aimed at in key design are: (a) small mass of moving arm, (b) use of a 'dead' metal for the arm, (c) optimum arm length, (d) small gap, and (e) contacts at center of percussion. Some detailed observations of keying methods are given, and the design of a new manipulating key is described.

**669.71+669.72]:621.3** 1233  
**Aluminium and Magnesium in the Electrical Industries**—B. J. Brajniff. (*Light Metals*, vol. 9, pp. 393–397 and 609–618; August and November, 1946.) The first article describes the application of aluminium in high-voltage capacitors with compressed gas insulation; it is based on the researches of B. M. Hochberg and co-workers at the Leningrad Physico-Technical Institute. The second article considers modern developments of high-voltage generation for nuclear physics, X-rays, or industrial applications; high electrical efficiency is obtainable by using aluminium in the construction of alternators.

**621.317.755** 1234  
**Principes de l'Oscillographe Cathodique [Book Review]**—R. Aschen and R. Gondry. (*Éditions Radio*, Paris, 88 pp., 100 fr. (*Toute la Radio*, vol. 13, p. 273; November, 1946.)

#### TELEVISION AND PHOTOTELEGRAPHY

**621.396.029.6** 1235  
**The Interstage Auto-Transformer at Television Frequencies**—Feldmann. (*See* 1027.)

**621.397.262** 1236  
**Approximate Method of Calculating Reflections in Television Transmission**—D. A. Bell. (*Jour. I.E.E. (London)*, part I, vol. 93, p. 605; December, 1946.) Summary of 553 of March.

**621.397.3** 1237  
**Choice of Definition in Television**—R. Barthélemy. (*Cah. Toute la Radio*, pp. 2–4; July, 1946.) Although a 900-line scan, with a frequency band of about 15 megacycles, causes a continuous appearance on the screen, both in the horizontal and vertical directions, in the case of some image structures the vertical definition is inferior to the horizontal. Without spoiling the definition, the use of interlacing produces a striation in the greater part of the picture equal to half the number of lines.

**621.397.331.2** 1238  
**New Cathode-Ray Tube for Television**—(*Radio en France*, no. 2, p. 46; 1946.) The tube has an over-all length of 35 centimeters and gives an image 12 by 16 centimeters. A directly heated filament takes under 1 ampere at 1.2 volts and the normal accelerating voltage is 2000 volts, which may be increased without danger to 3000 or 4000 volts.

**621.397.5** 1239  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.*, vol. 65, pp. 569–574; December, 1946.) Abstracts are given of the following papers read at the conference: "Color Television—Latest State of

the Art," by P. C. Goldmark; "Westinghouse Color Television Studio Equipment," by D. L. Balthis; "Television Transmitter for Black-and-White and Color Television," by N. Young; "Stratovision System of Communication," by C. E. Nobles and W. K. Ebel; "The Electrostatic Image Dissector," by H. Salinger; "The Use of Powdered Iron in Television Deflecting Circuits," by A. W. Friend; "Television Equipment for Guided Missiles," by C. J. Marshall and L. Katz; and "Results of Field Tests on Ultrahigh-Frequency (490 Mc/s) Color Television Transmission in the New York Metropolitan Area," by W. B. Lodge. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46–47, 111; December, 1946.

**621.397.5** 1240  
**All-Electronic Color Television**—(*Electronics*, vol. 19, pp. 140, 142; December, 1946.) A method of simultaneous color transmission is described. With a cathode-ray tube as light source, the image is split into three color components and directed to separate channels by a mirror system. Three separately actuated cathode-ray tubes project the received images to form a composite picture. The color transmissions will form monochrome pictures on existing receivers, which may alternatively be adapted for full-color reception.

**621.397.5(44)** 1241  
**The Conditions for the Development of Television in France and the Problem of the Line Standard**—M. Chauvierre. (*Radio en France*, no. 2, pp. 33–35; 1946.) Discusses the difficulties of starting a television service in France. It is concluded that industrial development based on a 500-line standard is essential immediately.

**621.397.5:515.6** 1242  
**Perspective and Television**—P. Philippe. (*Télévis. Franç.*, no. 12, pp. 9–11; April, 1946.) A discussion of perspective effects arising from differences of the points of view of camera and viewer.

**621.397.611** 1243  
**From the Iconoscope to the Isoscope and towards Large Screen Television**—G. Barret. (*Toute la Radio*, vol. 13, pp. 97–98; March–April, 1946.) Principles and construction of the iconoscope and of the more recently developed Radio Corporation of America's orthiconoscope, and the Compagnie des Compteurs isoscope, both of which are better adapted for screen projection.

**621.397.611:621.383** 1244  
**The Isoscope**—R. Barthélemy. (*Télévis. Franç.*, nos. 13 and 14, pp. 12–14 and 3–5, 23; May and June, 1946.) A method of picture analysis by means of slow electrons use modulation of the cathode beam, near the point of impact, by photoelectric charges created by the light projected on the transparent mosaic. A beat-frequency method produces high-frequency modulation of the beam, so that in the tuned output circuit the high-frequency current has 100 per cent modulation. See also 445 of 1946.

**621.397.62** 1245  
**Television Receiver Construction. Part I—Deflector Coils: General Principles**—(*Wireless World*, vol. 53, pp. 2–5; January, 1947.) Constructional details for those acquainted with sound-set construction and television circuit principles.

**621.397.62** 1246  
**R.C.A. Reveals First Electronic Color TV [Television]**—(*Elec. Ind.*, vol. 5, pp. 58–59, 103; December, 1946.) Three simultaneous color modulations are used to excite three small cathode-ray tubes and the light from these is

focused through appropriate filters on a translucent 15-inch by 20-inch screen.

- 621.397.62 1247  
**Problems on Theatre Television Projection Equipment. Parts 1 and 2**—A. H. Rosenthal. (*Jour. Telev. Soc.*, vol. 4, pp. 258–263 and 274–278; June and September, 1946.) Reprint of article abstracted in 461 of 1946.

- 621.397.62(44) 1248  
**The S.A.D.I.R. Receiver, Type R. 290**—(*Radio en France*, no. 2, pp. 43–45; 1946.) A short description of the 1943 television receiver with complete circuit diagram, and details of components used.

- 621.397.81 1249  
**A Modification to Ray Theory Allowing for Ground Contours**—Williams. (See 1184.)

## TRANSMISSION

- 621.394.652 1250  
**Telegraph Manipulating Key Design**—Wassell. (See 1232.)

- 621.395.4 1251  
**Wideband Phase Shift Networks**—Dome. (See 1021.)

- 621.396.61.029.58 1252  
**Medium Power Short-Wave Telephone-Telegraph Transmitter Type T.F.S.31**—C. R. Staines. (*Marconi Rev.*, vol. 9, pp. 89–101; July–September, 1946.) Designed for use in communication circuits where rapid frequency changing is of first importance, e.g., aerodrome ground stations, ship-to-shore services, and point-to-point circuits. The frequency range covered is 3 to 22.2 megacycles with output power 4.0 to 5.0 kilowatts on telegraphy, and carrier power 3.0 to 3.5 kilowatts on telephony.

- 621.396.619.1 1253  
**Cascade Phase Shift Modulator**—M. Marks. (*Electronics*, vol. 19, pp. 104–109; December, 1946.) The general requirements of phase-shift modulators are discussed, and it is shown that addition of the phase shifts of a number of stages in cascade allows a lower order of frequency multiplication. The design of a six-stage modulator with low-noise and distortion characteristics is described, and the procedure given for tuning the modulator and aligning the transmitter. Distortion less than 1 per cent is claimed.

- 621.396.619.14 1254  
**Phase Modulation**—V. O. Stokes. (*Marconi Rev.*, vol. 9, pp. 116–122; July–September, 1946.) Phase modulation may conveniently be applied to telegraph transmitters as an anti-fading system. A modulation method using a reactance tube in one of the driven stages is described. Curves showing the amplitude of the carrier and sidebands for various modulation indexes are given and a method of measuring the modulation index (for sinusoidal modulation) is described in detail with a circuit diagram of the monitor unit. The application of the system to a typical transmitter is explained. Unintentional phase modulation may be produced by alternating-current heating of tubes, etc.; a special monitor unit has been developed for measuring this unwanted modulation. This unit can also be used for the separate measurement of 'amplitude noise.'

## VACUUM TUBES AND THERMIONICS

- 621.317.7.085 1255  
**"Magic Eye" Indicators [with Positive Feedback]**—G. O. Thacker and R. Y. Walker. (*Wireless World*, vol. 53, pp. 30–31; January, 1947.) Criticizes the construction of wartime EM2 indicators and suggests that a short grid base and flat deflector vanes are necessary for high sensitivity.

- 621.383:621.391.64 1256  
**German Photo-Cells for the Infrared**—B.I.O.S. (*Jour. Telev. Soc.*, vol. 4, pp. 280–281; September, 1946.) An extract from a B.I.O.S. report by T. F. Johns, published by H. M. Stationery Office. The characteristics of lead sulphide and lead telluride photocells and German methods of producing them are briefly described.

- 621.385+621.396.694 1257  
**Magnetic Control of Anode Current**—C. R. Knight. (*Elec. Ind.*, vol. 5, pp. 72–73, 108; December, 1946.) Details of a new diode, type 2B23, in which the anode current is controlled by an external magnetic field. An illustration is given of its use as a voltage-control amplifier and a current-limit amplifier in an electronic motor control circuit. A more stable control circuit can be made using two such diodes differentially.

- 621.385 1258  
**Second National Electronics Conference, Chicago, Autumn, 1946**—(*Elec. Eng.*, vol. 65, pp. 569–574; December, 1946.) Abstracts are given of the following papers read at the conference: "Trends in Cathode-Ray Oscilloscope Design," by W. L. Gaines; "Production Test Facilities for High-Power Tubes," by W. L. Lyndon and B. Sheren; "The Cyclophon—a Multipurpose Beam Switching Tube," by J. J. Glauber, D. D. Grieg and S. Moskowitz; "An All Metal Tunable Squirrel Cage Magnetron," by F. H. Crawford, and "Bunching Conditions for Electron Beams with Space Charge," by L. Brillouin. For titles of other papers read, see other sections. For other abstracts see *Elec. Ind.*, vol. 5, pp. 46–47, 111; December, 1946.

- 621.385:389.6 1259  
**Service Valve Equivalents**—Incorporated Radio Society of Great Britain. List of commercial equivalents to service types. Booklet issued as supplement to *R.S.G.B. Bull.*, vol. 22, January, 1947.

- 621.385:533.5 1260  
**Evacuation of Mean and Low-Power Transmitting Tubes**—P. Plion. (*Le Vide* (Paris), vol. 1, pp. 71–78; May, 1946.) Discusses methods of degassing electrodes and envelopes and describes mass production evacuation methods used in France.

- 621.385.016.2 1261  
**Bettering Output from Power Tubes**—L. Dolinko. (*Elec. Ind.*, vol. 5, pp. 60–62, 104; December, 1946.) Improved performance by use of getter traps and graphite anodes.

- 621.385.029.63/.64 1262  
**The Beam Traveling-Wave Tube**—J. R. Pierce. (*Bell Lab. Rec.*, vol. 24, pp. 439–442; December, 1946.) For other accounts see 585 and 586 of March.

- 621.385.029.63/.64 1263  
**Broad-Band Tube**—(*Elec. Ind.*, vol. 5, pp. 57, 103; December, 1946.) A beam traveling-wave tube giving very high gain and a bandwidth about 80 times that hitherto practicable with other microwave tubes. See also 585 and 586 of March.

- 621.385.032.2:669.296 1264  
**Zirconium in Electron Tubes**—Foote Mineral Co. (See 1120.)

- 621.385.1.032.216 1265  
**Electronic Emission of Tungsten-Caesium and Tungsten-Thorium Cathodes**—C. Biguet. (*Le Vide* (Paris), vol. I, pp. 13–20 and 54–60; January and March, 1946.) The emissive properties of a thin layer of caesium, at most mono-atomic, obtained by progressive condensation on a tungsten filament, are deduced from De Boer's theory of adsorption. By analogy it is shown that for thorium on tungsten there is an optimum thickness corresponding to at least a

mono-atomic layer of thorium. Intensities corresponding to different carburization treatments are tabulated. Micrographs reveal the nature of the surface of thoriated tungsten, the formation processes and emission characteristics of which are discussed in detail.

- 621.385.2 1266  
**Emission-Limited Diode**—W. E. Benham. (*Proc. I.R.E. and Waves and Electrons*, vol. 34, p. 863; November, 1946.) Comment on 3885 of 1945. In the case where the ratio of the radii of the cylinders exceeds about 10, the transit time is best obtained from Scheibe's formula.

- 621.385.38:621.3.018.41 1267  
**Frequency Performance of Thyratrons**—H. H. Wittenberg. (*Trans. A.I.E.E. (Elec. Eng.*, December, 1946), vol. 65, pp. 843–848; December, 1946.)

- 621.385.832 1268  
**The Ion Trap in C.R. Tubes**—J. Sharpe. (*Electronic Eng.*, vol. 18, pp. 385–386; December, 1946.) An electron beam is always accompanied by negative ions of various types which have undesirable effects on the cathode-ray-tube screen. Three methods for separating the ions from the beam are described, which depend on the difference in the behavior of the ions from that of the electrons.

- 621.396.615.14:537.291 1269  
**Induction Currents Produced by Moving Electrons**—A. Colino. (*Marconi Rev.*, vol. 9, p. 123; July–September, 1946.) A "simple and intuitive" treatment of Ramo's formula (131 of 1940) for induction currents due to movement of electrons in tube interelectrode spaces.

- 621.396.615.141.2.032.21 1270  
**Magnetron Cathodes**—M. A. Pomerantz. (*Proc. I.R.E. and Waves and Electrons*, vol. 34, pp. 903–910; November, 1946.) Thermionic emission densities and their effect on magnetron performance are considered; the technique of emission measurement, and the interpretation of results in terms of the Richardson, Langmuir-Childs, and Schottky equations is discussed. Sparking, pulse temperature rise, and back bombardment are briefly considered. The essential requirements of magnetron cathodes which are thus made clear appear to be better met by a new 'sintor' (sintered thorium oxide) cathode than by existing types.

- 621.396.615.142.2 1271  
**The Klystron**—A. V. J. Martin. (*Toute la Radio*, vol. 13, pp. 270–271; November, 1946.)

- 621.396.615.142 1272  
**Elementary Treatment of Longitudinal Debunching in a Velocity Modulation System**—E. Feenberg. (*Jour. Appl. Phys.*, vol. 17, pp. 852–855; October, 1946.) Plane parallel electrode systems of unlimited extent, coaxial circular systems of infinite axial length, and concentric spherical systems give rectilinear motion of the electrons, with or without velocity modulation. In these cases a complete evaluation is given of the effect of space charge on the bunching process in the range of drift space where overtaking (crossing of orbits) does not occur.

- 621.396.645 1273  
**Load Conditions in Class A Triode Amplifiers**—Foster. (See 1044.)

- 621.396.645:621.396.822 1274  
**Background Noise in Amplifiers**—Zelbststein. (See 1200.)

- 621.385.3+621.396.694 1275  
**The Gas-Filled Triode [Book Review]**—G. Windred. Hulton Press, London, 1946, 72 pp., 2s. 6d. (*Nature* (London), vol. 158, p. 689; November 16, 1946.) "Practical applications in industrial control and trigger circuits" and "a complete list of models available at the present

time, with operating conditions and possible circuits."

### MISCELLANEOUS

061.5:621.38/.39 1276  
**Electrical Communication: 1940-1945: Part 2**—(*Elec. Commun.*, vol. 23, pp. 214-240; June, 1946.) Developments by Federal Telecommunication Laboratories and Federal Telephone and Radio Corporation in various fields including direction finding and aerial navigation communication equipment for radio, telephony, and telegraphy, tubes, rectifiers, quartz crystals, and transformers. For part 1 see 3135 of 1946; in this, the Universal Decimal Classification number should read as above.

061.6 1277  
**Work on the E.R.A. [British Electrical and Allied Industries Research Association]**—(*Electrician*, vol. 138, pp. 441-442; February 7, 1947. Editorial comment, p. 427.) A summary of the report for the twelve months ended September 30, 1946. Remarkable progress has been made in the fundamental theory of dielectric breakdown, and of the general shape of resonance lines. Inadequate financial support for the activities of the Association is feared as a result of the new Electricity Bill.

081 Rutherford 1278  
**The Rutherford Papers in the Library of the Cavendish Laboratory**—E. B. Bond and W. L. Bragg (*Nature* (London), vol. 158, p. 714; November 16, 1946.) These have been classified and cover Rutherford's scientific career from his first research papers to his last contribution in *Nature*. "A rich mine of information, not only about Rutherford himself, but also about many famous men of his time."

389.6(73) 1279  
**The American Standards Association—Our Colleague in Standardization**—W. H. Crew. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, pp. 874-875; November, 1946.)

41.316.3 1280  
**French Terms of English Origin**—E. Aisberg. (*Wireless World*, vol. 53, p. 21; January, 1947.) Some examples of French technical jargon and errors, e.g., "Courants de Eddy."

5+6:011 1281  
**Bulletin Analytique**—A monthly review of scientific and technical abstracts published by the Centre de Documentation du Centre National de la Recherche Scientifique, 18 rue Pierre-Curie, Paris (5<sup>e</sup>). Each month's issue is in two sections, the first dealing with mathematics, physics, chemistry, and their applications and the second with biology and medicine. Microfilm copies of all articles mentioned are available for loan. The annual subscription is 1800 francs per section.

5 Fleming 1282  
**Ambrose Fleming—His Life and Early Researches**—J. T. Macgregor-Morris. (*Jour. Telev. Soc.*, vol. 4, pp. 266-273; September, 1946.) The first Fleming Memorial Lecture.

5 Mandelstam 1283  
**In Memory of Academician L. I. Mandelstam, 1879-1944**—(*Bull. Acad. Sci.* (U.R.S.S.), sér phys., vol. 9, nos. 1/2, pp. 1-132; 1945. In Russian.) Double issue devoted to appreciations and surveys of the work of L. I. Mandelstam in various fields of physics and radio.

522.1(42) Greenwich 1284  
**Greenwich Observatory**—W. M. Wittchell. (*Weather* (London), vol. 2, pp. 23-29; January, 1947.) A sketch of the functions of the Royal Observatory and of its history from its foundation in 1676 to the present time.

537+538]:371.3 1285  
**Teaching Electricity and Magnetism**—V. P. Hessler. (*Trans. A.I.E.E. (Elec. Eng.*, December, 1946), vol. 65, pp. 828-833; December, 1946.) The importance of a full understanding of fundamental concepts is stressed. The order in which they should be taught is suggested, with careful distinction between definitions, experimental laws, and laws theoretically derived.

551.556.3 1286  
**Wind Energy: Its Value and the Search for [Installation] Sites**—P. Ailleret. (*Rev. Gén. Elec.*, vol. 55, pp. 103-108; March, 1946.)

621.3 1287  
**The Glasgow Technical Exhibition**—(*Electronic Eng.*, vol. 19, pp. 28-30; January, 1947.) A short review of some of the instruments, tools, and miscellaneous items shown at the Kelvin Hall, November, 1946.

621.3(44) 1288  
**The Role of the Société Française des Électriciens in French Electrical Activity**—R. Langlois-Berthelot. (*Bull. Soc. Franç. Elec.*, vol. 6, pp. 157-172; April, 1946.)

621.3.016.25 1289  
**Sign of Reactive Power**—F. B. Silsbee. (*Elec. Eng.*, vol. 65, pp. 598-599; December, 1946.) The definition of a quantity "quadergy," the time integral of the reactive power measured in kilovars, helps to reconcile the seemingly incompatible requirements that a vector diagram similar to admittance diagrams should be obtained and that an inductive circuit should be considered to absorb power. Comment on 971 of April.

621.315.3:389.6 1290  
**Views on the Wiring Codes**—"Supervisor." (*Electrician*, vol. 138, pp. 439-440; February 7, 1947.) Comment on two Codes of Practice concerning electrical installations now issued in draft form. It is felt that the Codes are too stringent for present conditions, and do not make the best use of available resources. Mains-borne radio interference should be suppressed at the source rather than by any choice of wiring system.

621.38/.39 1291  
**National Electronics Conference Papers**—(*Elec. Eng.*, vol. 65, pp. 569-574; December, 1946.) *Elec. Ind.*, vol. 5, pp. 46-47, 111; December, 1946.) Abstracts of papers to be published in the Proceedings of the second National Electronics Conference. For titles of papers read at the Conference, see other sections.

621.396 1292  
**Radio Amateur Call Book**—(*Wireless World*, vol. 53, p. 26; January, 1947.) Lists call signs, names and addresses in Great Britain, United States, and some seventy other countries. Quarterly publication has been resumed.

621.396:384 1293  
**The Radio Industry from an Economic Viewpoint**—D. A. Bell. (*Wireless World*, vol. 53, pp. 27-29; January, 1947.) A survey based on prewar figures. Postwar exports show a large increase. The economic need of the broadcast receiver industry at the present time is for more efficient organization of production.

621.396"20" 1294  
**Half a Century of Radio Communications (1896-1946)**—S. P. Chakravarti. (*Curr. Sci.*, vol. 15, pp. 299-305; November, 1946.)

678.1.02 1295  
**National Rubber Research at Stanford University**—(*Science*, vol. 104, p. 622; December

27, 1946.) An eight-month investigation to be carried out at Salinas, California, seeking to cultivate plants with high rubber content and develop economical processes of extracting the rubber.

744.34:621.3 1296  
**Cylindrical Draughting Machines for Electrical Diagrams, etc.**—A. M. Haworth. (*Electronic Eng.*, vol. 18, p. 387; December, 1946.) The paper is mounted on a cylinder which can be rotated about its horizontal axis. A straight edge is provided parallel to this axis. For drawing lines at right angles the cylinder is rotated and the pencil kept fixed.

6(02) 1297  
**Progress in Science [Book Review]**—W. L. Sumner. Blackwell, Oxford, 176 pp., 8s.6d. (*Nature*, (London), vol. 158, pp. 646-647; November 9, 1946.) Survey of technical developments during the last few years, including electron applications, the electron microscope, radar, television, atomic energy, and plastics. Future applications of present-day researches are discussed. The author "has never forgotten that he has been writing for those only slightly informed of matters scientific."

621.3:69 1298  
**Electricity in the Building Industry [Book Review]**—F. C. Orchard. Chapman and Hall, London, 232 pp., 15s. (*Electrician*, vol. 137, p. 1597; December 6, 1946.) Deals with questions of installation, workshop wiring, and lighting, power costs, maintenance, and research.

621.3.004.5/.6(023) 1299  
**Electrician's Maintenance Manual [Book Review]**—W. E. Steward. G. Newnes, London, 144 pp., 6s. (*Electronic Eng.*, vol. 18, p. 388; December, 1946.)

621.3.029.6(02) 1300  
**Hyper and Ultrahigh Frequency Engineering [Book Review]**—R. I. Sarbacher and W. A. Edson. John Wiley and Sons, New York, N. Y., 644 pp., \$6.00. (*Telegr. Teleph. Age*, vol. 64, p. 27; December, 1946.) "By avoiding complex mathematical technology, the authors have succeeded in giving in an easily understood manner a complete treatment on guided waves, Maxwell's equations, ultra-high-frequency generation, and all related equipment."

621.396 1301  
**Reference Data for Radio Engineers [Book Review]**—W. L. McPherson. Standard Telephones and Cables, London, second edition, 175 pp., 5s. (*Wireless World*, vol. 53, January 1947.)

621.396 1302  
**Reference Data for Radio Engineers [Book Review]**—Federal Telephone and Radio Corp., Publication Department, New York, N. Y., second edition, 336 pp., \$2.00. (*Telegr. Teleph. Age*, vol. 64, p. 31; November, 1946.) "Revised and enlarged to cover important radio technical data developed during the war."

621.396(031) 1303  
**The Radio Amateur's Handbook, 1946 Edition [Book Review]**—Headquarters Staff of the American Radio Relay League. American Radio Relay League, West Hartford, Conn., 1946, 468+208 pp., \$1.00 in the United States; \$1.50 elsewhere. (PROC. I.R.E. AND WAVES AND ELECTRONS, vol. 34, p. 865; November, 1946.) Comparison with the preceding edition shows an expansion of the treatment of wave guides and cavity resonators and of the ultra-high-frequency section, but about half the material on measurements is deleted.